

What studying mathematics has taught me about the impossible.

by HugoCardozaHC@

magic. I studied two careers just to pursue my love for maths. I have not escaped from my attraction to understand them. I choose a classic problem that is going to help me explain some lessons that I have learned flirting with mathematical knowledge.

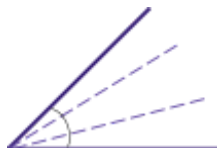
“Squaring a circle” is one of the most popular challenges in mathematics. Throughout history, it has seduced the mind of great mathematicians. It is the inspiration for art. The Vitruvian man from Da Vinci, plays with the concepts of the divine, a circle, and the mundane, a square. It explores how a human can be placed inside each of these terrains. For me, the problem is the culprit of countless minutes watching YouTube.

If it seems impossible, check your constraints

The problem of squaring a circle goes as follows: given a circle of a certain area, draw a square with the same area. Even an elementary student could answer this problem using modern algebra. We all know that creature called pi, π . But, you have to remember that algebra and pi are relatively new. Back in the day, they had fewer tools.

We now know that the problem is impossible using Euclidean Geometry where only a compass and a straightedge (an unmarked ruler) can be used. It has been proved in the 19th century that under this set of rules we can not access a certain family of numbers that are needed to perform the task.

A similar problem that is impossible using Euclidean geometry is trisecting an angle. This is splitting an angle into three equal parts. Funny enough, doing this with origami makes the problem quite possible. Adding the ability to fold the plane allows the geometry to reach the cubic root needed to solve this challenge.



So you might need to check the constraints or set of rules if you aim to reach new places.

Bias is one of the biggest constraints of the mind

You might be familiar with the problem of having a matrix of 9 dots and being asked to connect all of them using only 4 connected straight lines, in other words, without lifting the pen -or finger- connect 9 dots in a 3x3 table



Give it a try if you don't know the problem or try to remember if you do. The solution is thinking outside the box. An invisible box that our mind has placed.

Mathematics is a great place to learn modesty. The chances that we might be wrong are high. Think of the square root of 4. We can rapidly say the answer is 2, but we usually forget -2 is also part of the correct answer.

Error tolerance

If you recall your (hopefully fun) math classes, you know that the area of a circle is π times the radius time radius squared or πr^2 .

So if we have a circle with Area $A = \pi r^2$. We just need to build a rectangle with the same length so $L \times L = A$. so $L^2 = \pi r^2$. Then L has to be r times $\sqrt{\pi}$

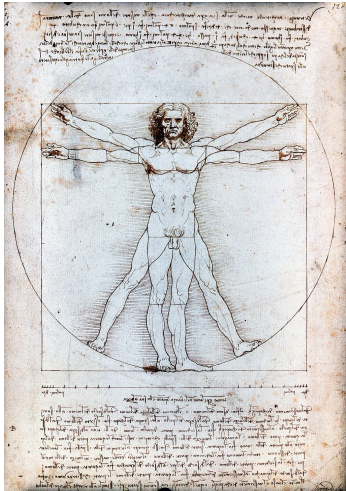
//circle-square animation here

But π and the $\sqrt{\pi}$ are numbers with infinite decimal points. **How can we even represent π in a computer?** This was a question that took me multiple courses of computer and mathematics in college to fully be able to answer it. Let me try to give you a short answer: discretization error. Given any small number, think 0.0001, there is a method to approximate π so that my Pi-Approximation (π^*) is smaller than the number you gave me. We usually call this small number epsilon.

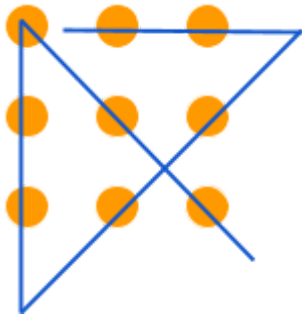
$$\pi^* - \pi < \epsilon$$

Even with our current state of maths it is impossible to fully accurately square a circle. There will always be a numerical error. But we can go as deep as needed. As mathematicians we are okay with this. We understand that precision comes at a cost and therefore we tolerate the always-present error as long. And as we are aware of it, we might be able to control it.

Images and material for the post



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- I created a circle to square animation to include in the article
<https://codepen.io/hugomosh/pen/qBjMjgr>



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