1.

Question	Answer	Marks	Guidance
11(a)	Substitute 4 to obtain gradient of curve is $\frac{1}{10}$	B1	
	Attempt equation of normal using $(4, 3)$ and $-\frac{1}{m}$ for their gradient	M1	
	$y = -10x + 43$	A1	
		3	

2.

Question	Answer	Marks	Guidance
4(a)	$[k =]4$	B1	
		1	
4(b)	$\left\{\frac{k}{4}x^4\right\} + \left\{\frac{2x^{-2+1}}{-2+1}\right\} [+c]$	B2,1,0FT	FT *their k* from **4(a)** or the letter *k*. B2 for both correct components and no other *x*-terms. B1 for one correct.
	$20 = \frac{\textit{their}\ k}{4} \times 2^4 - \frac{2}{2} + c$	M1	Substituting $x = 2$ and $y = 20$ into *their* integrated expression (defined by having at least one correct power and including a $'+c'$). Allow one slip.
	$c = 5$	A1	
	$[t =]4$	A1	
		5	

3.

Question	Answer	Marks	Guidance
3(a)	72.030004	B1	CAO. Not AWRT.
		1	
3(b)	30.3888	B1	CAO. Not AWRT.
		1	
3(c)	30[.0]	B1	CAO. 30 may be accompanied by 'around', 'approximately' etc.
		1	

4.

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx}=\left\{\frac{-8\times 3}{(3x-8)^2}\right\}\left\{+\frac{6}{(x-1)^2}\right\}$	B1 B1	B1 for each {} element.
	$\left\{\frac{-8\times 3}{(3.3-8)^2}\right\}\left\{+\frac{6}{(3-1)^2}\right\}\left[=\frac{-45}{2}\right]$	*M1	Using *their* differentiated expression, which must contain $(3x-8)^{-2}$ and $(x-1)^{-2}$, and $x=3$. This may be seen in *their* line equation.
	$y-5=\left(their\ \frac{-45}{2}\right)(x-3)$ or $5=\left(their\ \frac{-45}{2}\right)(3)+c \Rightarrow c=\left[\frac{145}{2}\right]$	*DM1	Correct form of a line equation with their gradient, but not the negative reciprocal, and $(3,\ 5)$.
	$\left[-8x-5=\frac{-45}{2}(x-3)\Rightarrow\right]\ 29x=145$	DM1	Replacing y with $-8x$ and collecting terms.
	$x=5, y=-40$	A1	Accept $(5,\ -40)$.
		6	

11(b)(i)	$\frac{-24}{(3x-8)^2}+\frac{6}{(x-1)^2}=0 \Rightarrow 24(x-1)^2=6(3x-8)^2$ $\left[\Rightarrow 4(x-1)^2=(3x-8)^2\right]$	*M1	Equate *their* $\frac{dy}{dx}$, which must contain $(3x-8)^{-2}$ and $(x-1)^{-2}$ to 0, and clear of fractions. Only condone ± errors.
	$5x^2-40x+60\ [=0]$ or $\pm 2(x-1)=3x-8$	DM1	OE Forming a three-term quadratic. Condone only ± errors. Or taking square roots. For this method, the ± must be present.
	2, 6	A1	Both values.
		3	
11(b)(ii)	$\frac{d^2y}{dx^2}=\left\{\frac{-24\times -2\times 3}{(3x-8)^3}\right\}+\left\{\frac{6\times -2}{(x-1)^3}\right\}$	B1FT B1FT	Correct differentials of the elements of *their* $\frac{dy}{dx}$, which must contain $(3x-8)^{-2}$ and $(x-1)^{-2}$.
	At $x=2$, $\frac{d^2y}{dx^2}=\frac{their\,144}{(6-8)^3}-\frac{their\,12}{(2-1)^3}$ and at $x=6$, $\frac{d^2y}{dx^2}=\frac{their\,144}{(18-8)^3}-\frac{their\,12}{(6-1)^3}$	M1	Replacing x with *their* 2 and *their* 6 in *their* $\frac{d^2y}{dx^2}$, which must contain $(3x-8)^{-3}$ and $(x-1)^{-3}$. Correct final answers.
	At $x=2$, $\frac{d^2y}{dx^2}=-30<0$, therefore max[imum] At $x=6$, $\frac{d^2y}{dx^2}=\frac{48}{10^3}\left[\frac{6}{125}\right]>0$, therefore min[imum]	A1	WWW Correct values, or working, and >0 and <0 are required.
		4	

	At $x=2$, $\frac{d^2y}{dx^2}=-30<0$, therefore max[imum] At $x=6$, $\frac{d^2y}{dx^2}=\frac{48}{10^3}\left[\frac{6}{125}\right]>0$, therefore min[imum]	A1	WWW Correct values, or working, and >0 and <0 are required.
		4	

5.

