

Q.1 What is Simple Harmonic Motion (SHM)? Write down characteristics of a body executing simple harmonic motion. What are the applications of SHM?

Simple Harmonic Motion or SHM is defined as a motion in which the restoring force is directly proportional to the displacement of the body from its mean position. The direction of this restoring force is always towards the mean position. The acceleration of a particle executing simple harmonic motion is given by, $a(t) = -\omega^2 x(t)$. Here, ω is the angular velocity of the particle.

Simple Harmonic, Periodic and Oscillation Motion

Simple harmonic motion can be described as an oscillatory motion in which the acceleration of the particle at any position is directly proportional to the displacement from the mean position. It is a special case of oscillatory motion.

All the Simple Harmonic Motions are oscillatory and also periodic but not all oscillatory motions are SHM. Oscillatory motion is also called the harmonic motion of all the oscillatory motions wherein the most important one is **simple harmonic motion (SHM)**.

In this type of oscillatory motion displacement, velocity and acceleration and force vary (w.r.t time) in a way that can be described by either sine (or) the cosine functions collectively called sinusoids.

Difference between Periodic, Oscillation and Simple Harmonic Motion

Periodic Motion

- A motion repeats itself after an equal interval of time. **For example, uniform circular motion.**
- There is no equilibrium position.
- There is no restoring force.
- There is no stable equilibrium position.

Oscillation Motion

- To and fro motion of a particle about a mean position is called an oscillatory motion in which a particle moves on either side of equilibrium (or) mean position is an oscillatory motion.
- It is a kind of periodic motion bounded between two extreme points. **For example**, Oscillation of Simple Pendulum, Spring-Mass System.
- The object will keep on moving between two extreme points about a fixed point is called mean position (or) equilibrium position along any path. (the path is not a constraint).
- There will be a restoring force directed towards equilibrium position (or) mean position.
- **In an oscillatory motion**, the net force on the particle is zero at the mean position.
- The mean position is a stable equilibrium position.

Simple Harmonic Motion or SHM

- It is a special case of oscillation along with straight line between the two extreme points (the path of SHM is a constraint).

- Path of the object needs to be a straight line.
- There will be a restoring force directed towards equilibrium position (or) mean position.
- Mean position in Simple harmonic motion is a stable equilibrium.

Conditions for SHM:

$$\vec{F} \propto -\vec{x}$$

$$\vec{a} \propto -\vec{x}$$

Types of Simple Harmonic Motion

SHM or Simple Harmonic Motion can be classified into two types:

- Linear SHM
- Angular SHM

Linear Simple Harmonic Motion

When a particle moves to and fro about a fixed point (called equilibrium position) along with a straight line then its motion is called linear Simple Harmonic Motion.

For Example: spring-mass system

Conditions for Linear SHM:

The restoring force or acceleration acting on the particle should always be proportional to the displacement of the particle and directed towards the equilibrium position.

$$\vec{F} \propto -\vec{x}$$

$$\vec{a} \propto -\vec{x}$$

- $\vec{x}$ – displacement of particle from equilibrium position.
- $\vec{F}$ – Restoring force
- $\vec{a}$ - acceleration

Angular Simple Harmonic Motion

When a system oscillates angular long with respect to a fixed axis then its motion is called angular simple harmonic motion.

Conditions to Execute Angular SHM:

The restoring torque (or) Angular acceleration acting on the particle should always be proportional to the angular displacement of the particle and directed towards the equilibrium position.

$$T\alpha - \theta \text{ or } \alpha\theta - T$$

Where,

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- T – Torque
- α angular acceleration
- θ – angular displacement

