A IAL Pure Maths 4
Integration
QP

1.Jan 2025 question 1

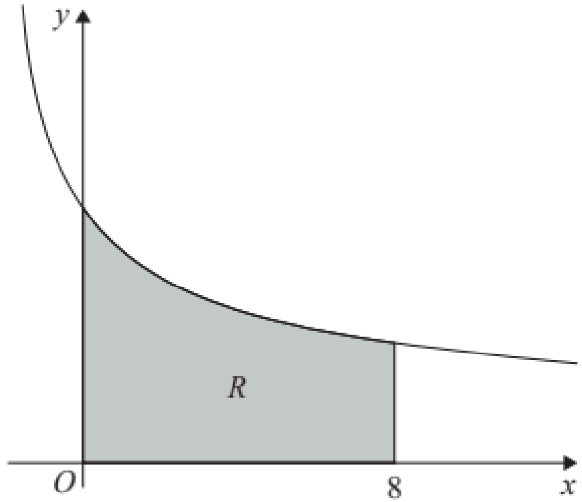

Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The curve shown in Figure 1 has equation

$$y = \frac{4}{x+2} \qquad x > -2$$

The region R, bounded by the curve, the y-axis, the x-axis and the line with equation $x = 8$ is shown shaded in Figure 1

Region R is rotated through 360 degrees about the x-axis.

Use calculus to find the exact value of the volume of the solid generated, writing your answer in simplest form.

(5)

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2.Jan 2025

5. (i) Find

$$\int x^2 e^{4x}\,dx$$

writing the answer in simplest form.

(4)

(ii) Use partial fractions and algebraic integration to show that

$$\int_4^7 \frac{2x+11}{(2x+1)(2-x)}\,dx = \ln k$$

where k is a fully simplified rational constant to be found.

(6)

3.Jan 2025

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Use the substitution $x = 4\sin\theta$ to find the exact value of

$$\int_{2}^{2\sqrt{3}} \frac{1}{\left(16 - x^2\right)^{\frac{3}{2}}}\,dx$$

(6)

4.Jan 2024

2. Given that

$$\frac{3x+4}{(x-2)(2x+1)^2} \equiv \frac{A}{x-2} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$$

(b) Hence find the exact value of

$$\int_7^{12} \frac{3x+4}{(x-2)(2x+1)^2}\,dx$$

giving your answer in the form $p\ln q + r$ where p, q and r are rational numbers.

(6)

5.Jan 2024

5. (a) Find $\int x^2 \cos 2x \, dx$

(4)

(b) Hence solve the differential equation

$$\frac{dy}{dt} = \left(\frac{t \cos t}{y}\right)^2$$

giving your answer in the form $y^n = f(t)$ where n is an integer.

(5)

6.Jan 2024

7. (a) Using the substitution $u = 4x + 2\sin 2x$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x}\cos^2 x\,dx = \frac{1}{8}(e^{2\pi} - 1)$$

(5)

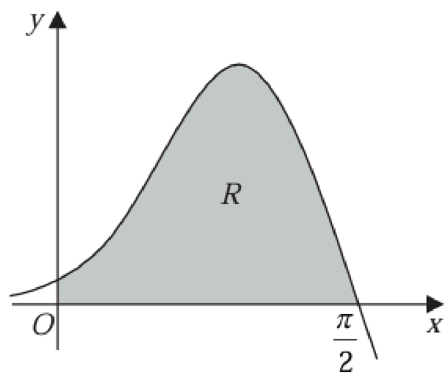

Figure 3

The curve shown in Figure 3, has equation

$$y = 6e^{2x+\sin 2x}\cos x$$

The region R, shown shaded in Figure 3, is bounded by the positive x-axis, the positive y-axis and the curve.

The region R is rotated through 2π radians about the x-axis to form a solid.

(b) Use the answer to part (a) to find the volume of the solid formed, giving the answer in simplest form.

(3)

7.June 2024

1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Find

$$\int_0^{\frac{\pi}{6}} x\cos 3x \, dx$$

giving your answer in simplest form.

(5)

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8.June 2024

5.