Question	Answer	Marks	Guidance
2(a)	[Gradient of tangent] = $4(2\times4-5)^3-9\times4^{\frac{1}{2}}\left[=90\right]$	M1	Substitute $x=4$ into $\frac{dy}{dx}$. $\frac{-11}{2}=4(2\times4-5)^3-9\times4^{\frac{1}{2}}$ is M0 unless they reach $-\frac{1}{90}$.
	[Gradient of normal] = $-\frac{1}{90}$	A1	AWRT -0.0111.
		2	
2(b)	$[y]=\left\{\frac{1}{2}(2x-5)^4\right\}\left\{-6x^{\frac{3}{2}}\right\}[+c]$	B1 B1	Accept unsimplified.
	$-\frac{11}{2}=\frac{1}{2}\times(2\times4-5)^4-6\times4^{\frac{3}{2}}+c$	M1	Sub $x=4, y=-\frac{11}{2}$ into an integrated expression and attempt to find c.
	$y=\frac{1}{2}(2x-5)^4-6x^{\frac{3}{2}}+2$	A1	Condone $c=2$ as final answer if '$y=\ldots$' seen previously. Fractions must be simplified. Accept f(x) in place of y.
		4	

6.

Question	Answer	Marks	Guidance
9(a)	$\left[\frac{dy}{dx}=\right]\frac{-24x^{-2}}{-2}\left[=\frac{12}{x^2}\right][+c]$	B1	May be unsimplified.
	$\frac{12}{(-2)^2}+c=0\ \left[\Rightarrow c=-3\right]$	M1	Substituting $x=-2$ into *their* $\frac{dy}{dx}$ of the form $kx^{-2}+c$, and setting = 0. Condone missing bracket.
	$\left[\frac{dy}{dx}=\right]\frac{12}{x^2}-3$ or $12x^{-2}-3$	A1	$\frac{-24}{-2}$ must be simplified to 12, but accept $c=-3$ as the final answer.
		3	

9(b)	$\left[\frac{12}{x^2}-3=0 \Rightarrow x^2=4 \Rightarrow x=\pm 2 \Rightarrow\right] x=2$	**B1**	May be done by inspection or calculator, but must be WWW. Condone ± 2.
	$\left[\frac{d^2y}{dx^2}=\right]-\frac{24}{(2)^3}$ or -3 or $<0 \Rightarrow$ maximum	**B1**	WWW. Or other valid method. Ignore any working for $x=-2$.
		2	
9(c)	$[y=]-12x^{-1}+\textit{their } cx\left[=-\frac{12}{x}-3x\right][+d]$	**B1FT**	May be unsimplified. FT *their* nonzero *c*.
	$19=-\frac{12}{-2}-3\times(-2)+d \; [\Rightarrow d=7]$	**M1**	Substituting $(-2, 19)$ into their integrated expression (defined by having at least one correct power) and including $+d$.
	$y=-\frac{12}{x}-3x+7$	**A1**	Accept $d=7$ as final answer if $y=...$ seen previously. Allow $'f(x)='$ in place of $'y='$.
		3	
9(d)	$\frac{12}{x^2}-3=-\frac{9}{4} \Rightarrow \frac{12}{x^2}=\frac{3}{4} \Rightarrow x=...\; [\Rightarrow x^2=16 \Rightarrow x=4, y=-8]$	***M1**	Setting *their* $\frac{dy}{dx}$ of the form $kx^{-2}+c=-\frac{9}{4}$, and correctly solving for *x* from their equation where $x^2>0$.
	Gradient of normal = $\frac{4}{9}$	**B1**	
	$\frac{y+8}{(x-4)}=\frac{4}{9}$	**DM1**	For using *their* stated positive *x* and *their* stated *y* and a changed gradient to correctly form a line equation; condone $-\frac{4}{9}$ or $\frac{9}{4}$.
	$4x-9y-88=0$	**A1**	OE in correct form.
		4	

7.