Simple Harmonic Motion Equation and its Solution

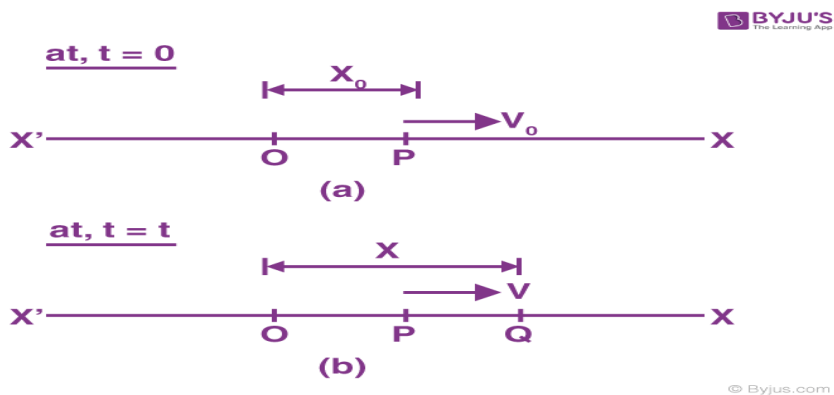
Consider a particle of mass (m) executing Simple Harmonic Motion along a path x o x ; the mean position at O .

Let the speed of the particle be v_0 when it is at position p (at a distance x_0 from O)

At $t = 0$ the particle at P (moving towards the right)

At $t = t$ the particle is at Q (at a distance x from O)

With a velocity (v)



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The restoring force $\vec{F}$ at Q is given by

$$\Rightarrow \vec{F} = -K\vec{x} \quad K - \text{is positive constant}$$

$$\Rightarrow \vec{F} = m\vec{a} \quad \vec{a} - \text{acceleration at Q}$$

$$\Rightarrow m\vec{a} = -K\vec{x}$$

$$\Rightarrow \vec{a} = -\left(\frac{K}{m}\right)\vec{x}$$

$$\text{Put, } \frac{K}{m} = \omega^2$$

$$\Rightarrow \omega = \sqrt{\frac{K}{m}}$$

$$\Rightarrow \vec{a} = -\left(\frac{K}{m}\right)\vec{x} = -\omega^2\vec{x} \quad \text{Since, } \left[\vec{a} = \frac{d^2\vec{x}}{dt^2}\right] \quad \frac{d^2\vec{x}}{dt^2} = -\omega^2\vec{x}$$

Q.2

a) If the velocity of a ripple is given by the relation $V_p = c_1 + c_2 \lambda$, where c_1 and c_2 are constants. Find the group velocity of the ripple?

Since the phase difference between the vibrations is continually changing, the specification of some initial nonzero phase difference is in general not of major significance in this case.

So we can suppose that the individual vibrations have an initial phase of 0, and hence can be written as:

$$E_1 = a \cos(\omega_1 t - k_1 z)$$

$$E_2 = a \cos(\omega_2 t - k_2 z)$$

Then the sum of these two waves is:

$$E = E_1 + E_2 = a[\cos(\omega_1 t - k_1 z) + \cos(\omega_2 t - k_2 z)]$$

Using the following triangular formula

$$\cos(\alpha) + \cos(\beta) = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$

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We get

$$E = 2a \cos \frac{1}{2}[(\omega_1 - \omega_2)t - (k_1 - k_2)z] \cos \frac{1}{2}[(\omega_1 + \omega_2)t - (k_1 + k_2)z]$$

We then introduce the notation of average angular frequency $\bar{\omega}$ and average wave number $\bar{k}$