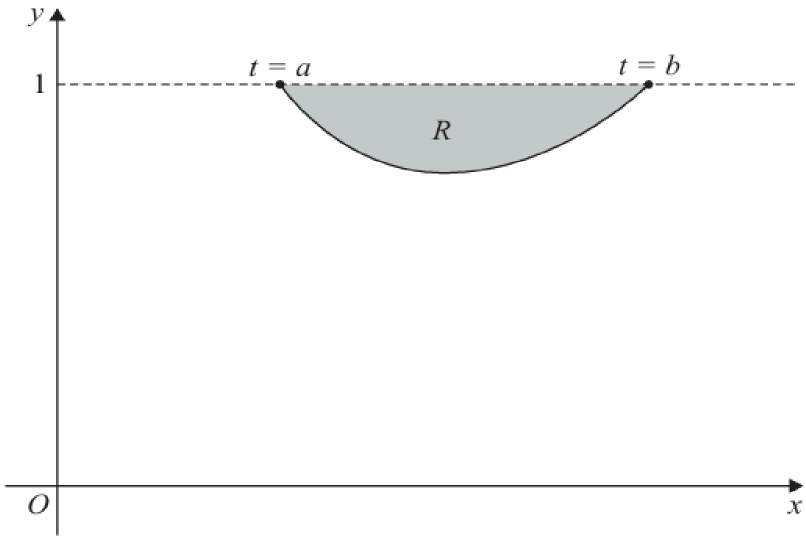

Figure 2

Figure 2 shows a sketch of the curve defined by the parametric equations

$$x = t^2 + 2t \qquad y = \frac{2}{t(3-t)} \qquad a \leqslant t \leqslant b$$

where a and b are constants.

The ends of the curve lie on the line with equation $y = 1$

(a) Find the value of a and the value of b

(2)

The region R, shown shaded in Figure 2, is bounded by the curve and the line with equation $y = 1$

(b) Show that the area of region R is given by

$$M - k\int_a^b \frac{t+1}{t(3-t)}\,\mathrm{d}t$$

where M and k are constants to be found.

(5)

(c) (i) Write $\frac{t+1}{t(3-t)}$ in partial fractions.

(ii) Use algebraic integration to find the exact area of R, giving your answer in simplest form.

(6)

9.June 2024

8. $$f(x) = (8 - 3x)^{\frac{4}{3}} \qquad 0 < x < \frac{8}{3}$$

(a) Show that the binomial expansion of f(x) in ascending powers of x up to and including the term in x^3 is

$$A - 8x + \frac{x^2}{2} + Bx^3 + \ldots$$

where A and B are constants to be found.

(4)

(b) Use proof by contradiction to prove that the curve with equation

$$y = 8 + 8x - \frac{15}{2}x^2$$

does **not** intersect the curve with equation

$$y = A - 8x + \frac{x^2}{2} + Bx^3 \qquad 0 < x < \frac{8}{3}$$

where A and B are the constants found in part (a).

(*Solutions relying on calculator technology are not acceptable.*)

(4)

10.June 2024

9.

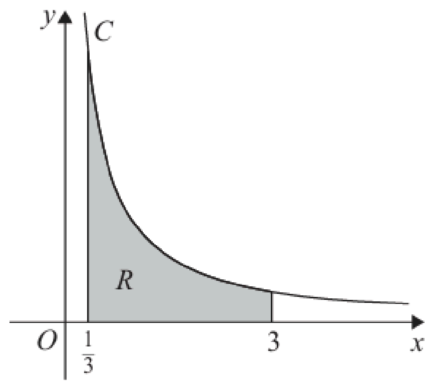

Figure 3

The curve C, shown in Figure 3, has equation

$$y = \frac{x^{-\frac{1}{4}}}{\sqrt{1+x}\left(\arctan\sqrt{x}\right)}$$

The region R, shown shaded in Figure 3, is bounded by C, the line with equation $x = 3$, the x-axis and the line with equation $x = \frac{1}{3}$

The region R is rotated through 360° about the x-axis to form a solid.

Using the substitution $\tan u = \sqrt{x}$

(a) show that the volume V of the solid formed is given by

$$k\int_a^b \frac{1}{u^2}\,\mathrm{d}u$$

where k, a and b are constants to be found.

(6)

(b) Hence, using algebraic integration, find the value of V in simplest form.

(3)

11.Oct 2024

6. Use the substitution $u = \sqrt{x^3 + 1}$ to show that

$$\int \frac{9x^5}{\sqrt{x^3 + 1}} \, dx = 2\left(x^3 + 1\right)^k \left(x^3 - A\right) + c$$

where k and A are constants to be found and c is an arbitrary constant.

(5)

12.Oct 2024

7.