Question	Answer	Marks	Guidance
1	$2+(12)(-2)x^{-3}$	**B1**	Correct differential but can be unsimplified.
	$2+(12)(-2)(-2)^{-3}\; [=5]$	***M1**	Substitute $x=-2$ into *their* differential, which must contain x^{-3}.
	Either $(\textit{their } 5)=\frac{y-(-1)}{x-(-2)}$ or $-1=(\textit{their } 5)\times(-2)+c \Rightarrow c=$	**DM1**	Attempt to find equation of tangent through $(-2, -1)$ with their numerical gradient obtained as described above.
	$y=5x+9$	**A1**	
		4	

8.

10(a)	$-9\times 2(2x-5)^{-2}+2$	B1	Correct differential.
	$\left(their\ -18(2x-5)^{-2}+2\right)=0$ and rearrange to form a quadratic. $\left[(2x-5)^2=9 \text{ or } 8x^2-40x+32=0\right]$	M1	Equating a two term $\frac{dy}{dx}$ to 0 and dealing correctly with the negative power. Their two term $\frac{dy}{dx}$ must contain $(2x-5)^{-2}$.
	$(1,\ -6)$ **and** $(4,6)$	A1, A1	A1 for two correct x-values or one correct point, second A1 for all correct.
		4	
10(b)	$-18\times -2\times 2(2x-5)^{-3}\ \left[=72(2x-5)^{-3} \text{ or } \frac{144x-360}{(2x-5)^4}\right]$	B1 FT	Following through on *their* first derivative which must contain $(2x-5)^{-2}$.
	Use $(their\ x=1$ and $x=4)$ in $(their\ 72(2x-5)^{-3})$ To determine the nature of both turning points.	M1	Substitute x-coordinate of each stationary point and determine their nature. Nature of the turning points must correctly follow from *their* values of $\frac{d^2y}{dx^2}$.
	For $x=1$, $\left[\frac{d^2y}{dx^2}=\right]\ -\frac{72}{27}$ or $<0 \Rightarrow$ maximum For $x=4$, $\left[\frac{d^2y}{dx^2}=\right]\ \frac{72}{27}$ or $>0 \Rightarrow$ minimum	A1	CWO
		3	

10(c)(i)	$(1,\ -13)$	B1	
		1	
10(c)(ii)	$[y=]\frac{9}{2(x\pm 3)-5}+2(x\pm 3)-5\pm 7$	M1	Application of $\begin{pmatrix}-3\\7\end{pmatrix}$ to the original expression for C but condone +/− sign errors.
	$[y=]-\left(\frac{9}{2(x\pm 3)-5}+2(x\pm 3)-5\pm 7\right)$	M1	**SC B1** for $[y=]-\left(\frac{9}{2x-5}+2x-5\right)$.
	$y=-\frac{9}{2x+1}-2x-8$	A1	Answer must be in this format; the '$y=$' can be implied by earlier inclusion.
		3	

9.

Question	Answer	Marks	Guidance
1	$[y=]\left\{\frac{12}{3\times 2}(2x-5)^3\right\}\left\{+\frac{8}{2}x^2\right\}[+c]$	B1 B1	OE Terms may be unsimplified. May see $[y=]\{16x^3\}\{-116x^2\}\{+300x\}[+c]$.
	$4=2\times(2\times 2-5)^3+4\times 2^2+c\ [\Rightarrow c=-10]$	M1	Sub (2, 4) correctly into *their* integrated expression with a '$+c$'.
	y or f or $f(x)=2(2x-5)^3+4x^2-10$	A1	May see $y=16x^3-116x^2+300x-260$. '$y=$' or 'f$(x)=$' or 'f $=$' can be implied if seen in working.
		4	

10.

Question	Answer	Marks	Guidance
5(a)	Attempt correct process for solving 3-term quadratic equation in $\sqrt{x}$	M1	Accept $8y^2 - 6y - 9 \rightarrow (2y - 3)(4y + 3)$, if $y = \sqrt{x}$ specified.
	Obtain at least $2\sqrt{x} - 3 = 0$ or equivalent	A1	Ignore $4\sqrt{x} + 3 = 0$. **SC B1** for $\sqrt{x} = \frac{3}{2}$ with no method shown for solving the 3-term quadratic.
	Conclude $x = \frac{9}{4}$ ignore $\frac{9}{16}$	A1	**SC B1** if no method shown for solving the 3-term quadratic.
	Alternative Method for Q5(a)		
	$3\sqrt{x} = 4x - \frac{9}{2} \rightarrow 16x^2 - 45x + \frac{81}{4}$ o.e and attempt correct process to solve	M1	
	Obtain $x = \frac{9}{4}$ or $\frac{9}{16}$	A1	**SC B1** if no method shown for solving the 3-term quadratic.
	$x = \frac{9}{4}$ ignore $\frac{9}{16}$	A1	**SC B1** if no method shown for solving the 3-term quadratic.
		3	

