9.4 Homogeneous Differential Equations

Ex 1 Separable or not?

a. $x^2y^3 = \frac{1}{\cos x}\frac{dy}{dx}$ b. $\frac{dy}{dx} = \frac{x+2y}{x}$

Differential Equations that are not separable might be homogeneous.

Separable differential equations take the form $y' = \frac{g(x)}{h(y)}$

Homogeneous differential equations take the form $y' = f\left(\frac{y}{x}\right)$

$\frac{dy}{dx} = \frac{x+2y}{x}$ can be written as $\frac{dy}{dx} = 1 + 2\frac{y}{x}$

We use the substitution $v = \frac{y}{x}$ so y = vx

so $\frac{d(vx)}{dx} = 1 + 2v$ and $\frac{dvx}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = 1 + 2v$ (which is now separable)

$x\frac{dv}{dx} = 1 + v$

$\frac{1}{1+v}dv = \frac{1}{x}dx$

$\ln|1+v| = \ln|x| + C$

$\ln|1 + \frac{y}{x}| = \ln|x| + C$

Let $A = e^{\pm c}$

$y = Ax^2 - x$

Ex 2 Solve $\frac{dy}{dx} = \frac{2x^2 + 3y^2}{4x^2}$ actually interesting!

Ex 3 Solve $x\frac{dy}{dx} - y = xy^2, \quad y(1) = 1$

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