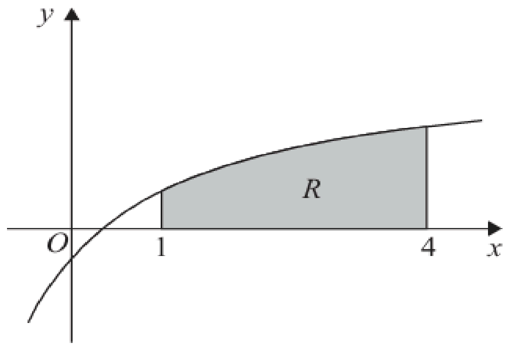

Figure 4

Figure 4 shows a sketch of part of the curve with equation

$$y = \frac{3x - 1}{x + 2} \qquad x > -2$$

The finite region R, shown shaded in Figure 4, is bounded by the curve, the line with equation $x = 4$, the x-axis and the line with equation $x = 1$

This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Use the answer to part (a) and algebraic integration to find the exact volume of the solid generated, giving your answer in the form

$$\pi(p + q\ln 2)$$

where p and q are rational constants.

(6)

13.Oct 2024

10.

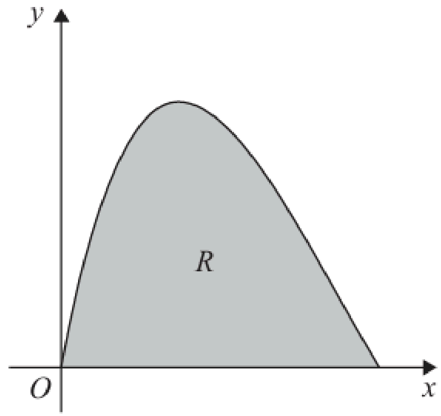

Figure 5

Figure 5 shows a sketch of the curve with parametric equations

$$x = 3t^2 \qquad y = \sin t \sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 5, is bounded by the curve and the x-axis.

(a) Show that the area of R is

$$k\int_0^{\frac{\pi}{2}} t\sin^2 t\cos t \,\mathrm{d}t$$

where k is a constant to be found.

(3)

(b) Hence, using algebraic integration, find the exact area of R, giving your answer in the form

$$p\pi + q$$

where p and q are constants.

(5)

14.Jan 2023

3.

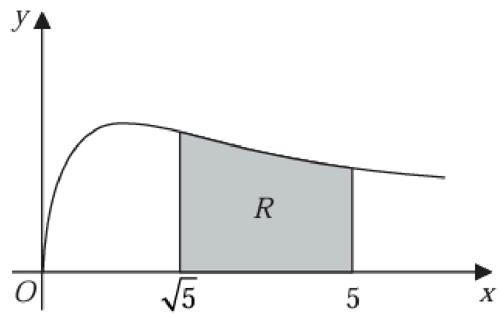

Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation

$$y = \sqrt{\frac{3x}{3x^2 + 5}} \qquad x \geqslant 0$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines with equations $x = \sqrt{5}$ and $x = 5$

The region R is rotated through 360° about the x-axis.

Use integration to find the exact volume of the solid generated. Give your answer in the form $a \ln b$, where a is an irrational number and b is a prime number.

(5)

15.Jan 2023

4. (a) Using the substitution $u = \sqrt{2x+1}$, show that

$$\int_{4}^{12} \sqrt{8x+4}\ \mathrm{e}^{\sqrt{2x+1}}\ \mathrm{d}x$$

may be expressed in the form

$$\int_{a}^{b} ku^2\mathrm{e}^{u}\ \mathrm{d}u$$

where a, b and k are constants to be found.

(4)

(b) Hence find, by algebraic integration, the exact value of

$$\int_{4}^{12} \sqrt{8x+4}\ \mathrm{e}^{\sqrt{2x+1}}\ \mathrm{d}x$$

giving your answer in simplest form.

(5)

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16.Jan 2023

8.

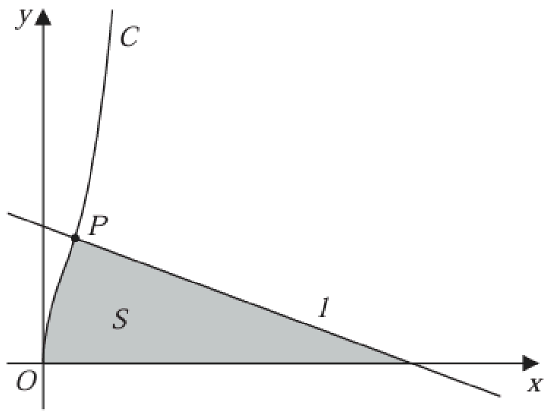

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = \sin^2 t \qquad y = 2\tan t \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P with parameter $t = \frac{\pi}{4}$ lies on C.

The line l is the normal to C at P, as shown in Figure 3.

(a) Show, using calculus, that an equation for l is

$$8y + 2x = 17$$

(5)

The region S, shown shaded in Figure 3, is bounded by C, l and the x-axis.