5(b)	Integrate to obtain form $k_1x^2 + k_2x^{\frac{3}{2}} + k_3x$ where $k_1k_2k_3 \neq 0$	M1	
	Obtain correct $2x^2 - 2x^{\frac{3}{2}} + x$ or equivalent	A1	Allow unsimplified.
	Substitute $x = 4$ and $y = 11$ to attempt value of c	M1	Dependent on at least 2 correct terms involving x.
	Obtain $y = 2x^2 - 2x^{\frac{3}{2}} + x - 9$	A1	Must be simplified. Allow 'f(x) ='. Allow y missing if y appears previously.
		4	

11.

Question	Answer	Marks	Guidance
5	Differentiate to obtain form $kx(2x^2 - 5)^{-2}$	M1	
	Obtain correct $-12x(2x^2 - 5)^{-2}$	A1	OE
	Substitute (2, 1) to obtain gradient $-\frac{24}{9}$	A1	OE e.g. $-\frac{8}{3}$. Allow -2.67.
	Apply negative reciprocal to *their* numerical gradient to obtain gradient of normal	*M1	Must have been some attempt at differentiation. Expect $\frac{3}{8}$
	Attempt equation of normal using *their* gradient of the normal and (2, 1)	DM1	Expect $y - 1 = \frac{3}{8}(x - 2)$.
	Obtain $3x - 8y + 2 = 0$ (allow multiples)	A1	Or equivalent of requested form e.g. $8y - 3x - 2 = 0$.
		6	

12.

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = -\frac{12}{x^4} + \frac{3}{x^2}$	B1	
	$\frac{dy}{dx} = -\frac{12}{x^4} + \frac{3}{x^2} = 0$ leading to $3x^4 - 12x^2 = 0$ or $-12 + 3x^2 = 0$	M1	Set = 0 or uses $<$, $\leqslant$ and simplifies. Must be from $\frac{dy}{dx} = \frac{A}{x^4} + \frac{B}{x^2}$.
	$3x^2(x^2 - 4) = 0$ leading to $x = \pm 2$ only	A1	**SC B1** for $x = \pm 2$ if M0 scored.
	$-2 < x < 0$ and $0 < x < 2$ or (-2, 0) and (0, 2) or $-2 < x < 2$ and $x \neq 0$	B1FT	Allow and/or.
		B1FT	Allow $-2 \leqslant x < 0$ and / or $0 < x \leqslant 2$ but only B1B0 if 0 included in either or both. Allow [–2, 0) and (0, 2]. Allow B1B0 for $-2 < x < 2$ or (–2, 2). Must be from $\frac{dy}{dx} = \frac{A}{x^4} + \frac{B}{x^2}$.
		5	B marks only available if $\frac{dy}{dx} = \frac{A}{x^4} + \frac{B}{x^2}$.
11(b)	[At $x = 1$] $y = 3$ and m tan $= -9$	*M1	Using *their* $\frac{dy}{dx}$.
	m norm $= -\frac{1}{-9} = \frac{1}{9}$	DM1	
	Equation of normal is $y - 3 = \frac{1}{9}(x - 1)$ $\left[\text{leading to } y = \frac{1}{9}x + \frac{26}{9}\right]$	A1	
	At $x = -1, y = 1, m = -9$	M1	
	Equation of tangent is $y - 1 = -9(x + 1)$ [leading to $y = -9x - 8$]	A1	
	Meet when $\frac{1}{9}x + \frac{26}{9} = -9x - 8$ $\left[\text{leading to } x = -1.19512\ldots, \frac{-49}{41}\right]$	M1	Equates *their* tangent and *their* normal.
	Area $= \frac{1}{2} \times$ *their* $1.19512\ldots \times$ *their* $\left(\frac{26}{9} + 8\right)$	M1	If $\int y_2 - y_1$ is used integration must be correct and substitution shown.
	6.51	A1	AWRT Accept fraction wrt 6.51
		8	

13.