$$\bar{\omega} = \frac{1}{2}(\omega_1 + \omega_2)$$

$$\bar{k} = \frac{1}{2}(k_1 + k_2)$$

And modulation frequency ω_m and modulation wave number k_m

$$\omega_m = \frac{1}{2}(\omega_1 - \omega_2)$$

$$k_m = \frac{1}{2}(k_1 - k_2)$$

We then get

$$E = 2a \cos(\omega_m t - k_m z) \cos(\bar{\omega} t - \bar{k} z)$$

We can make

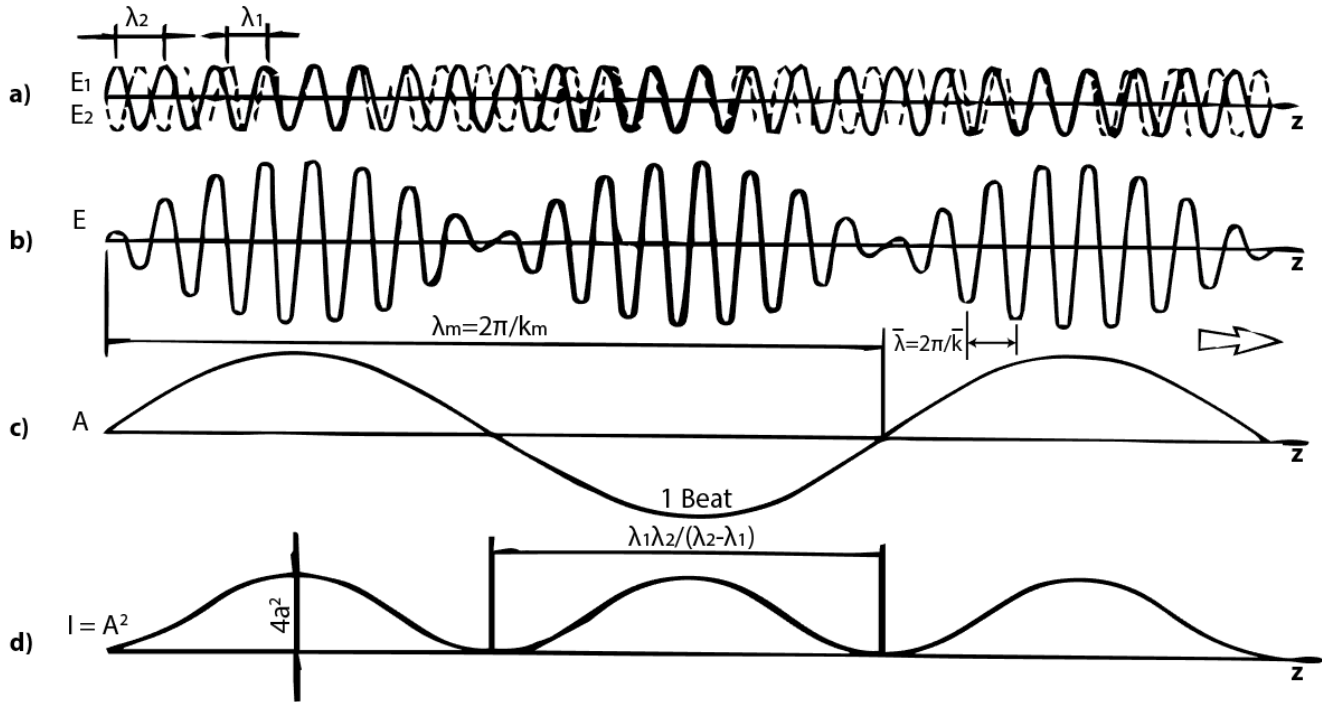
$$A = 2a \cos(\omega_m t - k_m z)$$

Then we get

$$E = A \cos(\bar{\omega} t - \bar{k} z)$$

This means that the resultant superposed wave has an angular frequency $\bar{\omega}$, and its amplitude varies between **0** and **2a** with time **t** and position **z**.

The following picture shows the superposition result. Since light waves have very high frequency, if $\omega_1 \approx \omega_2$, then $\bar{\omega} \gg \omega_m$, which means that **A** varies slowly but **E** varies extremely fast.



Intensity **I** of the superposed wave is proportional to A^2 , we have

$$I = A^2 = 4a^2 \cos^2(\omega_m t - k_m z)$$

or

$$I = A^2 = 2a^2 [1 + \cos 2(\omega_m t - k_m z)]$$

So intensity **I** varies between **0** and **$4a^2$** with time **t** and position **z**. This phenomenon is called “**beat**”. From the last formula we can see that the beat frequency is 2 times of modulation frequency ω_m , from ω_m ’s definition $\omega_m = (\omega_1 - \omega_2)/2$, we can see that the beat frequency equals to $\omega_1 - \omega_2$.