(b) Find, using calculus, the exact area of S.

(6)

17.June 2023

3. $$f(x) = \frac{8x - 5}{(2x - 1)(4x - 3)} \qquad x > 1$$

(a) Express $f(x)$ in partial fractions.

(3)

(b) Hence find $\int f(x)\,dx$

(3)

(c) Use the answer to part (b) to find the value of k for which

$$\int_{k}^{3k} f(x)\,dx = \frac{1}{2}\ln 20$$

(5)

18.June 2023

5. (i) Find

$$\int x^2 e^x \, dx$$

(4)

(ii) Use the substitution $u = \sqrt{1 - 3x}$ to show that

$$\int \frac{27x}{\sqrt{1-3x}} \, dx = -2(1-3x)^{\frac{1}{2}}(Ax + B) + k$$

where A and B are integers to be found and k is an arbitrary constant.

(6)

19.June 2023

8.

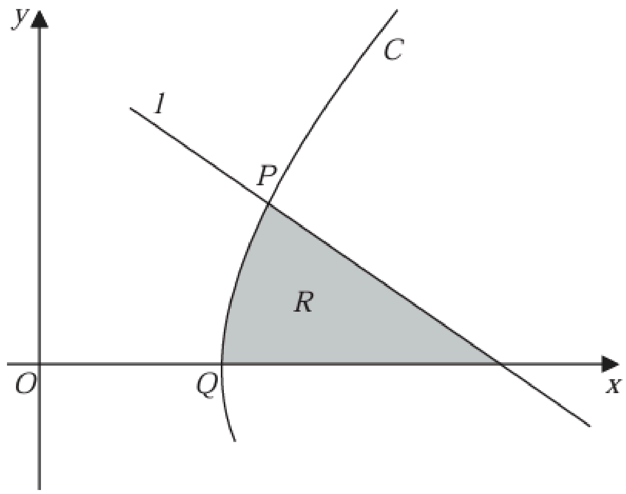

Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = t + \frac{1}{t} \qquad y = t - \frac{1}{t} \qquad t > 0.7$$

The curve C intersects the x-axis at the point Q.

The region, R, shown shaded in Figure 2 is bounded by the curve C, the line l and the x-axis.

(d) Using algebraic integration, find the exact volume of the solid of revolution formed when the region R is rotated through 2π radians about the x-axis.

(7)

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20.Oct 2023

3. **In this question you must show all stages of your working.**

Solutions based on calculator technology are not acceptable.

(i) Use integration by parts to find the exact value of

$$\int_0^4 x^2 e^{2x} \, dx$$

giving your answer in simplest form.

(5)

(ii) Use integration by substitution to show that

$$\int_3^{\frac{21}{2}} \frac{4x}{(2x-1)^2} \, dx = a + \ln b$$

where a and b are constants to be found.

(7)

21.Oct 2023

8.

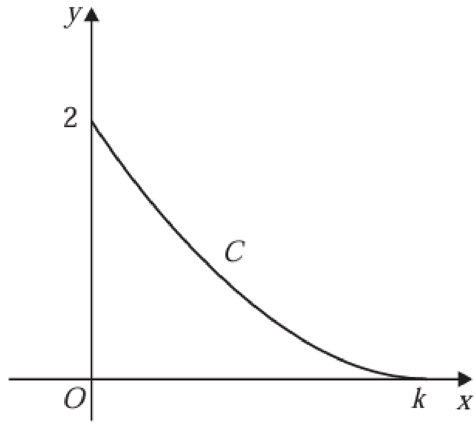

Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 6t - 3\sin 2t \qquad y = 2\cos t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The curve meets the y-axis at 2 and the x-axis at k, where k is a constant.

The region bounded by the curve, the x-axis and the y-axis is rotated through 2π radians about the x-axis to form a solid of revolution.

(d) (i) Show that the volume of this solid is given by

$$\int_0^{\alpha} \beta(1 - \cos 4t)\,dt$$

where α and β are constants to be found.

(ii) Hence, using algebraic integration, find the exact volume of this solid.

(6)

22.Jan 2022

5.

