

Physics C Study Guide 10

10.1 - Electrostatics with Conductors

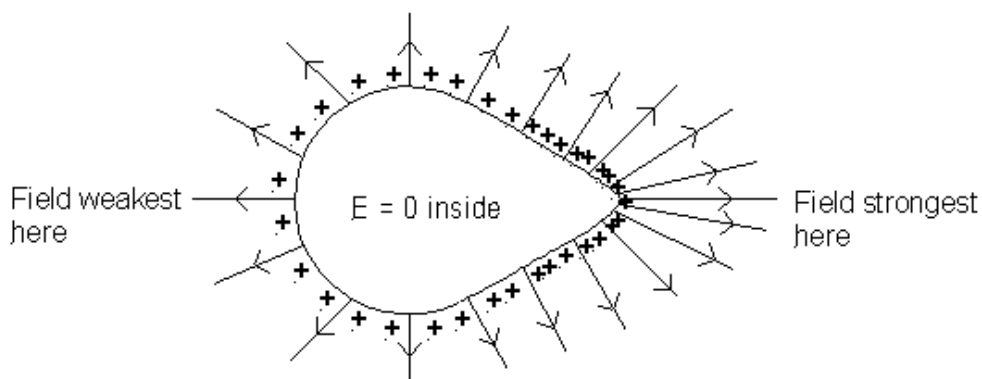
First, a few quick facts:

- The electric field within a conductor is always zero.
- A charged object, made of conductive material, has all its charge distributed on its surface.
- Surface charge density is highest at “pointy” points on a surface (like the pointed end of an egg).
- When multiple charged objects are present, charge will redistribute on a surface based on like charges repelling and opposite charges attracting.
- Electric potential is constant throughout the surface of a conductor.
- Electric potential is given by $V = k \frac{q}{r} = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$
 - r can mean the distance between a measurement point and a point charge, or r can mean the radius of a charged, conductive object, in which case V is the electric potential at the surface of the object.

Electric Field

In the case of a solid object made of a conducting material, the electric field *within* the object is always zero. As an example, if Earth were metal and were charged, the instant you went underground, the electric field would be zero.

This even goes for theoretically two-dimensional objects. The electric field *on the surface* (within the material) is zero. On a uniform surface (like a sphere), charge is evenly distributed, but on a non-uniform surface (like a cube), charge is built up at corners.



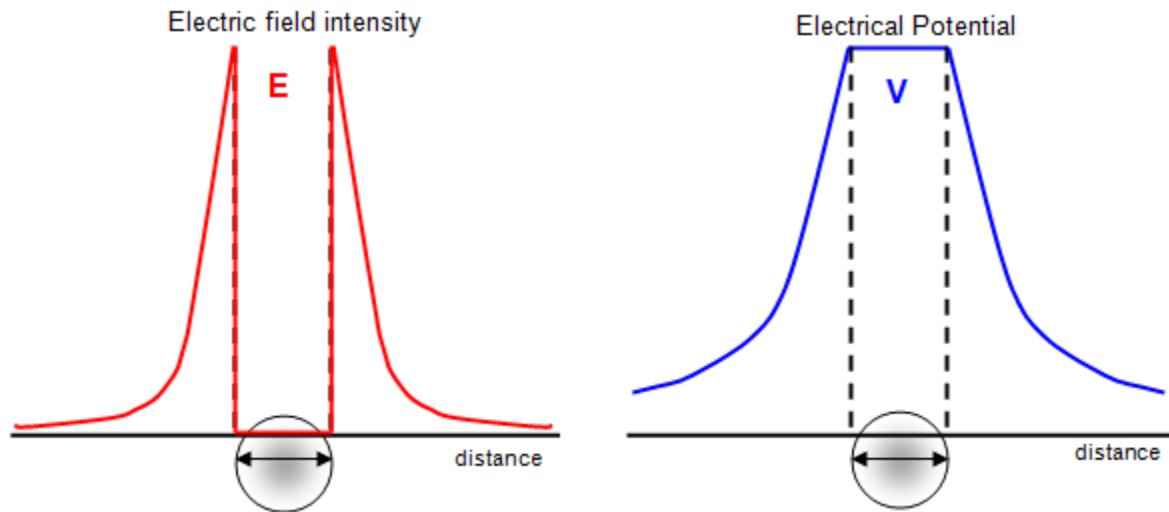
(d) Electric field & charge distribution around a pear-shaped conductor

