

1. Matrices and Transformations

1. a) (i) On the grid provided, with the same scale on both axes, draw the square S whose vertices are (0, 0), (2, 0), (2,2) and (0, 2). (1 mk)

(ii) Find the coordinates and draw the image T of S under the transformation whose matrix A maps a point (x, y) onto (x', y')

where; $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2x - y \\ x + 2y \end{pmatrix}$ (3 mks)

(iii) Draw the image U of S under the transformation whose matrix is

$B = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$ (2 mks)

(b) (i) Find the product AB and draw the image V of S under the transformation whose matrix is AB (3 mks)

(ii) Describe the single transformation that maps S onto V (1 mk)

2. On the grid provided, draw triangle PQR with P(2,3), Q(1,2) and R(4,1). On the same axes draw triangle $P^{11}Q^{11}R^{11}$ with vertices $P^{11}(-2,3)$, $Q^{11}(-1,2)$ and $R^{11}(-4,1)$. (2mks)

(a) Describe fully a single transformation which will map triangle PQR onto triangle $P^{11}Q^{11}R^{11}$. (1mk)

(b) On the same plane, draw triangle $P^1Q^1R^1$ the image of triangle PQR under reflection in the line $y = -x$. (2mks)

(c) Describe fully a single transformation which maps triangle $P^1Q^1R^1$ onto triangle $P^{11}Q^{11}R^{11}$. (2mks)

(d) Draw triangle $P^{111}Q^{111}R^{111}$ such that it can be mapped onto triangle PQR by a position quarter about (0,0) (2mks)

(e) State all pairs of triangles that are oppositely congruent. (1mk)

3. a) Given the transformation matrices

$T_1 = \begin{bmatrix} & \\ & \end{bmatrix}$ and $T_2 = \begin{bmatrix} & \\ & \end{bmatrix}$

and that transformation T_1 followed by T_2 can be replaced by a single transformation T, write down the matrix for T. (3 marks)

a) Find the inverse of matrix T (2 marks)

- b) The points $A^{11}(7,-11)$, $B^{11}(-7,-13)$, $C^{11}(-8,16)$ and $D^{11}(8,8)$ are the images of points A, B, C and D respectively under transformation T_1 followed by T_2

- Write down the co-ordinates of A, B, C, and D. (5 marks)
4. A(3, 7), B(5, 5), C(3, 1), D(1, 5)
- a) On the grid provided in the next page, plot ABCD on a Cartesian plane (2mks)
- b) A'B'C'D' is the image of ABCD under a translation $T \begin{pmatrix} -6 \\ -9 \end{pmatrix}$. Plot A'B'C'D' and state its coordinates. (2mks)
- c) Plot A''B''C''D'', the image of A'B'C'D' after a rotation about (-1, 0) through a positive quarter turn. State its coordinates. (3mks)
- d) A'''B'''C'''D''' is the image of A''B''C''D'' after a reflection in the line $y=x+2$. Plot A'''B'''C'''D''' and state its coordinates (3 mks)
5. A transformation represented by the matrix $\begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$ maps P(0,0), Q(2,0), R(2,3) and S(0,3) onto P', Q', R', S'
- a) On the grid provided draw the quadrilateral PQRS and P'Q'R'S' (4mks)
- b) (i) Determine the area of PQRS (1mk)
- (ii) Hence or otherwise find the area of P'Q'R'S' (2mks)
- c) A transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ maps P'Q'R'S' onto P''Q''R''S''. Determine the matrix of transformation that would map P''Q''R''S'' onto PQRS (3mks)
6. A translator T maps P (8, -2) onto P¹ (-2, -3). Find the image of Q (6, -2) under the same translation. (3 mks)
7. The vertices of a triangle are A(2,5), B(4,3) and C(2,3). H represents a half turn rotation about the point (0,2).
- a) Draw triangle ABC and A', B', C' under H (4 marks)
- b) T represents a reflection in the line $x=0$ and K represent a translation $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$. Find the coordinates of A'', B'', C'' of A', B', C' under TK. Hence draw A'', B'', C'' (4 marks)
- c) Describe a single transformation that maps ABC onto A'', B'', C'' (2 marks)
8. Given triangle ABC with vertices A (-6, 5), B(-4, 1) and C(3, 2) and that A(-6, 5) is mapped onto A¹(-6, -4) by a shear with y-axis in variant. On the grid provided below;
- (i) draw triangle ABC
- (ii) draw triangle A¹B¹C¹, the image of triangle ABC, under the shear
- (iii) determine the matrix representing the shear
- (b) Triangle A¹B¹C¹ is mapped onto A¹¹B¹¹C¹¹ by a transformation defined by the matrix $\begin{pmatrix} & \\ & \end{pmatrix}$
- (i) Draw triangle A¹¹B¹¹C¹¹ on the same grid as ABC and A¹B¹C¹
- (ii) Describe fully a single transformation that maps A¹¹B¹¹C¹¹

