2023 UAB Middle School Math Tournament
8th Grade Division
Written Test

1. 60 minutes will be allowed to complete this examination. The proctor will keep time. Students must stay in the room for the full 60 minutes. If you must leave the room, you must turn in your scantron before leaving since re-entry will not be permitted.
2. No calculators, books, notes, or other aides may be used. The proctor will supply scratch paper; you may not bring your own. If you need more scratch paper or a pencil during the test, raise your hand and the proctor will bring it to you.
3. You will receive 4 points for each correct answer and -1 for each incorrect answer on the 25 multiple choice questions. Questions left blank will be worth 0 points. There are 3 tie breakers at the end of the test. Correct answers on the tie breakers are worth 0.1 point. Your score on the written test will be the sum of the scores from the multiple choice and tie breaker sections. Please write tiebreakers in the appropriate spaces on your answer sheet.
4. If the correct answer is not listed a-d, then mark e. NOTA (None of the Above)
5. If there are ties after the scores are computed as described above, ties will be broken by comparing answers counting backwards from number 25, then 24, and so on, using these questions as tie breakers.
6. Please give the proctor your answer sheet before you leave the classroom. You may keep the copy of the test. We suggest that you mark your answers on this sheet as well so that you may check your answers outside of the testing rooms.

1. Simplify: (2+7r)(5-3r)

A. $10 + 29r - 21r^2$
B. $10 – 11r – 21r^2$
C. $15 + 41r – 21r^2$
D. $15 – 11r + 21r^2$
E. NOTA

2. Prateek buys 29 items for a total of $14.60. He buys 17 erasers, 10 pencils, and 2 pens. Knowing the pens are 50 cents each and the erasers are 40 cents each, how much does each pencil cost?

A. $1.04
B. $0.44
C. $0.83
D. $0.69
E. NOTA

3. You shuffle a deck of cards thoroughly. What is the minimum number of cards would you need to draw in order to guarantee you get 2 of the same suit?

A. 4
B. 3
C. 7
D. 5
E. NOTA

4. What is the sum of all the cubes between 0 and 500?

A. 944
B. 849
C. 736
D. 788
E. NOTA

5. Solve for x: $(9^2-3^2)(10x-15) = 18\sqrt{(100x+3600)}$

A. $0, \frac{7}{4}, \frac{49}{16}$
B. $0,\frac{7}{4}$
C. $\frac{7}{4}$
D. $0, \frac{49}{16}$
E. NOTA

6. What's the 6th term in the following sequence: 10, , …

A. $-\frac{5}{1024}$
B. $-\frac{5}{512}$
C. $\frac{5}{512}$
D. $\frac{5}{1024}$
E. NOTA

7. Given $x + \frac{1}{x} = 5$, what is the value of x?

A. $\frac{1\pm\sqrt{77}}{2}$
B. $\frac{5\pm\sqrt{17}}{2}$
C. $\frac{5\pm\sqrt{21}}{2}$
D. $\frac{1\pm\sqrt{17}}{2}$
E. NOTA

8. Solve 8! – 3!

A. 30414
B. 40314
C. 41034
D. 34043
E. NOTA

9. A triangle has side lengths of 10 and 12. What is a possible side length of the 3rd side?

A. 23
B. 1.5
C. 2
D. 22
E. NOTA

10. If $f(x) = x^3 + 7x$, what is $f(x+y)$?

A. $x^3 + 3x^2y + 3xy^2 + y^3 + x + y$
B. $x^3 + y^3 + 7x + 7y$
C. $x^2 + xy + y^2 + 7x + 7y$

D. $x^3 + x^2y + xy^2 + y^3 + 7x + 7y$
E. NOTA

11. Hannah has 2 6-sided dice. What's the probability that if she rolls it twice, her dice will add up to a sum greater than or equal to 7?

