The Relationship Between the Size of a Hole in a Bucket, and the Time for Half the Water to Leave it

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Contents:

Background:

The tedious and meticulous task of continuously refilling your water bottle throughout the day has plagued my and many others' daily lives, wishing the water would just flow faster. This idea is called volumetric flow rate and in physics, it regards the volume of fluid that flows per unit time. The genius and monumental figure in the physics world, Daniel Bernoulli piqued interest in the ideas surrounding fluid mechanics, leading to his renowned discovery of its principles which we still study today. The flow rate equation Q = V/T in which Q is flow rate, V is volume, and "t" is time relates to this idea and is essential in my research to find the relationship between area, flow rate, and time. The equation, Q = Av is also applicable and expresses the connection between flow rate and area most clearly, while also relating the velocity. This is why if you are draining a bucket of water, a larger hole should make it drain faster. If the flow rate increases, it must lower the time in Q = V/T due to their ratio between V/T. The relationship between Q and T indicates that the higher the flow rate the lower the time.

$$Q = \frac{V}{T}$$

$$Q = Av$$

$$Av = \frac{V}{T}$$

$$T = \frac{V}{Av} \quad T = \frac{V}{Q}$$

By setting the two flow rate equations equal to each other, we can represent time with volume, area, and velocity. As presented, time is equal to volume divided by the area multiplied by the velocity. If we increase the volume, the time would increase in relation. This makes sense because if there is more liquid to flow out, it would take longer for that to occur. This can be applied to the equation's denominator as increasing the area or velocity would decrease the time, mathematically indicating that the larger the area is for a liquid to flow out, the less time it would take to do so.

Introduction:

The purpose of this investigation is to determine the relationship between the hole diameter, and the time it takes half the water to flow from a bucket. Using a bucket with different-sized holes drilled into it, we can determine this relationship. The independent variable is the diameter of the holes which is represented by different drill bit sizes ranging from the smallest hole being drilled by a 7/64 inch drill bit and increasing by 1/64 inch each interval until the largest hole being drilled by a ¼ inch bit. The dependent variable is the time it takes for the water to half.

I believe that the time it takes the water to flow out of the bucket to half will gradually decrease as the drilled hole in the bucket increases. This will happen because as the area of the hole increases, it creates more of an opening for a larger amount of water to flow out, therefore increasing the flow rate and decreasing the amount of time for the water to half.

Method:

Before proceeding, let's reassure our readers that no buckets were harmed in the making of this experiment. All buckets involved have willingly volunteered for their liquid-dispensing duties and will be returned to their storage spots in pristine condition post-investigation. The use of the drill was supervised by a certified adult expert (my dad) and the process was accompanied by the use of safety goggles, gloves, and appropriate clothing. Apart from the risk of using a drill, the rest of the experiment poses no ethical or safety concerns.

Materials:

- Large bucket with inside measurements
- Adjustable drill with a set of different-sized drill bits (7/64 inch increasing by 1/64 inch each interval until ¼ inch bit)
- Duct tape
- Sink
- Dish Rack
- Timer

I got a bucket that was labeled with liters and I marked the six-liter and three-liter lines to make timing easier. Using a drill, I drilled ten different-sized holes using different drill bits. These holes were drilled one inch apart and all on the same level to ensure accurate results. I closed each hole with duct tape and poured in water to ensure no leakage was occurring. By keeping the temperature constant, using water from the same source, having an elevated rack to catch any possible leakage, and using the same bucket throughout the entire experiment, I can eliminate the most influencing variable and ensure my controlled variables remain constant and gather the most accurate data I can. Aiming the smallest hole towards the sink, I tore the piece of tape holding the water from spilling and simultaneously started my stopwatch and waited until the water reached the three-liter marking and pressed stop on the watch. I marked down the results and repeated them two more times for three trials per hole. Using three trials, the precision of my results dramatically increased while remaining efficient when gathering the rest of the data. Repeating this for the nine other holes resulted in thirty trials and ten accurate data points that I would use further for the rest of my research.