Question	Answer	Marks	Guidance
10(a)	$\frac{dy}{dx} = 3x^2 \ [+c]$	B1	
	$3 \times 2^2 + c = 0$	M1	Substitute $x = 2$ and $\frac{dy}{dx} = 0$ into an integral (c must be present).
	$\frac{dy}{dx} = 3x^2 - 12$	A1	
		3	
10(b)	$y = x^3 - 12x \ [+k]$	B1 FT	FT on *their* non-zero c (dependent on c being found at some stage).
	$-10 = 2^3 - 12 \times 2 + k$	M1	Substitute $x = 2$, $y = -10$ (k present).
	$y = x^3 - 12x + 6$	A1	Must be $y =$ (unless $y = x^3 - 12x + k$ stated earlier).
		3	
10(c)	$3x^2 - 12 = 0$ [leading to $x = -2$]	M1	Set *their* two term $\frac{dy}{dx} = 0$. Expect $x = -2$. Ignore $x = 2$ given in addition.
	$y = (-2)^3 - 12 \times (-2) + 6 = 22$ leading to $(-2, 22)$	A1	
	When $x = -2$, $\frac{d^2y}{dx^2} < 0$ (or -12) hence Maximum	A1	Can be from correct conclusion from $\frac{dy}{dx}$ **sign** diagram if $\frac{dy}{dx}$ calculated correctly. Do not allow concave downward for final A1. Can be awarded if the only error is incorrect or missing y-coordinate.
		3	
10(d)	At $x = 0$, $\frac{dy}{dx} = -12$, $y = 6$	M1	Both required. FT on *their* $\frac{dy}{dx}$ and y.
	$y - 6 = -12x$	A1	OE
		2	

14.

Question	Answer	Marks	Guidance
3(a)	[Gradient of normal =] $\dfrac{-1}{\text{Their}\,\frac{11}{2}}$ $\left[\dfrac{-1}{\frac{11}{2}}=-\dfrac{2}{11}\right]$	M1	Tangent gradient must come from $x = 2$ substituted into the given expression.
	$\dfrac{y-8}{x-2}=-\dfrac{2}{11}$ or $11y+2x=92$ or $y=-\dfrac{2x}{11}+\dfrac{92}{11}$	A1	OE
		2	
3(b)	$[y=]\left\{\dfrac{1}{2}x^2\div 2\right\}\left\{+\dfrac{72}{x^3}\div -3\right\}$ $[+c]$ $\left[\dfrac{x^2}{4}-\dfrac{24}{x^3}+c\right]$	B1, B1	One mark for each correct unsimplified { }.
	$8=\dfrac{1}{4}\times 4-\dfrac{24}{8}+c$	M1	Substitution of $x = 2$, $y = 8$ into *their* integrated expression, defined by at least one correct power. Two terms and $+ c$ needed.
	$y=\left(\dfrac{1}{4}\text{ or }0.25\right)x^2-\dfrac{24}{x^3}+10$	A1	Both coefficients must be simplified but allow x^{-3}. Condone $c = 10$ as line as long as either y or $f(x) =$ is seen elsewhere.
		4	

15.

Question	Answer	Marks	Guidance
1	$[y]=\left\{\dfrac{x^2}{2}\right\}$ $\left\{\dfrac{-3x^{\frac{1}{2}}}{\frac{1}{2}}\right\}$ $[+c]$	B1 B1	Any unsimplified correct form, ISW for extra terms, allow lists.
	$1 = 8 - 12 + c$	M1	Substitute (into an integrated expression) $x = 4$, $y = 1$. c must be present. Expect $c = 5$.
	$y=\dfrac{1}{2}x^2-6x^{\frac{1}{2}}+5$, allow $f(x) =$	A1	Condone $c = 5$ as the final line so long as '$y =$' present.
		4	

16.