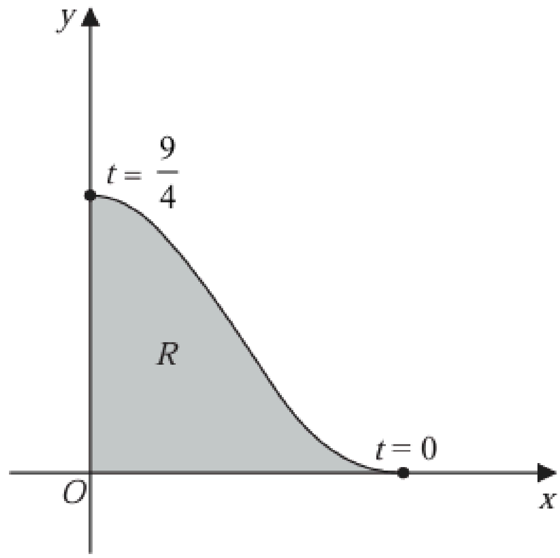

Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = \sqrt{9 - 4t} \qquad y = \frac{t^3}{\sqrt{9 + 4t}} \qquad 0 \leqslant t \leqslant \frac{9}{4}$$

The curve touches the x-axis when $t = 0$ and meets the y-axis when $t = \frac{9}{4}$

The region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the y-axis.

(a) Show that the area of R is given by

$$K\int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81 - 16t^2}}\,dt$$

where K is a constant to be found.

(4)

(b) Using the substitution $u = 81 - 16t^2$, or otherwise, find the exact area of R.

(*Solutions relying on calculator technology are not acceptable.*)

(6)

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23.Jan 2022

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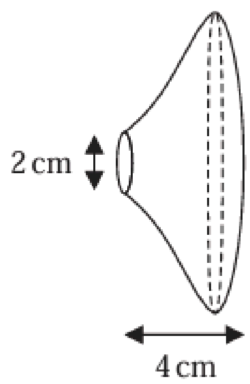

Figure 3

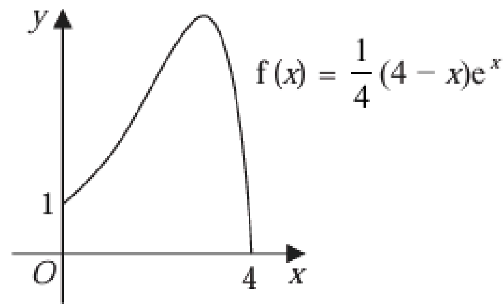

Figure 4

Figure 3 shows the design of a doorknob.

The shape of the doorknob is formed by rotating the curve shown in Figure 4 through 360° about the x-axis, where the units are centimetres.

The equation of the curve is given by

$$f(x) = \frac{1}{4}(4 - x)e^{x} \qquad 0 \leqslant x \leqslant 4$$

(a) Show that the volume, V cm^3, of the doorknob is given by

$$V = K\int_0^4 \left(x^2 - 8x + 16\right)e^{2x}\,dx$$

where K is a constant to be found.

(3)

(b) Hence, find the exact value of the volume of the doorknob.

Give your answer in the form $p\pi(e^q + r)$ cm^3 where p, q and r are simplified rational numbers to be found.

(5)

24.Jan 2022

9. (a) Find the derivative with respect to y of

$$\frac{1}{(1+2\ln y)^2}$$

(2)

(b) Hence find a general solution to the differential equation

$$3\operatorname{cosec}(2x)\frac{dy}{dx} = y(1+2\ln y)^3 \qquad y>0 \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(4)

(c) Show that the particular solution of this differential equation for which $y = 1$ at $x = \frac{\pi}{6}$ is given by

$$y = e^{A\sec x - \frac{1}{2}}$$

where A is an irrational number to be found.

(5)

25.June 2022

5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Use the substitution $x = 2\sin u$ to show that

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,dx = \int_0^p \left(\frac{3}{2}\sec u \tan u + \frac{1}{2}\sec^2 u\right) du$$

where p is a constant to be found.

(4)

(b) Hence find the exact value of

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,dx$$

(4)

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26.June 2022

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

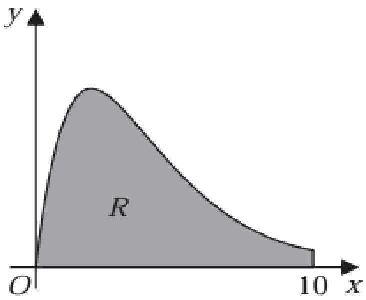

Figure 2

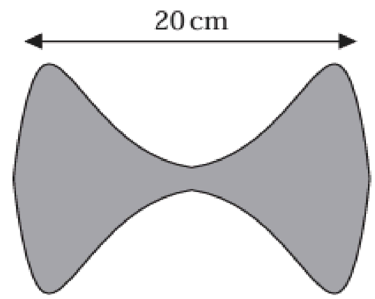

Figure 3

Figure 2 shows the curve with equation

$$y = 10xe^{-\frac{1}{2}x} \qquad 0 \leqslant x \leqslant 10$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line with equation $x = 10$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the volume, V, of this solid is given by

$$V = k\int_0^{10} x^2e^{-x}\,dx$$

where k is a constant to be found.