This process, as a purely mathematical result, can be carried out for any values of ω_1 and ω_2 . But its description as a “beat” phenomenon is physically meaningful only if $|\omega_1 - \omega_2| \ll \omega_1 + \omega_2$.

PHASE VELOCITY AND GROUP VELOCITY

Clearly the velocity of a monochrome light equals its equiphase surface propagation velocity. However, in the case of a superposed wave, we need to carefully define its propagation velocity.

Let’s continue using the superposed wave equation from above:

$$E = 2a \cos(\omega_m t - k_m z) \cos(\bar{\omega} t - \bar{k} z)$$

The superposed wave has two propagation velocities: equiphase surface propagation velocity (called Phase Velocity **V_p**), and equiamplitude surface propagation velocity (called Group Velocity **V_g** as defined by Rayleigh).

b) For a light source of 60 watts, what is the intensity of light 10m away from the source?

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$$P = 60 \text{ watts}$$

$$r = 10 \text{ m}$$

$$I = P / A$$

$$I = P / 4\pi r^2$$

$$I = 60 / 4\pi \times 10^2$$

$$I = 60 / 1256$$

$$I = 0.047 \text{ W m}^{-2}$$

Q.3 Derive relativity of mass, length and time using Lorentz transformation. (Complete mathematical derivation).

The general rule (and handy sanity-check for calculations) is that the visible frequency should change by the same ratio as the photographable length, so blueshifted rulers appear lengthened and redshifted rulers appear shortened (in the direction of motion). If an approaching object is seen to be ageing twice as quickly, then its apparent length should be doubled. If it is receding and its frequency appears to be halved, then its photographed length should appear halved, too.

There are two basic ("pre-SR") sets of equations for these photographable propagation-based apparent length-changes and Doppler shifts due to relative motion, depending on whether the speed of light is assumed to be globally constant for (1) the observer or (2) the emitter. For simple recession (with v as recession velocity), these are

$$(1) \dots \text{freq}'/\text{freq} = \text{plength}'/\text{plength} = c/(c+v)$$

and

$$(2) \dots \text{freq}'/\text{freq} = \text{plength}'/\text{plength} = (c-v)/c$$

Notice that the ratio between these two predictions is $1-v^2/c^2$, the square of the Lorentz ratio.

Special relativity retains the rule that visible lengths appear to change by the same ratio as received frequencies but removes the conflict between the two older sets of predictions by using their geometric mean or "root product", instead

$$(3) \text{freq}'/\text{freq} = \text{plength}'/\text{plength} = \text{SQRT}[(c-v)/(c+v)]$$

, and this new intermediate prediction for apparent frequency or length can then be expressed as either of the two initial propagation-based predictions, appropriately multiplied or divided by a Lorentz factor ("time dilation", "length contraction").

Written descriptions of special relativity would suggest that we use (1) for calculating the propagation effects and then multiply in a Lorentz length contraction and redshift, but it turns out that the math for objects moving at arbitrary angles is often easier if you start from (2) (where the denominator is simpler), and multiply an /inverse/ Lorentz factor - so that's what people tend to do, out of pragmatism.

Tricks to justify using the more convenient form of the equations include doing them from the emitter's point of view (convoluted and IMO maybe a bit "iffy", but the math still works), or using wavelength rather than

frequency. Another is to not use the frequency or rate of events as a variable, but to instead use the amount of time that those events are seen to take place in, which, again, inverts the relationship (double the frequency, halve the time seen for the event stream to happen).

Authors have a fair amount of choice over which combination of variables and conventions that they use to describe a problem, and they often pick whatever seems most familiar, or whatever makes the math easier.