Charge Distribution

Gauss's Law ($\Phi = \frac{q_{enc}}{\epsilon_0} = E \cdot A$) states an enclosed charge is proportional to the product of the field and the surface area. If any one of those is zero (like the amount of charge), the whole thing is zero (like the field). Therefore, since a conductor has zero field inside the surface, there's also no charge within the surface. **All charge on a conductor is distributed on the surface, and a field can't pass through a conductor.**

The reason all charge is distributed on the surface is because opposite charges attract and like charges repel. If a metal sphere had a net positive charge, all the positive charges would distribute themselves as far apart as possible, on the outside surface of the sphere. They'd also be attracted to the neutral charges outside the sphere, since at least they're less positive than the sphere itself.

Electric field lines are drawn positive-to-negative. Since the entire surface has positive charges distributed throughout, there are no field lines within the object, since they'd have to be positive-to-positive.



Everything said above, regarding electric field and charge distribution, is true whether the conductor is solid or hollow.

Electric potential of conductors

Electric potential on the surface of a sphere (hollow or solid) is given by:

$$V = k \frac{q}{r}$$

Where r is the radius of the sphere. More charge q and less radius r both result in greater electric potential V . For electric potential at a point outside the sphere, r is the distance from the point of measurement to the center of the sphere.

Electric potential throughout the surface of a conductor is constant. If another conductor is brought into contact, the two objects are electrically the “same point,” and the electric potential is constant throughout the surface of both objects.

When two conductors come into contact, charge will redistribute until electric potential is constant throughout a system, if possible. This remains true after the two objects are no longer in contact.

10.2 - Redistribution of Charge Between Conductors

Charge and electric potential after contact

When two conductors come into contact, charge will redistribute in order to equalize the electric potentials of both objects. If there are two objects, that means $V_1 = V_2$, or $\frac{kQ_1}{r_1} = \frac{kQ_2}{r_2}$. After contact, the charges on each object may be different, but their electric potentials are equal. After contact, the charges on each object may be different, but their electric potentials are equal. This remains true even after the two objects are no longer in contact.

No work is done moving a charge along the surface of a charged conductor, nor through the middle to another point on the surface, because the field inside it is zero and the electric potential is the same at all points on the surface. Therefore, there's no change in potential energy.

Total charge within a system is conserved, so the sum of charge on each object must be the same and the sum of charge before contact (often only one charge).

Because $V = k\frac{q}{r}$, electric potential V is directly proportional to charge q and is inversely proportional to radius r . In order to keep electric potential constant within a system, objects with a larger radius must also have more charge.

Example: A sphere with radius $r_1 = 2$ cm has an initial charge $q_0 = 12$ nC. It's brought into contact with a sphere with a radius $R_2 = 4$ cm. Find the charge and electric potential of each sphere after contact. Because the larger sphere has twice as much radius, it'll have twice as much charge as the smaller sphere:

$$Q_2 = 2q_1$$

The total charge must equal 12 nC:

$$q_1 + Q_2 = q_1 + 2q_1 = 12 \text{ nC}$$

$$q_1 = 4 \text{ nC}$$

$$Q_2 = 2q_1 = 8 \text{ nC}$$

We can now use the charges with $V = k\frac{q}{r}$ to find the electric potentials.

$$V_1 = k\frac{q_1}{r_1} = (9 \cdot 10^9) \frac{4 \cdot 10^{-9}}{0.02} = 1,800 \text{ V}$$

The electric potential of the larger sphere must be equal, but we can calculate it separately just in case:

$$V_2 = k\frac{Q_2}{R_2} = (9 \cdot 10^9) \frac{8 \cdot 10^{-9}}{0.04} = 1,800 \text{ V}$$

10.3 - Capacitors

A *capacitor* is a device which can store charge. This is different from a battery, which stores electric potential (voltage). A capacitor's defining characteristic is *capacitance*, measured in farads (F).

Capacitance is the ability to store charge, which is given by:

$$C = \frac{Q}{\Delta V}$$

For any given capacitor, capacitance C is a fixed value. Therefore, in order to store more charge Q , electric potential ΔV must also be increased by the same factor.

This formula can also be written as $Q = CV$, which can come in handy, as we'll see later.