9. (a) Under a certain rotation $A(2,0)$ is mapped onto $A^1(-4, 2)$ and $B(0,5)$ is mapped onto $B^1(-9, 0)$
 (i) On the grid provided plot the lines AB and A^1B^1 on the same axes
 (ii) Hence determine by construction the co-ordinates of the centre and angle of rotation
 (b) Under a quarter positive turn about the origin O , A^1 is mapped onto A^{11} and B^1 is mapped onto B^{11} . Determine the co-ordinates of A^{11} and B^{11}
 (c) Describe fully a single transformation which would map A to A^{11} and B to B^{11}
10. A transformation T is represented by the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and transformation $U \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ by the matrix. Given that a rectangle has co-ordinates at $A(1,2)$, $B(6, 2)$, $C(6, 4)$ and $D(1, 4)$ and that under T the image of $ABCD$ is $A_1B_1C_1D_1$ and under U the image of $A_1B_1C_1D_1$ is $A_2B_2C_2D_2$:
 (a) Find the co-ordinates of $A_1B_1C_1D_1$ and $A_2B_2C_2D_2$
 (b) On the grid provided, plot $ABCD$, $A_1B_1C_1D_1$ and $A_2B_2C_2D_2$
 (c) Describe the transformation represented by:-
 (i) U
 (ii) UT
 (d) If $A_2B_2C_2D_2$ were to be transformed by a transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ to map onto $A_3B_3C_3D_3$. What would be the area of $A_3B_3C_3D_3$
11. The vertices of a quadrilateral are $A(2,2)$, $B(8,2)$, $C(8,6)$ and $D(6,4)$ under a rotation the images of vertices A and D are $A(0,8)$ and $D^1(-2, 12)$.
 (a) On the grid provided and using the same axes draw the quadrilateral $ABCD$ and the points A^1 and D^1
 (b) Determine the centre and angle of rotation
 (c) Locate the points B^1 and C^1 under the rotation and complete the quadrilateral
12. A translation maps the point $P(5, -3)$ onto $P^1(2, -5)$
 (a) Determine the translation vector T
 A Point R^1 is the image of $R(-2, -3)$ under the same translation in (a) above, find the magnitude of P^1R^1 (b)
13. Triangle ABC has vertices at $A(0, -1)$, $B(4, 3)$ and $C(2,2)$.
 (a) Find the coordinates of image triangle $A^1B^1C^1$ of triangle ABC under translation $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$
 (b) Given that triangle $A^{11}B^{11}C^{11}$ is the image of triangle $A^1B^1C^1$ under an enlargement scale factor 3, centre $O(0,0)$, find the coordinates of A^{11} , B^{11} and C^{11}
 (c) If the area of triangle $A^1B^1C^1$ is 24 cm^2 , calculate the area of triangle $A^{11}B^{11}C^{11}$
 (d) Find the matrix that maps triangle $A^{11}B^{11}C^{11}$ onto triangle ABC
14. a) The triangle ABC where $A(2,-1)$, $B(1, 2)$ and $C(4, 4)$ is reflected in the line $X = 4$ to give triangle $A_1B_1C_1$. Draw the two triangles on the graph provided and state the co-ordinates of $A_1B_1C_1$
 b) Draw the triangle $A_2(5,6)$, $B_2(2,7)$ and $C_2(0,4)$. Given that triangle $A_2B_2C_2$ is the image of triangle $A_1B_1C_1$ under rotation, determine the centre and angle of this rotation
 c) Show the image of triangle $A_2B_2C_2$ under an enlargement centre $(0, 6)$ scale factor -1
15. (a) Find the co-ordinates for the image of point $P(6, -2)$ under the transformation defined by :-
 $x^1 = x - 3y$
 $y^1 = 2x$
 (b) (i) A quadrilateral $ABCD$ has vertices $A(4, -3)$, $B(2, -3)$, $C(4, -1)$ and $D(5, -4)$. On the grid provided, draw the quadrilateral $ABCD$

(ii) $A^1B^1C^1D^1$ is the image of ABCD under a rotation through $+90^\circ$ about the origin.

On the same axes, draw $A^1B^1C^1D^1$ under the transformation

(c) $A^2B^2C^2D^2$ is the image of $A^1B^1C^1D^1$ under another transformation by the matrix $\begin{pmatrix} & \\ & \end{pmatrix}$

(i) Determine the co-ordinates of $A^2B^2C^2D^2$ and plot it on the same axes

(ii) Describe the transformation that maps $A^1B^1C^1D^1$ onto $A^2B^2C^2D^2$

(d) Find a single matrix of transformation that would map $A^2B^2C^2D^2$ onto ABCD

16. (a) Triangle XYZ has vertices X(2, -1) Y(4, -1) and Z(4,2). Triangle XYZ maps onto triangle $X^1Y^1Z^1$ under transformation $T_1 = \begin{pmatrix} & \\ & \end{pmatrix}$. Draw triangles XYZ and its image $X^1Y^1Z^1$ on the grid provided