(2)

(b) Find $\int x^2e^{-x}\,dx$

(3)

Figure 3 represents an exercise weight formed by joining two of these solids together.

The exercise weight has mass 5 kg and is 20 cm long.

Given that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

and using your answers to part (a) and part (b),

(c) find the density of this exercise weight. Give your answer in grams per cm^3 to 3 significant figures.

(5)

27.Oct 2022

2. (a) Express $\dfrac{3x}{(2x-1)(x-2)}$ in partial fraction form.

(3)

(b) Hence show that

$$\int_{5}^{25} \frac{3x}{(2x-1)(x-2)}\,dx = \ln k$$

where k is a fully simplified fraction to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

28.Oct 2022

5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

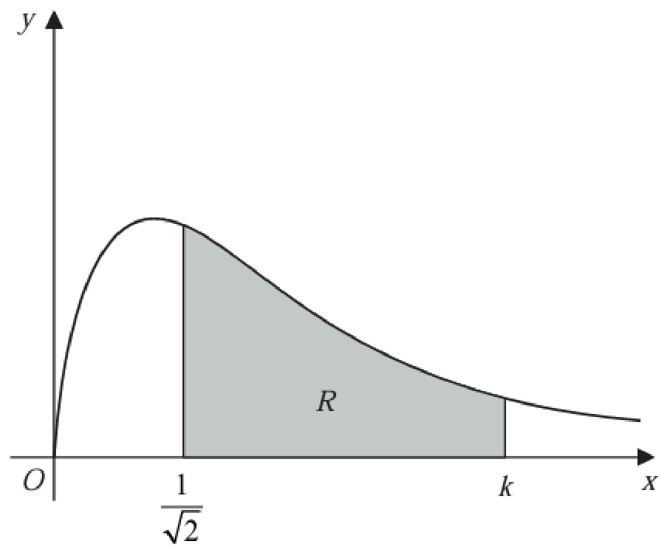

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = \frac{12\sqrt{x}}{(2x^2 + 3)^{1.5}}$$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = \frac{1}{\sqrt{2}}$, the x-axis and the line with equation $x = k$.

This region is rotated through 360° about the x-axis to form a solid of revolution.

Given that the volume of this solid is $\frac{713}{648}\pi$, use algebraic integration to find the exact value of the constant k.

(6)

29.Oct 2022

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Use the substitution $u = e^x - 3$ to show that

$$\int_{\ln 5}^{\ln 7} \frac{4e^{3x}}{e^x - 3}\,dx = a + b\ln 2$$

where a and b are constants to be found.

(7)

(ii) Show, by integration, that

$$\int 3e^x \cos 2x\,dx = pe^x \sin 2x + qe^x \cos 2x + c$$

where p and q are constants to be found and c is an arbitrary constant.

(5)

30.Jan 2021

5. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Using the substitution $u = 3 + \sqrt{2x-1}$ find the exact value of

$$\int_{1}^{13} \frac{4}{3 + \sqrt{2x-1}}\,dx$$

giving your answer in the form $p + q\ln 2$, where p and q are integers to be found.

(8)

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31.Jan 2021

7.

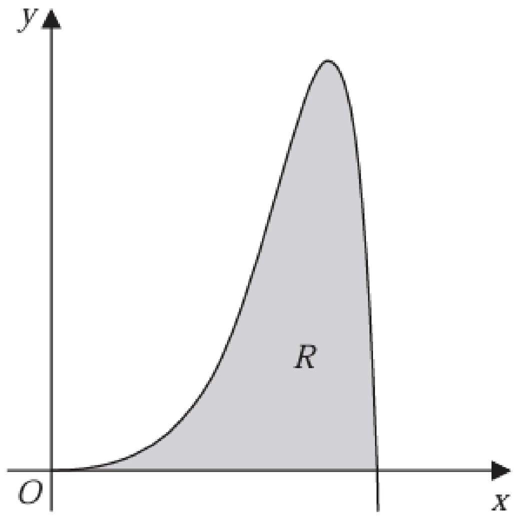

Figure 2

(a) Find $\int e^{2x} \sin x \, dx$

(5)

Figure 2 shows a sketch of part of the curve with equation

$$y = e^{2x} \sin x \qquad x \geqslant 0$$

The finite region R is bounded by the curve and the x-axis and is shown shaded in Figure 2.