Suppose you have a particle which moves with a constant velocity w.r.t. to the LAB system S (you could also take any kind motion, because you could consider small intervals in time during which V -constant). Then we can have a clock moving with the particle which measures the intervals of time between events which occur with particle. We can now think of a system S' moving along with particle with velocity V constant; in S' our particle is in rest. Let $\Delta t = t_2 - t_1$ be the interval of time measured in S during which particle moves from (x, y, z) to $(x + \Delta x, y + \Delta y, z + \Delta z)$, and $\Delta t' = t_2' - t_1'$ the time interval measured by clock in S' , during which (i.e. $\Delta t'$) in S' particle is in rest, thus $\Delta x' = \Delta y' = \Delta z' = 0$. Therefore, it is important to use the inverse Lorentz transform, since the terms $\Delta x' = x_2' - x_1' = 0$ and so they cancel out.

Q.4 What is electromagnetic spectrum? Write down useful and harmful effects of different regions of electromagnetic spectrum.

Electromagnetic spectrum, the entire distribution of electromagnetic radiation according to frequency or wavelength. Although all electromagnetic waves travel at the speed of light in a vacuum, they do so at a wide range of frequencies, wavelengths, and photon energies. The electromagnetic spectrum comprises the span of all electromagnetic radiation and consists of many subranges, commonly referred to as portions, such as visible light or ultraviolet radiation. The various portions bear different names based on differences in behaviour in the emission, transmission, and absorption of the corresponding waves and also based on their different practical applications. There are no precise accepted boundaries between any of these contiguous portions, so the ranges tend to overlap.

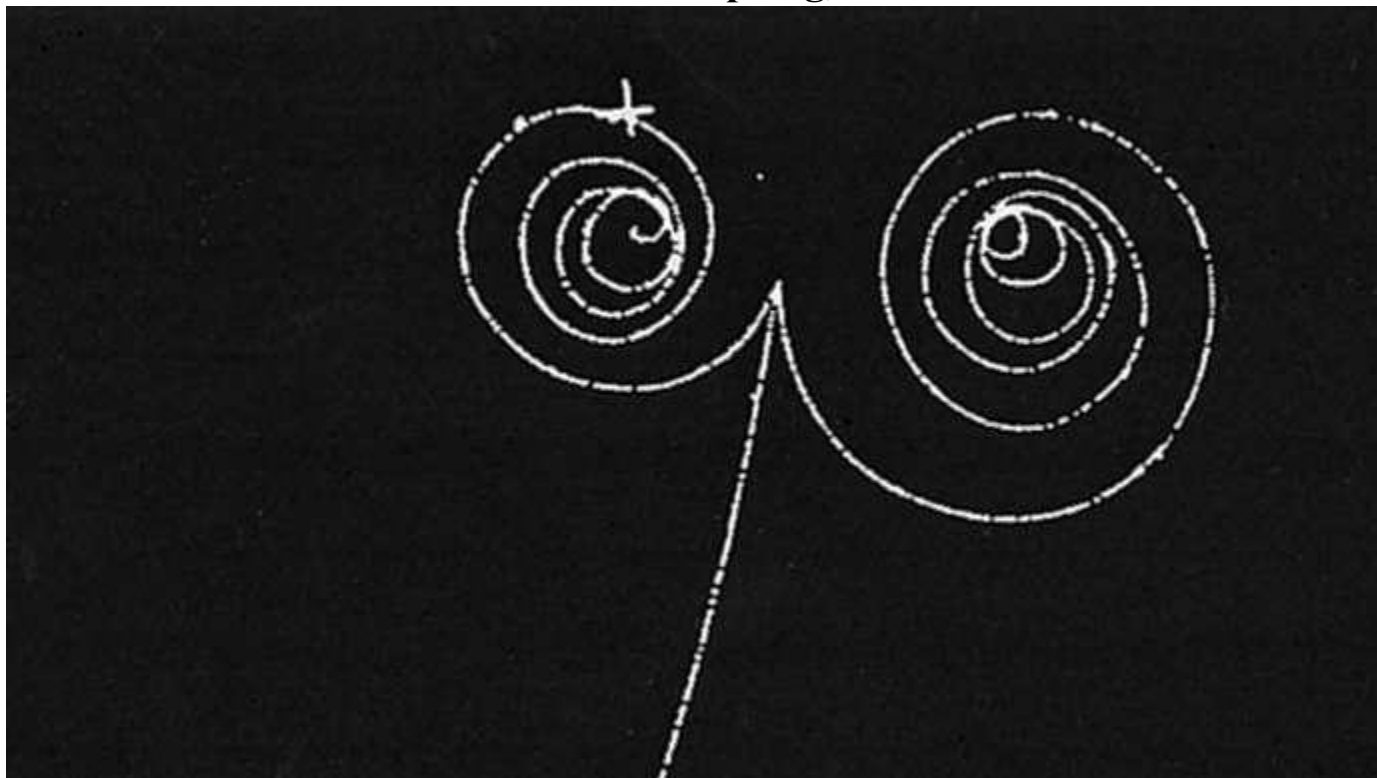
The entire electromagnetic spectrum, from the lowest to the highest frequency (longest to shortest wavelength), includes all radio waves (e.g., commercial radio and television, microwaves, radar), infrared radiation, visible light, ultraviolet radiation, X-rays, and gamma rays. Nearly all frequencies and wavelengths of electromagnetic radiation can be used for spectroscopy.

