— steps expected to show

Calculus — Reasons/Explanations to help students understand
Name:
Date: (not required steps)
Period:
— could also be steps that may be useful but not required

2.1-2.4 Review

Show your steps and simplify answers. No calculators will be allowed on the quiz.

1-12: Find the derivative of each function.

Product Rule: If $h(x)=(f(x))(g(x))$ then $h'(x)=(f'(x))(g(x))+(g'(x))(f(x))$

times ⇒ need Product Rule

1. $f(x) = 10x^{2}\cdot\cos(x)$

Product Rule setup $f'(x)=(10x^2)'(\cos x) + (\cos x)'(10x^2)$

Derive $f'(x)=(20x)(\cos x) + (-\sin x)(10x^2)$

Reformatted $f'(x)=20x\cos x - 10x^2\sin x$

ok to leave in this form

An equivalent answer is below

Factored out $10x$ $f'(x)=10x(2\cos x - x\sin x)$

2. $g(x) = 10x^3\sin(x)$

$g'(x)=(10x^3)'(\sin x)+(\sin x)'(10x^3)$

$g'(x)=(30x^2)(\sin x)+(\cos x)(10x^3)$

$g'(x)=30x^2\sin x+10x^3\cos x$

OR

Factored out $10x^2$ $g'(x)=10x^2(3\sin x + x\cos x)$

Division ⇒ use Quotient Rule

3. $f(x) = \tan x = \frac{\sin x \leftarrow Hi}{\cos x \leftarrow Lo}$

4. $f(x) = \cot x = \frac{\cos x \leftarrow Hi}{\sin x \leftarrow Lo}$

Quotient Rule: $\left(\frac{Hi}{Lo}\right)' = \frac{LodHi - HidLo}{LoLo}$

$f'(x)=\frac{(\cos x)(\sin x)'-(\sin x)(\cos x)'}{(\cos x)(\cos x)}$

Quotient Rule setup $f'(x)=\frac{(\sin x)(\cos x)'-(\cos x)(\sin x)'}{(\sin x)(\sin x)}$

$f'(x)=\frac{(\cos x)(\cos x)-\sin x(-\sin x)}{(\cos x)^2}$

Derive $f'(x)=\frac{(\sin x)(-\sin x)-(\cos x)(\cos x)}{(\sin x)^2}$

$f'(x)=\frac{(\cos x)^2+(\sin x)^2}{(\cos x)^2}$

simplify $f'(x)=\frac{-(\sin x)^2-(\cos x)^2}{(\sin x)^2}$

$f'(x)=\frac{-1(\sin^2 x+\cos^2 x)}{\sin^2 x}=\frac{-1(1)}{\sin^2 x}$

Pythagorean Identity: $(\cos x)^2+(\sin x)^2=1$

$f'(x)=\frac{1}{(\cos x)^2}$

$f'(x)=\frac{-1}{(\sin x)^2}$

Trig Definition

$f'(x)=(\sec x)^2$

OR $f'(x)=\sec^2 x$

~~short way of writing~~ Equivalent form either is fine

$f'(x)=-(\csc x)^2$

$f'(x)=-\csc^2 x$

$(8x^4)$ is an internal function so we need chain rule

5. $f(x) = \sin(8x^4)$

$$f'(x) = (\cos(8x^4))(32x^3)$$

Derived "sin" part

$(8x^4)'$ which is the internal rate of change

either is fine

$$f'(x) = 32x^3\cos(8x^4)$$

Equivalent answer in "typical format"

6. $g(x) = \cos(5x^3)$

$$g'(x) = (-\sin(5x^3))(15x^2)$$

Derived "cos" part

$(5x^3)'$ which is the internal rate of change

either is fine

$$g'(x) = -15x^2\sin(5x^3)$$

Division $\Rightarrow$ Quotient Rule : $\left(\frac{Hi}{Lo}\right)' = \frac{Lo\,dHi - Hi\,dLo}{LoLo}$

7. $f(x) = \frac{x^2-1}{x^3+5x+3}$

$$f(x) = \frac{x^2-1}{x^3+5x+3}$$

$$f'(x) = \frac{\underbrace{(x^3+5x+3)}_{Lo}\underbrace{(2x)}_{dHi} - \underbrace{(x^2-1)}_{Hi}\underbrace{(3x^2+5)}_{dLo}}{\underbrace{(x^3+5x+3)}_{Lo}\underbrace{(x^3+5x+3)}_{Lo}}$$

$$f'(x) = \frac{2x^4+10x^2+6x-(3x^4+5x^2-3x^2-5)}{(x^3+5x+3)^2}$$

$$f'(x) = \frac{2x^4+10x^2+6x-3x^4-5x^2+3x^2+5}{(x^3+5x+3)^2}$$

$$f'(x) = \frac{-x^4+8x^2+6x+5}{(x^3+5x+3)^2}$$

8. $g(x) = \frac{x^2+5x-2}{\cos x}$

$$g'(x) = \frac{(\cos x)(x^2+5x-2)' - (x^2+5x-2)(\cos x)'}{(\cos x)(\cos x)}$$

$$f'(x) = \frac{(\cos x)(2x+5) - (x^2+5x-2)(-\sin x)}{(\cos x)^2}$$

$f'(x) =$ ~~2xcosx+5cosx~~

$$f'(x) = \frac{2x\cos x + 5\cos x - (-x^2\sin x - 5x\sin x + 2\sin x)}{(\cos x)^2}$$

$$f'(x) = \frac{2x\cos x + 5\cos x + x^2\sin x + 5x\sin x - 2\sin x}{(\cos x)^2}$$

times ⇒ product Rule

internal function ⇒ need Chain Rule to derive this part

9. $f(x) = x^2\sqrt{x^3+1}$

$f(x) = x^2(x^3+1)^{1/2}$ ← $\sqrt{n} = n^{1/2}$

$f'(x) = (x^2)'(x^3+1)^{1/2} + \underbrace{\left((x^3+1)^{1/2}\right)'}_{\text{chain Rule}}(x^2)$ Product Rule