(b) Show that the exact area of R is $\dfrac{e^{2\pi} + 1}{5}$

(*Solutions relying on calculator technology are not acceptable.*)

(2)

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32.Jan 2021

9.

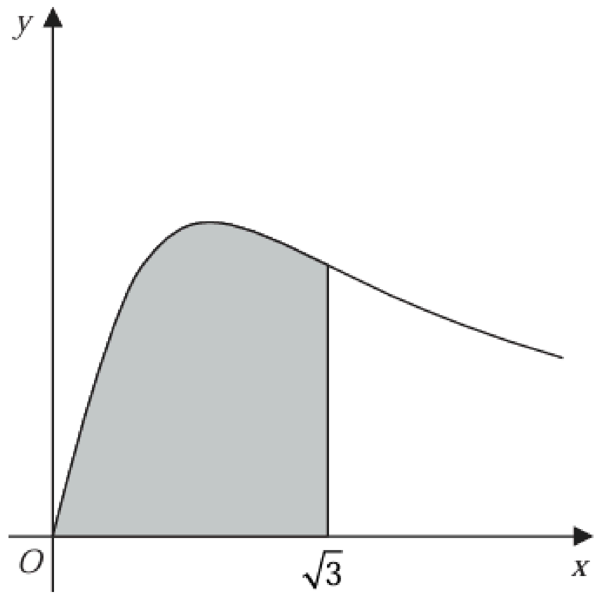

Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = \tan\theta \qquad y = 2\sin 2\theta \qquad \theta \geqslant 0$$

The finite region, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = \sqrt{3}$

The region is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the exact volume of this solid of revolution is given by

$$\int_0^k p(1 - \cos 2\theta)\, d\theta$$

where p and k are constants to be found.

(7)

(b) Hence find, by algebraic integration, the exact volume of this solid of revolution.

(3)

33.June 2021

2.

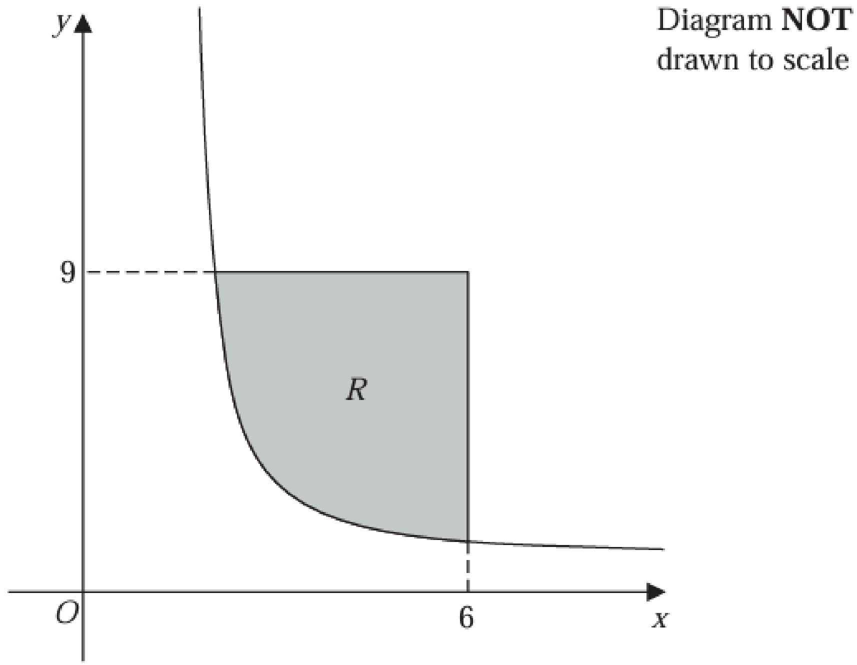

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \frac{9}{(2x-3)^{1.25}} \qquad x > \frac{3}{2}$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $y = 9$ and the line with equation $x = 6$

This region is rotated through 2π radians about the x-axis to form a solid of revolution.

Find, by algebraic integration, the exact volume of the solid generated.