Question	Answer	Marks	Guidance
10(a)	$-\frac{3}{2}=\frac{1}{2}+k$ leading to $k=-2$	B1	**AG** Need to see $4^{-\frac{1}{2}}$ evaluated as $\frac{1}{4^{\frac{1}{2}}}$ or better.
		1	
10(b)	$[y]=2x^{\frac{1}{2}}-2x \quad [+c]$	M1 A1	Allow $\frac{x^{\frac{1}{2}}}{1/2}-2x$.
	$-1=4-8+c$	M1	Substitute $x=4$, $y=-1$ (c present) Expect $c=3$.
	$y=2x^{\frac{1}{2}}-2x+3$ or $y=2\sqrt{x}-2x+3$	A1	Allow if f(x) = or y = anywhere in the solution.
		4	
10(c)	$x^{-1/2}-2=0$	M1	Set *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ to zero.
	$x=\frac{1}{4}$	A1	If $\left(\frac{1}{2}\right)^2=\pm\frac{1}{4}$ max of M1A1 if $\left(\frac{1}{4},3\frac{1}{2}\right)$ seen.
	(¼, 3½)	A1	
		3	

Question	Answer	Marks	Guidance
10(d)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{1}{2}x^{-\frac{3}{2}}$	B1	
	<0 (or -4) hence Maximum	DB1	WWW Ignore extra solutions from $x=-\frac{1}{4}$.
		2	

17.

Question	Answer	Marks	Guidance
11(a)	$6a^2-30a+6a=0$ $[\Rightarrow 6a(a-4)=0]$	B1	Sub $x=a$ into $\frac{\mathrm{d}y}{\mathrm{d}x}=0$. May see $a^2-5a+a=0$.
	$a=4$ only	B1	
		2	

11(b)	$\frac{d^2y}{dx^2} = 12x - 30$ or correct values of $\frac{dy}{dx}$ either side of $x = 4$	M1	Differentiate $\frac{dy}{dx}$ (mult. by power or dec. power by 1) M0 if no values of $\frac{dy}{dx}$, only signs.
	At $x = 4, \frac{d^2y}{dx^2} > 0 \therefore$ minimum or $\frac{d^2y}{dx^2} = 18 \therefore$ minimum or concludes minimum from $\frac{dy}{dx}$ values	A1	WWW A0 XP if $a = 4$ obtained incorrectly in (a) Must see 'minimum'. If M0, **SC B1** for 'minimum' from $\frac{dy}{dx}$ sign diagram.
		2	
11(c)	$[y =]\ \frac{6}{3}x^3 - \frac{30}{2}x^2 + 6(\textit{their}\ a)x[+c]$	B1 FT	Expect $2x^3 - 15x^2 + 24x[+c]$. B1 poss. even if uses 'a' – no value in (a) – max 1/3.
	$-15 = 2(\textit{their}\ "4")^3 - 15(\textit{their}\ "4")^2 + 6(\textit{their}\ "4")^2 + c$	M1	Sub $x = \textit{their}\ "4"$, $y = -15$ into integral (must incl $+c$) Look for $-15 = 128 - 240 + 96 + c$ $[\Rightarrow c = 1]$.
	$y = 2x^3 - 15x^2 + 24x + 1$	A1	Coefficients must be correct and simplified. Need to see '$y =$' or '$f(x) =$' in the working.
		3	
11(d)	$\frac{dy}{dx} = 6x^2 - 30x + 6(\textit{their}\ "4")[=0]$ If correct, $[6](x-1)(x-4)[=0]$ or $\frac{30 \pm \sqrt{(-30)^2 - 4(6)(24)}}{12}$	M1	OE Forming a 3-term quadratic using the given $\frac{dy}{dx}$ and solving by factorisation, formula or completing the square. Check for working in **(b)**.
	Coordinates $(1,12)$	A1	Allow $x = 1, y = 12$ (ignore $x = 4$ if present). If M0, award **SC B1** for $(1,12)$.
		2	

18.

Question	Answer	Marks	Guidance
10(a)	$\left[\frac{dy}{dx} =\right]\{9\} + \left\{-\frac{3}{2}(2x+1)^{1/2} \times 2\right\}$	B1, B1	Including '+c' makes the second term B0.
	$9 - 3(2x+1)^{1/2} = 0$ leading to $2x + 1 = 9$	M1	Set differential to zero and solve by squaring SOI. Beware $9^2 - 3^2(2x+1) = 0$ M0A0. $2x+1 = \sqrt{3}\ or\ 2x+1 = \pm 9$ get M0.
	Max point = (4, 9)	A1	WWW y = 9 must come from original equation.
		4	
10(b)	When $x = 1½$, shows substitution or $\frac{dy}{dx} = 3$	M1	Substituting $x = 1½$ into their $\frac{dy}{dx}$.
	Gradient of AB is $\frac{5½ - 3½}{1½ - 7½}\left[= \frac{-1}{3}\right]$	M1	Substituting into a correct expression for m_{AB}.
	$-\frac{1}{3}x3 = -1$. [Hence AB is the normal]	A1	
	Alternative method for Question 10(b)		
	When $x = 1½$ $\frac{dy}{dx} = 3$,[perpendicular gradient is -1/3]	M1	
	Perpendicular through A has equation $y = \frac{-x}{3} + 6$ which contains B(7.5,3.5) leading to AB is a normal to the curve at A	M1 A1	
		3	