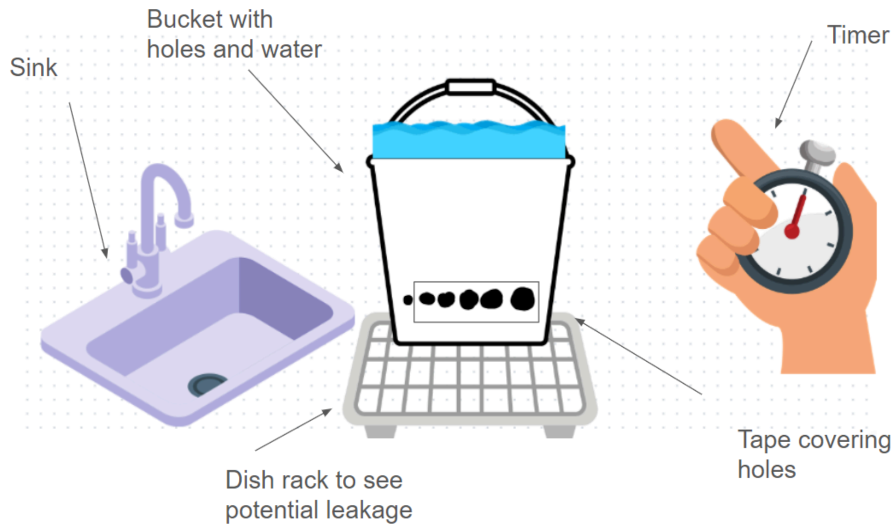


Figure 1: Diagram of Experiment

Results: .:. Data File

Drill Bit Size (in.)					
+ /- .002	Time			Sec	Sec
X / in	Trial 1	Trial 2	Trial 3	Average	Uncertainty
7/64	653.44	650.98	653.12	652.51	1.07
1/8	495.43	494.98	496.35	495.59	0.7
9/64	385.32	383.78	382.99	384.03	1.17
5/32	304.52	303.7	304.93	304.38	0.62
11/64	254.92	254.11	253.17	254.07	0.88
3/16	211.97	211.82	212.89	212.23	0.54
13/64	185.07	184.66	184.91	184.88	0.2
7/32	160.51	161.87	159.48	160.62	1.2
15/64	139.37	138.66	140.21	139.41	0.78
1/4	122.47	121.72	124.01	122.73	1.15

Data Table 1: Raw Data of 3 Trials for 10 Different Diameters

My data table demonstrates all of my results in reference to my experiment. It shows each of my trials and the time it took for the water to be half with each size hole. The results between the three trials were very similar at each area increment; as such, the uncertainties were quite low.

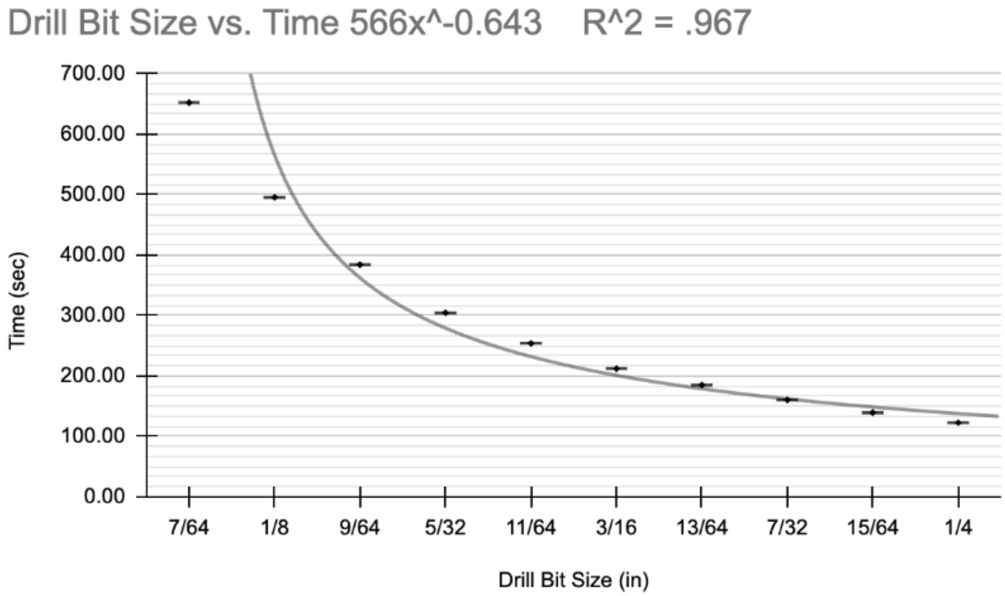


Figure 2: Raw Data Graph Modeling Relationship Between Time for Water to Half and Hole Diameter