(7)

34.June 2021

4. Use algebraic integration and the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{10}{5x + 2x\sqrt{x}}\,dx$$

Write your answer in the form $4\ln\left(\frac{a}{b}\right)$, where a and b are integers to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(8)

35.Oct 2021

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

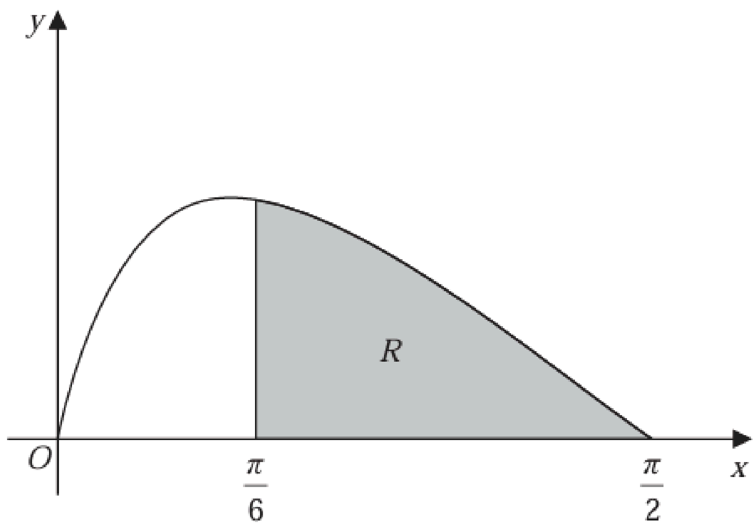

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{16\sin 2x}{(3 + 4\sin x)^2} \qquad 0 \leqslant x \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line with equation $x = \frac{\pi}{6}$

Using the substitution $u = 3 + 4\sin x$, show that the area of R can be written in the form $a + \ln b$, where a and b are rational constants to be found.

(7)

36.Oct 2021

8. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

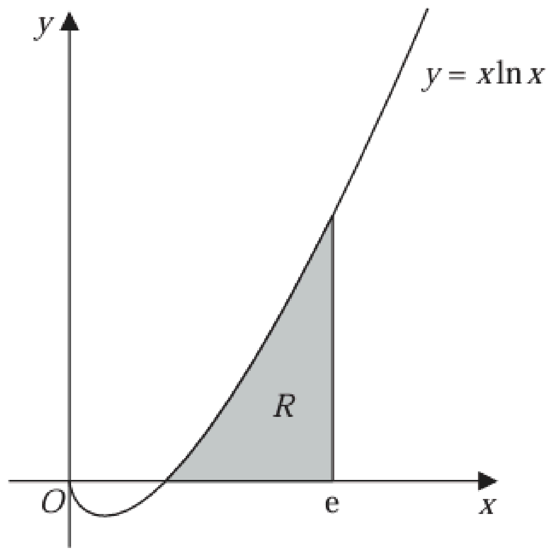

Figure 3

(a) Find $\int x^2 \ln x \, dx$

(3)

Figure 3 shows a sketch of part of the curve with equation

$$y = x \ln x \qquad x > 0$$

The region R, shown shaded in Figure 3, lies entirely above the x-axis and is bounded by the curve, the x-axis and the line with equation $x = \mathrm{e}$.

This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact volume of the solid formed, giving your answer in simplest form.

(4)

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5.

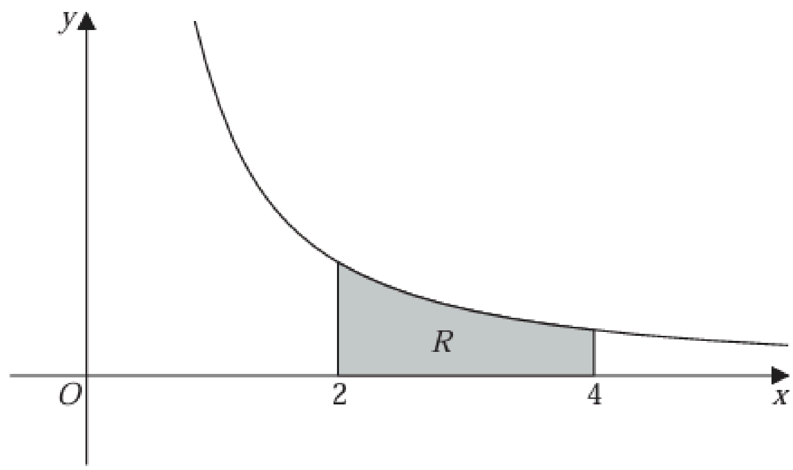

Figure 3

(a) Find $\int \frac{\ln x}{x^2}\,\mathrm{d}x$

(3)

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{3 + 2x + \ln x}{x^2} \qquad x > 0.5$$

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

(b) Use the answer to part (a) to find the exact area of R, writing your answer in simplest form.

(4)

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7. (i) Using a suitable substitution, find, using calculus, the value of

$$\int_1^5 \frac{3x}{\sqrt{2x-1}}\,dx$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(6)

(ii) Find

$$\int \frac{6x^2-16}{(x+1)(2x-3)}\,dx$$

(6)