A. $\frac{17}{36}$
B. $\frac{1}{2}$
C. $\frac{5}{12}$
D. $\frac{7}{12}$
E. NOTA

12. Evaluate $81x^2$ where $x = 4$

A. 1331
B. 1296
C. 1221
D. 1016
E. NOTA

13. What is the sum of all prime numbers from 0 to 50

A. 324
B. 326
C. 343
D. 327
E. NOTA

14. If $0.21x = 42$, what is $0.56x$?

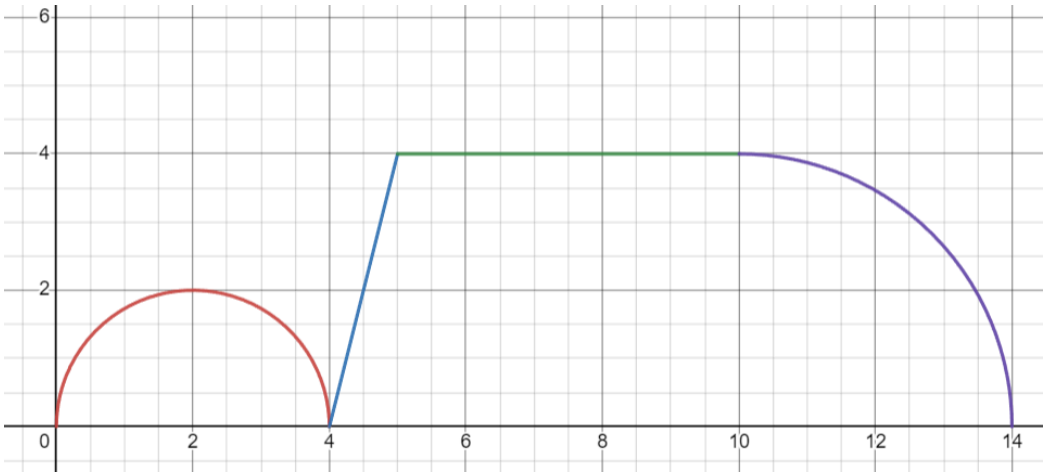
A. 112
B. 120
C. 108
D. 110
E. NOTA

15. The area of a circle is 256π, what would its circumference be?

A. 32π
B. 48π
C. 16π
D. 8π
E. NOTA

16. Ethan has been struggling in math for the past few weeks. At the beginning of the semester, he did well, making 100, 95, 100, and 95 on his first four tests. However, he's been caught slacking with his exam scores being 90, 80, 85, and 90 on the next 4 tests. His parents threaten that Ethan won't get a Bugatti if he doesn't get an A (93 or above) in the class and will force him to ride his little sister's pink bicycle to school. What is the sum of the scores Ethan needs on his remaining two tests in order to pass the class?

A. 180
B. 190
C. 165
D. 195
E. NOTA

17. 

What's the area under the curve for the shape between $0 \leq x \leq 14$

A. $6\pi + 28$
B. $8\pi + 30$
C. $4\pi + 26$
D. $4\pi + 32$
E. NOTA

18. For the segments above, what is the total length of these segments?

A. $4\pi + 2\sqrt{5} + 5$
B. $8\pi + 2\sqrt{5} + 5$
C. $6\pi + \sqrt{19} + 5$

D. $4\pi + \sqrt{19 + 5}$
E. NOTA

19. What's the radius of a cone with a slant height of 30 and an angle of 30 degrees at the base?

A. $15\sqrt{2}$
B. $20\sqrt{2}$
C. $20\sqrt{3}$
D. $15\sqrt{3}$
E. NOTA

20. Simplify $\sqrt{\frac{81a^3b^9c^4}{49a^{-2}b^{-3}c^2}}$

A. $9a^5b^{12}c^6$
B. $\frac{9}{4}a^5b^6c^3$
C. $\frac{9}{7}a^5b^6c^3$
D. $\frac{9}{7}a^2b^3c$
E. NOTA

21. It has been determined that the amount of carbon-14 in a dinosaur's bone is around 2.5%. Scientists have determined that the percentage of carbon-14 in the atmosphere back then was around 40%. Knowing that the half-life of carbon-14 is 5730 years, how long ago did that fossil live.