The graph of my raw data presents a power function represented by the trendline presented. My graph utilizes the mean from the three trials per data point that corresponds to the respective drill bit sizes. The R^2 value being exceptionally high for the results indicates a strong trend toward the regression polynomial. The uncertainties were also too small to show the error bars.

$$T = \frac{V}{Q}$$

$$Q = Av$$

$$T = \frac{V}{Av}$$

$$T = \frac{V}{(\pi r^2 v)}$$

By setting time to the equation seen above, we can see that the exponent of r^2 is surrounded by constants. The volume and velocity stay consistent throughout as the volume of water stays between six and three litres and the velocity is dependent on the height of water flow which is consistent in my experiment. Letting “k” be set equal to the constants, we can represent time in the equation seen below.

$$t = kr^{-2}$$

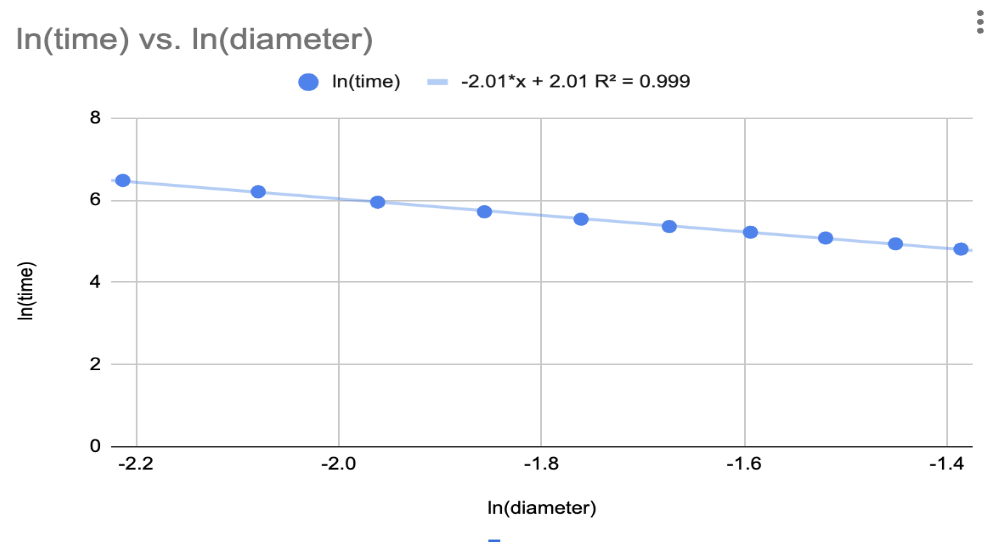


Figure 3: Linearized Graph Comparing Natural Log of Data

Because my data can be represented as a power function, by taking the natural log of both the x and y functions, we can linearize it to create an easier analysis of data. Power functions are represented as $y = Ax^n$ and we can deduce the value of the coefficient A and the power n using this linearized graph. The value of n would be the slope of my line which is -2.01 which can be rounded to just -2. A is equal to e to the power of my y-intercept which is 2.01 which can also be rounded to 2.

$$e^2 = 7.3890561$$

This gives us the equation of $y = 7.3890561x^{-2}$ with y representing time and x representing the drill bit size. The equation found through the log-log graphs perfectly matches my predicted equation as the exponents are the same being -2 and we learn that the constant "k" that represents V, v, and π is equal to 7.3890561. I learned that this exponent of -2 indicated an inverse square relationship between the radius and time