Finding capacitance

Many capacitors are simply a pair of parallel conductive plates, and the capacitance can be found with:

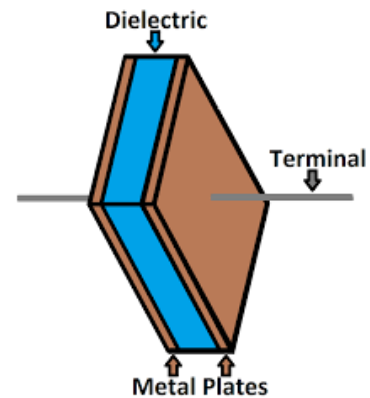
$$C = \frac{\kappa \epsilon_0 A}{d}$$

κ : dielectric constant (if not given, assume it's 1)

$$\epsilon_0 = 8.85 \cdot 10^{-12} \frac{C^2}{Nm^2} \quad //this is on your formula sheet$$

A : area of the two plates, in m^2

d : the distance between the two plates, in m

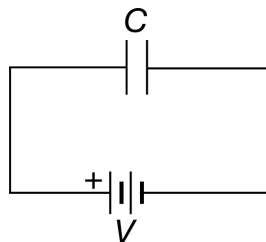


Capacitors with large plates close together have high values of capacitance, allowing them to store large amounts of charge Q at modest values of electric potential ΔV .

Because capacitance can be expressed in two ways ($C = \frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d}$), you may have to set these expressions equal to each other in order to solve for a given variable, like electric potential or distance between the plates.

Isolated vs. connected to voltage

Depending on the circumstances, either Q or V may be held constant. Q is held constant when the capacitor is "isolated." In this case, electrons are not allowed to move freely, so the charge is constant. As a result, C and V will have an inverse relationship.



If the capacitor is connected to a steady voltage source (like a battery, see left image), V is constant, and electrons are free to move, so charge is not necessarily constant. In this situation, Q and C have a linear, direct relationship.

Potential energy

The electric potential across a capacitor means potential energy is present, and it's given by:

$$U = \frac{1}{2}CV^2$$

Given $Q = CV$, a quick substitution and some algebra gives:

$$U = \frac{Q^2}{2C}$$

Given $C = \frac{\kappa\epsilon_0 A}{d}$, another substitution and more algebra yields:

$$U = \frac{dQ^2}{2\kappa\epsilon_0 A} = \frac{Q^2}{2C} = \frac{1}{2}CV^2$$

Remember, work is change in energy, and work can be negative!

Connected to battery

Voltage V is constant

C and Q have direct relationship

U is directly related to C

U is directly related to Q (thanks to C)

Separating plates takes negative work

Isolated

Charge Q is constant

C and V have inverse relationship

U is inversely related to C

U is directly related to V (thanks to C)

Separating plates takes positive work

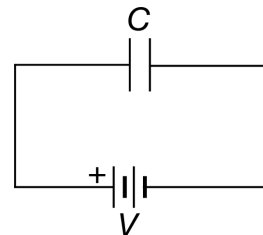
Keep these in mind when things like κ , A , and d change, all of which change capacitance C . A change in capacitance may or may not change V or Q , depending on whether the capacitor is connected or isolated.

Long example (6 parts): In the image at right, the voltage source has a voltage of 9 V. The capacitor's plates each have an area of 9 cm², and they're held 2 cm apart, separated by air.

1. How much charge is stored in the capacitor?

$$C = \frac{\kappa\epsilon_0 A}{d} = \frac{1(8.85 \cdot 10^{-12})(0.0009)}{0.02} = 3.9825 \cdot 10^{-13} \text{ F}$$

$$Q = CV = (3.9825 \cdot 10^{-13}) \cdot 9 = \mathbf{3.584 \cdot 10^{-13} \text{ C}}$$



2. The plates are pulled to a distance of 5 cm apart. How much charge is *now* stored in the capacitor?

$$C = \frac{\kappa\epsilon_0 A}{d} = \frac{1(8.85 \cdot 10^{-12})(0.0009)}{0.05} = 1.593 \cdot 10^{-13} \text{ F}$$

$$Q = CV = (1.593 \cdot 10^{-13}) \cdot 9 = \mathbf{1.434 \cdot 10^{-12} \text{ C}}$$

Voltage is constant, so the only thing that changes is the capacitance, due to change in d .