10(c)	$\left\{\frac{9x^2}{2}\right\}+\left\{\frac{-(2x+1)^{\frac{5}{2}}}{\frac{5}{2}\times 2}\right\}$	B1 B1	Integrating y with respect to x.
	$\left\{\frac{9}{2}7.5^2-\frac{1}{5}(2\times 7.5+1)^{2.5}\right\}-\left\{\frac{9}{2}1.5^2-\frac{1}{5}(2\times 1.5+1)^{2.5}\right\}$ or $\left(\frac{9}{2}\times\frac{225}{4}-\frac{1024}{5}\right)-\left(\frac{81}{8}-\frac{32}{5}\right)$ or $\frac{1933}{40}-\frac{149}{40}$ or $48.325-3.725$	M1	OE Apply limits 1½ to 7½ to an integral. Working must be seen. Expect 44.6 .
	$\frac{1}{2}\left(5\frac{1}{2}+3\frac{1}{2}\right)\times 6$ or $\int_{\frac{3}{2}}^{\frac{15}{2}}(\frac{-1}{3}x+6)\mathrm{d}x=$ $\left(\frac{-1}{6}\times\left(\frac{15}{2}\right)^2+6\times\frac{15}{2}\right)-\left(\frac{-1}{6}\times\left(\frac{3}{2}\right)^2+6\times\frac{3}{2}\right)$ or $\frac{285}{8}-\frac{69}{8}$ [= 27]	B1	SOI Area of trapezium. May be seen combined with the area under the curve integral.
	[Shaded area = 44.6 – 27 =] 17.6	A1	**SC B1** if no substitution of the limits seen.
		5	

10(c)	**Alternative method for Question 10(c)**		
	$A=\int_{\frac{3}{2}}^{\frac{15}{2}}((9x-(2x+1)^{\frac{3}{2}})-\left(\frac{-1}{3}x+6\right))\mathrm{d}x=\int_{\frac{3}{2}}^{\frac{15}{2}}((\frac{28}{3}x-(2x+1)^{\frac{3}{2}}-6)\mathrm{d}x$	M1	Finding the equation of AB and subtracting from the equation of the curve.
	$=\left\{\frac{28}{3\times 2}x^2-6x\right\}+\left\{\frac{-(2x+1)^{\frac{5}{2}}}{\frac{5}{2}\times 2}\right\}$	A1 A1	
	$\frac{127}{10}-\frac{-49}{10}$	M1	Apply limits 1½ to 7½ to an integral. Working must be seen.
	17.6	A1	**SC B1** if no substitution of limits seen.
		5	

19.

Question	Answer	Marks	Guidance
11(a)	$6a^2-30a+6a=0$ [$\Rightarrow 6a(a-4)=0$]	B1	Sub $x=a$ into $\frac{\mathrm{d}y}{\mathrm{d}x}=0$. May see $a^2-5a+a=0$.
	$a=4$ only	B1	
		2	