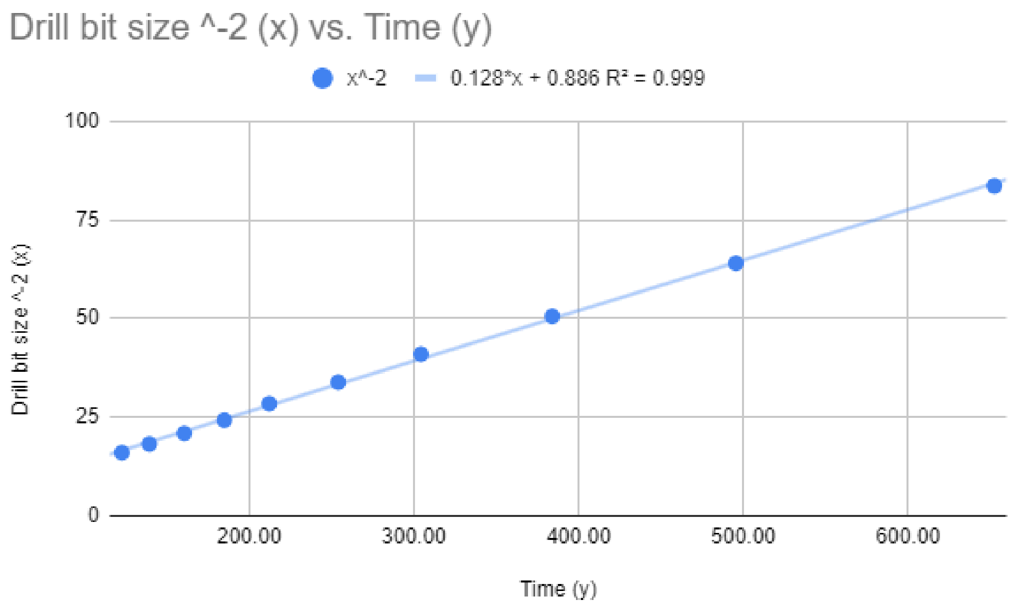


Figure 4: Graph to Model Predicted Equation to find Linearized Graph

Finally, once I deduced that I needed to raise my x values to the power of negative two in order to linearize my data, I was given a linear trendline that proved the inverse square root relationship of time and drill bit size.

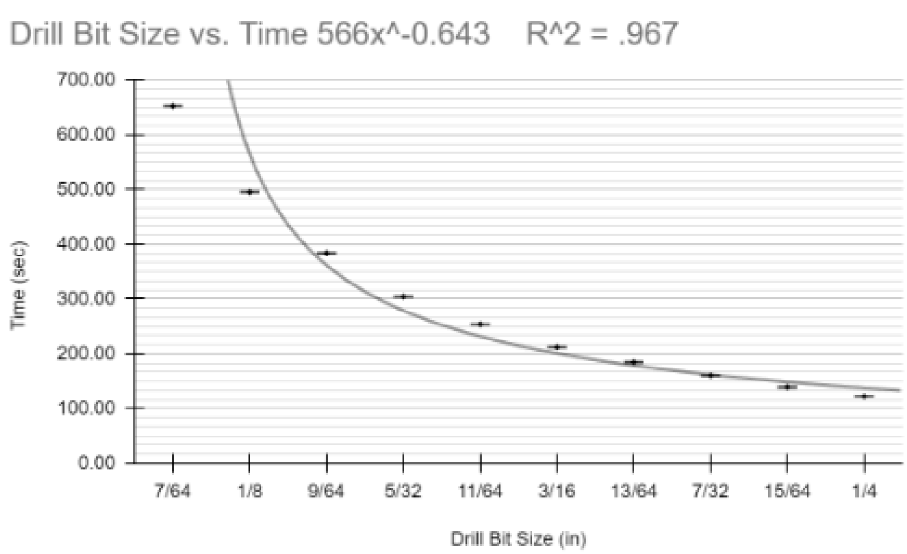
Conclusion:

The purpose of this experiment was to determine the relationship, if any, between the area of a hole that water is flowing through and the time it takes for that water to half in volume. The data gathered clearly supports my hypothesis, as increasing the area the water flows through resulted in a decrease in time. I initially believed that if the area of the holes increased, then the water would flow out faster but I did not account for a power function relationship. Our investigation revealed a compelling inverse square relationship between hole size and the time it took for water to reach half its initial level. The mathematical model $T=Kr^{-2}$ captured this relationship, emphasizing the significant impact of hole size on the efficiency of water drainage. Utilizing the equation $y = 7.3890561x^{-2}$ that we found previously, we can estimate the time using the drill bit size and compare it to the actual data we found.