$f'(x) = 2x(x^3+1)^{1/2} + \left(\underbrace{\tfrac{1}{2}(x^3+1)^{-1/2}}_{\text{derived } n^{1/2}}\underbrace{(3x^2)}_{(x^3+1)' \text{ i.e internal rate of change}}\right)(x^2)$ Derive

$f'(x) = 2x(x^3+1)^{1/2} + \dfrac{3x^4}{2(x^3+1)^{1/2}}$ simplify

$f'(x) = \dfrac{2x(x^3+1)^{1/2}}{1} \cdot \dfrac{2(x^3+1)^{1/2}}{2(x^3+1)^{1/2}} + \dfrac{3x^4}{2(x^3+1)^{1/2}}$ Get Common Denominator

$f'(x) = \dfrac{4x(x^3+1)}{2(x^3+1)^{1/2}} + \dfrac{3x^4}{2(x^3+1)^{1/2}}$

$f'(x) = \dfrac{4x^4+4x}{2(x^3+1)^{1/2}} + \dfrac{3x^4}{2(x^3+1)^{1/2}} = \dfrac{7x^4+4x}{2(x^3+1)^{1/2}}$ simplify

10. $g(x) = 5x^4\sqrt[3]{x^2-1}$

$\sqrt[3]{n} = n^{1/3}$

$g(x) = 5x^4(x^2-1)^{1/3}$

$g'(x) = (5x^4)'(x^2-1)^{1/3} + \left((x^2-1)^{1/3}\right)'(5x^4)$

$g'(x) = (20x^3)(x^2-1)^{1/3} + \left(\underbrace{\tfrac{1}{3}(x^2-1)^{-2/3}}_{\text{Derived } n^{1/3} \text{ part}}\underbrace{(2x)}_{(x^2-1)' \text{ i.e. internal rate of } \Delta}\right)(5x^4)$

$g'(x) = 20x^3(x^2-1)^{1/3} + \dfrac{10x^5}{3(x^2-1)^{2/3}}$

$g'(x) = \dfrac{20x^3(x^2-1)^{1/3}}{1} \cdot \dfrac{3(x^2-1)^{2/3}}{3(x^2-1)^{2/3}} + \dfrac{10x^5}{3(x^2-1)^{2/3}}$

$g'(x) = \dfrac{60x^3(x^2-1)}{3(x^2-1)^{2/3}} + \dfrac{10x^5}{3(x^2-1)^{2/3}}$

$g'(x) = \dfrac{60x^5-60x^3}{3(x^2-1)^{2/3}} + \dfrac{10x^5}{3(x^2-1)^{2/3}}$

$g'(x) = \dfrac{70x^5-60x^3}{3(x^2-1)^{2/3}}$

11. $f(x) = (5x^6+2)^8$

$f(x) = (5x^6+2)^8$

Think $(n^8)' = 8(n)^7 \cdot \underbrace{(5x^6+2)'}_{\text{internal rate of } \Delta}$

$f'(x) = 8(5x^6+2)^7(30x^5)$

$f'(x) = 240x^5(5x^6+2)^7$

12. $g(x) = \cos^2(5x^2)$

$g(x) = \cos^2(5x^2)$

$g(x) = (\cos(5x^2))^2$ meaning of $\cos^2\theta$

$g'(x) = \underbrace{2(\cos(5x^2))}_{\text{Derive squared part outside}}\underbrace{(-\sin(5x^2))}_{\text{derived cos part}}\underbrace{(10x)}_{\text{derived } 5x^2 \text{ part}}$

$g'(x) = -20x(\cos(5x^2))(\sin(5x^2))$

$5x^2$ was an internal function of cosine and cosine was an internal function of $(\ )^2$ ⇒ internal internal rate of change

Need a point and a slope

13. Find the equation of the line tangent to $f(x) = 2x^3 - 5x + 7$ when x = 2.

Point:

$f(x) = 2x^3 - 5x + 7$ when $x=2$

$f(2) = 2(2)^3 - 5(2) + 7 = 13$

16 −10 +7

Point $(2, 13)$

Slope f′ finds slope of f

$f'(x) = 6x^2 - 5$

$f'(2) = 6(2)^2 - 5 = 19$

24 −5

$\boxed{y - 13 = 19(x-2)}$ Answer in point slope form

any of the equivalent forms are correct

Equivalent answers below:

$y = 19(x-2) + 13$

$y = 19x - 38 + 13$

$y = 19x - 25$ ← slope intercept form

14. Find the equation of the line tangent to $g(x) = x\sin x$ when $x = \pi$.

Need a point and a slope

Point

$g(x) = x\sin x$ when $x = \pi$

$g(\pi) = \pi \sin \pi = \pi(0) = 0$

$(\pi, 0)$

Slope g′ finds slope of g

Need Product Rule

$g(x) = x \cdot \sin x$

Product Rule setup $g'(x) = (x)'(\sin x) + (\sin x)'(x)$

$g'(x) = 1(\sin x) + (\cos x)(x)$

$g'(x) = \sin x + x\cos x$

$g'(\pi) = \sin \pi + \pi \cos \pi$

$g'(\pi) = 0 + \pi(-1) = -\pi$

any one of the equivalent forms is correct

$y - 0 = -\pi(x - \pi)$ } answer in point slope form

or

$y = -\pi(x - \pi)$

$y = -\pi x + \pi^2$ } answer in slope intercept form