(b) Another triangle $X^{11}Y^{11}Z^{11}$ is the image of $X^1Y^1Z^1$ after transformation $T_2 = \begin{pmatrix} & \\ & \end{pmatrix}$. Draw triangle $X^{11}Y^{11}Z^{11}$ on the same set of axes

(c) Find the single transformation matrix T that maps triangle XYZ on to the final image $X^{11}Y^{11}Z^{11}$

(d) Given that the area of triangle XYZ is 15cm^2 , find the area of the triangle $X^{11}Y^{11}Z^{11}$

17. The quadrilateral A (2,1), B (4,1), C (4,4) and D (2,4) is mapped onto $A^1B^1C^1D^1$ by a matrix M_1 such that A^1 (8,7), B^1 (14,7), C^1 (14,16) and D^1 (8,16).

a) Draw both ABCD and $A^1B^1C^1D^1$ on the same plane

b) Find the matrix of transformation that mapped ABCD onto $A^1B^1C^1D^1$ and describe it fully

c) $A^1B^1C^1D^1$ underwent another matrix transformation at N which is a translation that gave the image $A^{11}B^{11}C^{11}D^{11}$, Where A^{11} (7,9), B^{11} (13,9), C^{11} (13,18) and D^{11} (7,18).

The transformation N is a translation. Find the translation

d) Draw $A^{11}B^{11}C^{11}D^{11}$ on the same axes where ABCD and $A^1B^1C^1D^1$ were drawn

18. a) On the grid provided. Plot the points A(2, -1) B(0, -3) C(2, -4) and D(4, -2) and join them to form a quadrilateral ABCD. What is the name of this quadrilateral?

b) The points A^1 (1, 2) B^1 (3, 0) C^1 (4, 2) and D^1 (2, 4) are the images of A, B, C and D under a certain transformation T_1 . On the same grid draw quadrilateral $A^1B^1C^1D^1$ and describe transformation T_1 fully.

c) The points A^{11} (-2, -4) B^{11} (-6, 0) C^{11} (-8, -4) and D^{11} (-4, -8) are the images of $A^1B^1C^1D^1$ under transformation T_2 . On the same grid draw quadrilateral $A^{11}B^{11}C^{11}D^{11}$ and describe the transformation T_2 fully.

d) On the same grid draw quadrilateral $A^{111}B^{111}C^{111}D^{111}$, the image of $A^{11}B^{11}C^{11}D^{11}$ under a reflection in the x-axis. State the co-ordinates of $A^{111}B^{111}C^{111}D^{111}$.

19. The Points A^1B^1 and C^1 are the images of A(4, 1), B(0, -2) and C(-2, 4) respectively under a transformation represented by the matrix;

$$M = \begin{pmatrix} & \\ & \end{pmatrix}$$

(a) Write down the coordinates of A^1B^1 and C^1

(b) $A^{11}B^{11}$ and C^{11} are the images of A^1B^1 and C^1 under another transformation whose Matrix is

$$N = \begin{pmatrix} & \\ & \end{pmatrix} \quad \begin{matrix} 2 & -1 \\ & \end{matrix}$$

Write down the coordinates of $A^{11}B^{11}$ and C^{11}

(c) Transformation M followed by N can be represented by a single transformation P. Determine the matrix for P

$$\begin{pmatrix} & \\ & \end{pmatrix}$$

(d) A matrix **P** is given by $\begin{pmatrix} 8 & 7 \\ 4 & 5 \end{pmatrix}$

Find P^{-1}

20. Triangle $A^1B^1C^1$ is the image of triangle ABC under a transformation represented by matrix $T = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. If the area of triangle $A^1B^1C^1$ is 25.6cm^2 , find the area of the object

21. A point P(2, -4) is mapped into $P^1(4, 0)$ under a translation.
Determine the image of point Q(-1, 2) under the same translation

22. The points A (2, 6), B (1, 1), C (2, 3) and D (4,0) are the vertices of quadrilateral ABCD.
(a) On graph paper plot the points A, B, C, and D and join them to form quadrilateral ABCD.

(b) The points A, B, C and D are the images of A^1 , B^1 , C^1 and D^1 respectively under an enlargement centre the origin and scale factor -2. On the same grid draw the image quadrilateral $A^1B^1C^1D^1$.

(c) The points A^{11} , B^{11} , C^{11} and D^{11} are the images of ABCD respectively under reflection in the x – axis. On the same grid, locate the points A^{11} , B^{11} , C^{11} and D^{11} and draw the second image quadrilateral $A^{11}B^{11}C^{11}D^{11}$.

(d) Quadrilateral $A^{111}B^{111}C^{111}D^{111}$ is the image of ABCD under a certain transformation T.
Describe transformation T fully.

$$\begin{pmatrix} 5x & 2 \\ x & -3 \end{pmatrix}$$

23. T is a transformation represented by the matrix $\begin{pmatrix} 5x & 2 \\ x & -3 \end{pmatrix}$. Under T, a square of area 10cm^2 is mapped onto a square 110cm^2 . Find the values of x