Drill Bit Size (x)	Time (y)
7/64	652.51
1/8	495.59
9/64	384.03
5/32	304.38
11/64	254.07
3/16	212.23
13/64	184.88
7/32	160.62
15/64	139.41
1/4	122.73

Drill Bit Size (x)	Time (sec) y = 7.3890561x^-2
7/64	617.66
1/8	472.90
9/64	373.65
5/32	302.66
11/64	250.13
3/16	210.18
13/64	179.09
7/32	154.42
15/64	134.51
1/4	118.22

Data Table 2: Raw Data vs. Estimated Data



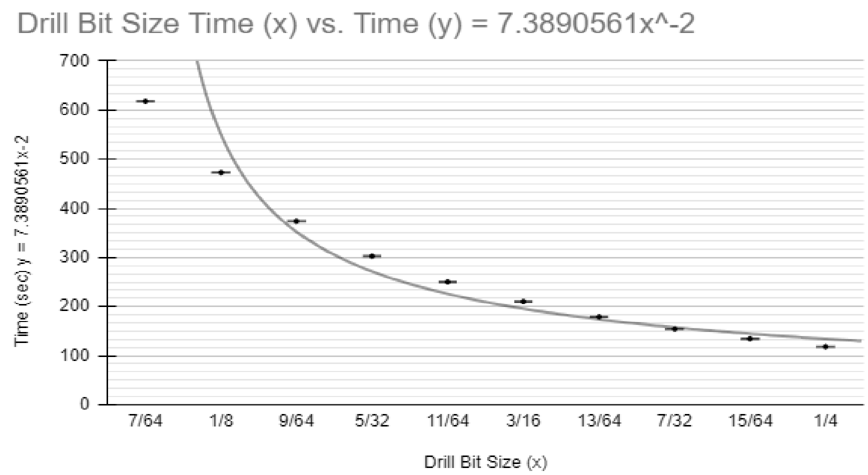


Figure 5: Raw Data Graph vs. Estimated Data Graph

As seen in the data tables and graphs above, the equation $y = 7.3890561x^{-2}$ accurately matches our original graph modeling the relationship between the area and the time it takes for water to flow to half. The numbers are quite close and the shapes of the graphs are especially related. Changing the radius of the hole and squaring that radius accordingly, gives the factor that is then divided by the constant "k". As the size of the hole increases, so does the cross-sectional area for water flow.

Limitations:

One possible limitation that may have skewed my data was the human error that could be applied when drilling out holes in my bucket. I first drew a horizontal line to level all of the holes which may not have been exactly perfect and the holes drilled in relation to this line may not have been perfectly aligned. When drilling the holes, I may not have drilled it in and out with precise straightness which may alter the size of the holes. Having exactly 6 liters of water in the bucket after each trial was also difficult as it was based on the marking inside the bucket and it would be impossible to have the exact amount of water for each trial. Timing the trials could be another potential problem as like the hole sizes, it employs human error. Starting and stopping the timer as soon as the water flows and reaches 3 liters would be nearly impossible to do perfectly by a human.

Improvements:

To combat these potential limitations and create a more accurate and efficient experiment for next time, I could have used a jig or guide for the drill. A jig is a specialized tool or device used to control, guide, or hold a workpiece during something like drilling. Jigs are designed to ensure accuracy, and precision in tasks such as cutting, drilling, or shaping materials which would mitigate the human error by handrilling holes. I could have also used a scale to accurately measure out 6 liters of water before each trial to ensure a consistent experiment. Having some sort of automatic timing mechanism to start and stop precisely with sensors or triggers activated by the water reaching a specific level would dramatically increase the accuracy of my results, but would be difficult to maintain and create. A simpler way I could have improved the timing would be using video recording to precisely time and measure when the water reached the 3-liter mark.

Further Research Suggestion:

If I were to perform this experiment again, it would be interesting to see how different the results would be with more precise instruments and minimize the sources of error. While recreating this experiment, I am also curious to see how different shapes affect the time it takes for water to flow, rather than the area. Although I would keep the area constant, I am intrigued by the idea that different shapes like squares, diamonds, circles, etc. would have different flow rates and therefore would decrease the time. The practicality of this experiment would be beyond valuable in the field of hoses and water fountains, assisting in all our daily lives.

Related Links:

https://en.wikipedia.org/wiki/Volumetric_flow_rate: Wikepedia page, defines flow rate and give examples
https://www.valin.com/resources/blog/introduction-pressure-part-ii-velocity-flow-rate-and-reaction-force#:~:text=In%20order%20to%20determine%20the,v%20is%20its%20average%20velocity: Relates area with flow rate
https://skybrary.aero/articles/bernoullis-principle : Explains Bernouillis equation
https://www.cuemath.com/data/bernoulli-trials/ : Bernoullis equation and experiment with results
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