3. How much work is done pulling the plates from 2 cm to 5 cm apart?

$$U_1 = \frac{1}{2}C_1V^2 = \frac{1}{2}(3.9825 \cdot 10^{-13}) \cdot 9^2 = 1.6129 \cdot 10^{-11} \text{ J}$$

$$U_2 = \frac{1}{2}C_2V^2 = \frac{1}{2}(1.593 \cdot 10^{-13}) \cdot 9^2 = 6.4517 \cdot 10^{-12} \text{ J}$$

$$W = \Delta U = U_2 - U_1 = (6.4517 \cdot 10^{-12}) - (1.6129 \cdot 10^{-11}) = \mathbf{-9.677 \cdot 10^{-12} \text{ J}}$$

When V is constant, it takes negative work to separate plates and positive work to bring them together.

4. The capacitor is isolated from the voltage source, and then the plates are pushed from a distance of 5 cm apart to a distance of 3 cm apart. How much charge is stored in the capacitor?

$$Q = 1.434 \cdot 10^{-12} \text{ C}$$

When the capacitor is isolated, charge is constant, and voltage is potentially variable. The charge will remain the same as it was before isolation, found in part (2).

5. What is the potential difference across the capacitor?

$$C = \frac{\kappa \epsilon_0 A}{d} = \frac{1(8.85 \cdot 10^{-12})(0.0009)}{0.03} = 2.655 \cdot 10^{-13} \text{ F}$$

$$Q = CV$$

$$1.434 \cdot 10^{-12} \text{ C} = (2.655 \cdot 10^{-13})V$$

$$V = \frac{1.434 \cdot 10^{-12}}{2.655 \cdot 10^{-13}} = 5.401 \text{ V}$$

When the capacitor is isolated, charge is constant, and voltage is potentially variable. Because the capacitance increases, voltage must decrease by the same factor to make up for it.

6. How much work is done pushing the plates from 5 cm to 3 cm apart?

$$U_1 = 6.4517 \cdot 10^{-12} \quad // \text{equal to } U_2 \text{ in part (3)}$$

$$U_2 = \frac{1}{2} C_2 V^2 = \frac{1}{2} (2.655 \cdot 10^{-13}) (5.401)^2 = 3.8726 \cdot 10^{-12}$$

$$W = \Delta U = U_2 - U_1 = (3.8726 \cdot 10^{-12}) - (6.4517 \cdot 10^{-12}) = -2.579 \cdot 10^{-12} \text{ J}$$

When charge is constant, it takes negative work to bring the plates closer together and positive work to separate them.

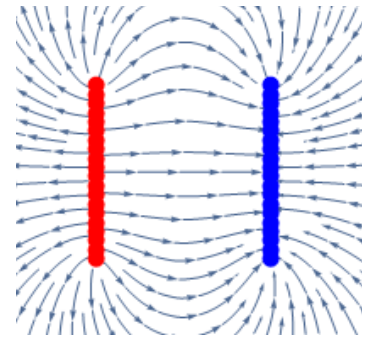
Electric field within a capacitor

Electric field within a capacitor is given by:

$$E = \frac{\sigma}{\epsilon_0}$$

Where σ is the surface charge density of the plates, given by $\sigma = \frac{Q}{A}$.

Increasing charge will therefore increase the electric field, but essentially no other variable has any effect on the electric field within a capacitor.



The electric field within a capacitor is theoretically uniform, but near the edges, there are edge effects which cause differences in both magnitude and direction of the electric field.

Electric field within a conductor is zero, so if a conductive material is placed between the plates, the distance between them is effectively reduced, which increases the capacitance of the capacitor.

10.4 - Dielectrics

About that κ thing

As we saw earlier, the formula for capacitance is:

$$C = \frac{\kappa \epsilon_0 A}{d}$$

κ is the dielectric constant, determined by the dielectric, which is a material between the two plates of a capacitor. Air has a κ -value of 1, and conductors have extremely high (theoretically infinite) κ -values. Essentially any dielectric will have a value greater than 1. κ has no units.

κ is not affected by d , or in other words, a dictionary and a piece of paper have the same κ -value, but different d -values.

Notably, κ is directly (i.e. linearly) related to capacitance. If you were to increase κ by a factor of 4.3, C would also increase by a factor of 4.3.

Because κ changes the value of C , it also changes the values of charge, potential difference, potential energy, and electric field, as respectively shown:

Charge: $Q = VC$

Potential difference: $V = \frac{Q}{C} = \frac{Qd}{\kappa \epsilon_0 A}$

Potential energy: $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{dQ^2}{2\kappa \epsilon_0 A}$

Electric field: $E = \frac{V}{d} = \frac{Q}{\kappa \epsilon_0 A}$

Most of these expressions are simply substitutions and derivations of one another. It's handy to know how to express any variable in terms of almost any other combination of variables, depending on what information is provided.