A. 22.9 millenia
B. 22.8 millenia
C. 22.7 millenia
D. 23.1 millenia
E. NOTA

22. What is 1011011 in binary

A. 91
B. 87
C. 166
D. 182
E. NOTA

23. Sri is a pokemon trainer whose purpose is to catch them all. He sees a shiny Venusaur 10 meters away from him and is determined to catch it. His Charmeleon's Flamethrower does more damage the closer it is to the enemy in a quadratic pattern. At 15 meters away, Charmeleon does 0 damage (vertex). At 10 meters away, Charmelon does 50 damage. Assuming Venusaur has 160 health, how close does Charmeleon have to get in order to knock out Venusaur in a move rounded to the nearest whole number.

A. 6 meters
B. 7 meters
C. 4 meters
D. 8 meters
E. NOTA

24. Find the inverse of the following matrix and determine what number would be located in the top right corner

$$\begin{pmatrix} -2 & 4 \\ 2 & 2 \end{pmatrix}$$

A. $\frac{1}{4}$
B. $-\frac{1}{4}$
C. $\frac{2}{3}$
D. $-\frac{2}{3}$
E. NOTA

25. What's the value of $2sin(\theta)cos(\theta)$ at 30°

A. $\frac{\sqrt{3}}{2}$
B. $\frac{\sqrt{2}}{2}$
C. $\frac{1}{2}$
D. 1
E. NOTA

TB1: A Lamborghini and a Ferrari are racing each other to see who can beat each other in a 10 mile race. The Lamborghini accelerates from 0 to 120 MPH in 30 seconds and keeps at that speed for the rest of the race. The Ferrari has an engine failure and takes 2 minutes to fix it. Then, it speeds up to 200 MPH in 20 seconds and maintains that speed for the rest of the race. Who wins and what time does the winning car have? Use

Lambo:

$$\Delta x = \left(\frac{v_i + v_f}{2}\right)t = \left(\frac{0+120}{2}\right)\left(\frac{30}{3600}\right) = 60 * \frac{1}{120} = 0.5\ miles$$

$$\frac{9.5\ miles}{120\ miles/hr} = \frac{19}{240} hr * 60\frac{min}{hr} = \frac{19}{4}\ minutes = 4.75\ minutes\ (after\ initial\ 30\ seconds)$$

Lamborghini total time: 5.25 minutes

Ferrari:

$$\Delta x = \left(\frac{v_i + v_f}{2}\right)t = \left(\frac{0+200}{2}\right)\left(\frac{20}{3600}\right) = 100 * \frac{1}{180} = \frac{5}{9}\ miles, \frac{85}{9}\ miles\ left$$

$$\frac{85/9\ miles}{200\ miles/hr} = \frac{17}{360} hr * 60\frac{min}{hr} = \frac{17}{6}\ minutes = 2.83\ minutes\ (after\ initial\ 2.33\ minutes)$$

Ferrari total time: 5.17 minutes

Ferrari wins @ 5 minutes and 10 seconds, Lamborghini loses @ 5 minutes and 15 seconds

TB2: Suppose you have a knight and a rook at the origin of a coordinate plane. The rook can move along any axis for a total of 5 units. The knight can take 3 moves (moves in an L shape, <2,1> or <1,2> in vector terms). What is the maximum distance the two pieces can have between each other? Write in the simplest form.

√130

TB3: You've been teleported into the One Piece Realm, and you've been given the choice between two fruits to eat. One fruit is in the shape of a cylinder whose radius is half of its height. Another fruit is in a spherical shape where the radius is ¾ the height of the other fruit. Knowing that the fruits are notoriously disgusting, what is the difference in volume between the two fruits, knowing the diameter of the spherical fruit is 3 ft and which would you eat?

$\frac{5}{2}\pi$