Question	Answer	Marks	Guidance
11(b)	$\frac{d^2y}{dx^2} = 12x - 30$ or correct values of $\frac{dy}{dx}$ either side of $x = 4$	M1	Differentiate $\frac{dy}{dx}$ (mult. by power or dec. power by 1) M0 if no values of $\frac{dy}{dx}$, only signs.
	At $x = 4, \frac{d^2y}{dx^2} > 0 \therefore$ minimum or $\frac{d^2y}{dx^2} = 18 \therefore$ minimum or concludes minimum from $\frac{dy}{dx}$ values	A1	WWW A0 XP if $a = 4$ obtained incorrectly in (a) Must see 'minimum'. If M0, **SC B1** for 'minimum' from $\frac{dy}{dx}$ sign diagram.
		2	
11(c)	$[y =]\ \frac{6}{3}x^3 - \frac{30}{2}x^2 + 6(\textit{their } a)x[+c]$	B1 FT	Expect $2x^3 - 15x^2 + 24x[+c]$. B1 poss. even if uses 'a' – no value in (a) – max 1/3.
	$-15 = 2(\textit{their}\ "4")^3 - 15(\textit{their}\ "4")^2 + 6(\textit{their}\ "4")^2 + c$	M1	Sub $x = \textit{their}\ "4"$, $y = -15$ into integral (must incl $+c$) Look for $-15 = 128 - 240 + 96 + c$ [$\Rightarrow c = 1$].
	$y = 2x^3 - 15x^2 + 24x + 1$	A1	Coefficients must be correct and simplified. Need to see '$y =$' or '$f(x) =$' in the working.
		3	
11(d)	$\frac{dy}{dx} = 6x^2 - 30x + 6(\textit{their}\ "4")[=0]$ If correct, $[6](x-1)(x-4)[=0]$ or $\frac{30 \pm \sqrt{(-30)^2 - 4(6)(24)}}{12}$	M1	OE Forming a 3-term quadratic using the given $\frac{dy}{dx}$ and solving by factorisation, formula or completing the square. Check for working in **(b)**.
	Coordinates $(1,12)$	A1	Allow $x = 1, y = 12$ (ignore $x = 4$ if present). If M0, award **SC B1** for $(1,12)$.
		2	

20.

Question	Answer	Marks	Guidance
9(a)	$[y =]\ \{x\}\ \{+(x-1)^{-2}\}\ \ [+c]$	B1 B1	May be unsimplified.
	Sub $x = 0$, $y = 3$ leading to $3 = 0 + 1 + c$	M1	Substitution into an integral, expect $c = 2$.
	$y = x + (x-1)^{-2} + 2$ or $f(x) = x + (x-1)^{-2} + 2$	A1	$\frac{-2}{(-2)(x-1)^2}$ or $\frac{-2(x-1)^{-2}}{-2}$ must be simplified.
		4	
9(b)	[Gradient of tangent =] $f'(0) = 3$	B1	
	Equation of tangent is $y - 3 = \textit{their}$ gradient at $x = 0(x - 0)$	M1*	Expect $y = 3x + 3$, normal gets M0.
	Intersection given by $3x + 3 = x + (x-1)^{-2} + 2$	DM1	FT *their* equation from part **(a)**.
	$2x + 1 = \frac{1}{(x-1)^2} \rightarrow (2x+1)(x-1)^2 - 1 = 0$ or solve equation before given form reached and show solution ($x = 3/2$) satisfies given result	A1	WWW AG
		4	

9(c)	Substitute $x=\frac{3}{2}$ leading to $(2x+1)(x-1)^2-1$ leading to $4\times¼-1=0$. Hence $x=\frac{3}{2}$ If shown in **(b)** must be referenced here (in part **(c)**)	**B1**	Evaluation of each bracket must be shown. Allow $\left(\frac{1}{2}\right)^2$ for second bracket. Solution of $(2x+1)(x-1)^2-1=0$ is acceptable.
	When $x=\frac{3}{2}$ $y=7½$	**B1**	
		2	

21.

Question	Answer	Marks	Guidance
2(a)	$12\left(\frac{1}{2}\times6-1\right)^{-4}\left[=12(2)^{-4}=\frac{3}{4}\right]$	**M1**	Substitute $x=6$ into $\frac{dy}{dx}$ SOI by gradient $\frac{3}{4}$ used.
	$y-4=\frac{3}{4}(x-6)$ OR evaluates $c=-\frac{1}{2}$	**A1**	OE e.g. $y=\frac{3}{4}x-\frac{1}{2}$ or evaluates c in $y=\frac{3}{4}x+c$ using (6, 4) and gradient $\frac{3}{4}$. ISW
		2	

2(b)	$[y=]\left(\frac{12\left(\frac{1}{2}x-1\right)^{-3}}{-3}\right)\div\frac{1}{2}\left[=-8\left(\frac{1}{2}x-1\right)^{-3}\right]$	**B2, 1, 0**	
	$4=\frac{12\times\left(\frac{1}{2}\times6-1\right)^{-3}}{\frac{1}{2}\times-3}+c\left[\Rightarrow 4=-8\times2^{-3}+c\right]\Rightarrow c=[5]$	**M1**	Must have $+c$. Substitute $y=4$, $x=6$ and solve for c in an integrated expression. May be unsimplified.
	$[y=]-8\left(\frac{1}{2}x-1\right)^{-3}+5$	**A1**	OE Must see '$y=$' or '$\mathrm{f}(x)=$' in the working.
		4	