If κ increases, the resultant changes depend on how the capacitor is hooked up:

Connected to voltage

V is constant

C increases

Q increases

U increases

E is constant

Isolated

Q is constant

C increases

V decreases

U decreases

E decreases

All increases and decreases are by the same factor as κ .

Long example (13 parts): An air-gapped capacitor has a plate area of 36 cm^2 and a plate spacing of 8 mm . If the potential between the plates exceeds 300 V , the capacitor will fail and discharge.

1. What is the capacitance of the capacitor?

$$A = 36 \text{ cm}^2 = 0.0036 \text{ m}^2$$

$$d = 8 \text{ mm} = 0.008 \text{ m}$$

$$\kappa = 1$$

$$C_1 = \frac{1(8.85 \cdot 10^{-12})(0.0036)}{0.008} = \mathbf{3.983 \cdot 10^{-12} \text{ F}}$$

2. What is the maximum charge the capacitor can hold?

$$Q_1 = CV = (3.9825 \cdot 10^{-12}) \cdot 300 = \mathbf{1.195 \cdot 10^{-9} \text{ C}}$$

3. What is the maximum electric field possible between the plates?

$$E_1 = \frac{V}{d} = \frac{300}{0.008} = \mathbf{37,500 \frac{V}{m}}$$

4. What is the maximum potential energy between the two plates?

$$U_1 = \frac{1}{2}CV^2 = \frac{1}{2}(3.983 \cdot 10^{-12})(300)^2 = \mathbf{1.792 \cdot 10^{-7} \text{ J}}$$

example, continued: A dielectric with a κ value of 3.2 is inserted into the capacitor. What are the new values for capacitance, maximum charge, maximum electric field, and maximum potential energy? Maximum voltage is constant, so C , Q , and U will all increase by a factor of κ . E will be constant, since neither V nor d change.

$$5. C_2 = C_1 \cdot \kappa = (3.983 \cdot 10^{-12}) \cdot 3.2 = \mathbf{1.274 \cdot 10^{-11} \text{ F}}$$

$$6. Q_2 = Q_1 \cdot \kappa = (1.195 \cdot 10^{-9}) \cdot 3.2 = \mathbf{3.823 \cdot 10^{-9} \text{ C}}$$

$$7. E_2 = E_1 = \mathbf{37,500 \frac{V}{m}}$$

$$8. U_2 = U_1 \cdot \kappa = (1.792 \cdot 10^{-7}) \cdot 3.2 = \mathbf{5.735 \cdot 10^{-7} \text{ J}}$$

example, still continued: With the 3.2 dielectric still inserted, the capacitor is charged to its maximum potential difference of 300 V, allowed to reach steady state, and then isolated. The value of the dielectric is then increased from 3.2 to 4.8. What are the new values for capacitance, charge, potential difference, electric field, and potential energy?

κ changed from 3.2 to 4.8, which is a factor of 1.5.

Because the capacitor is isolated, charge is constant. Capacitance will increase by a factor of 1.5 (the same as κ), while potential difference, potential energy, and electric field will decrease by a factor of 1.5.

$$9. C_3 = C_2 \cdot 1.5 = (1.274 \cdot 10^{-11}) \cdot (1.5) = \mathbf{1.912 \cdot 10^{-11} \text{ F}}$$

$$10. Q_3 = Q_2 = \mathbf{3.823 \cdot 10^{-9} \text{ C}}$$

$$11. V_3 = \frac{V_2}{1.5} = \frac{300}{1.5} = \mathbf{200 \text{ V}}$$

$$12. E_3 = \frac{E_2}{1.5} = \frac{37,500}{1.5} = \mathbf{25,000 \frac{V}{m}}$$

$$13. U_3 = \frac{U_2}{1.5} = \frac{5.735 \times 10^{-7}}{1.5} = \mathbf{3.823 \cdot 10^{-7} \text{ J}}$$

Dielectrics and electric field

Dielectrics are polarized opposite to the polarization of the capacitor (since opposites attract and likes repel), despite being electrically neutral (the dielectric itself contains no charge).

The polarization of the dielectric causes a small electric field to run within the dielectric in the direction opposite to the field within the capacitor. The sum of the capacitor's field and the dielectric's field partially cancel out, leaving a smaller net field E as a result.