Gamma rays are produced in the disintegration of radioactive atomic nuclei and in the decay of certain subatomic particles. The commonly accepted definitions of the gamma-ray and X-ray regions of the electromagnetic spectrum include some wavelength overlap, with gamma-ray radiation having wavelengths that are generally shorter than a few tenths of an angstrom (10^{-10} metre) and gamma-ray photons having energies that are greater than tens of thousands of electron volts (eV). There is no theoretical upper limit to the energies of gamma-ray photons and no lower limit to gamma-ray wavelengths; observed energies presently extend up to a few trillion electron volts—these extremely high-energy photons are produced in astronomical sources through currently unidentified mechanisms.

The term gamma ray was coined by British physicist Ernest Rutherford in 1903 following early studies of the emissions of radioactive nuclei. Just as atoms have discrete energy levels associated with different configurations of the orbiting electrons, atomic nuclei have energy level structures determined by the configurations of the protons and neutrons that constitute the nuclei. While energy differences between atomic energy levels are typically in the 1- to 10-eV range, energy differences in nuclei usually fall in the 1-keV (thousand electron volts) to 10-MeV (million electron volts) range. When a nucleus makes a transition from a high-energy level to a lower-energy level, a photon is emitted to carry off the excess energy; nuclear energy-level differences correspond to photon wavelengths in the gamma-ray region.

When an unstable atomic nucleus decays into a more stable nucleus (see radioactivity), the “daughter” nucleus is sometimes produced in an excited state. The subsequent relaxation of the daughter nucleus to a lower-energy state results in the emission of a gamma-ray photon. Gamma-ray spectroscopy, involving the precise measurement of gamma-ray photon energies emitted by different nuclei, can establish nuclear energy-level structures and allows for the identification of trace radioactive elements through their gamma-ray emissions. Gamma rays are also produced in the important process of pair annihilation, in which an electron and its antiparticle, a positron, vanish and two photons are created. The photons are emitted in opposite directions and must each carry 511 keV of energy—the rest mass energy (see relativistic mass) of the electron and positron. Gamma rays can also be generated in the decay of some unstable subatomic particles, such as the neutral pion. Gamma-ray photons, like their X-ray counterparts, are a form of ionizing radiation; when they pass through matter, they usually deposit their energy by liberating electrons from atoms and molecules. At the lower energy ranges, a gamma-ray photon is often completely absorbed by an atom and the gamma ray’s energy transferred to a single ejected electron (see photoelectric effect). Higher-energy gamma rays are more likely to scatter from the atomic electrons, depositing a fraction of their energy in each scattering event (see Compton effect). Standard methods for the detection of gamma rays are based on the effects of the liberated atomic electrons in gases, crystals, and semiconductors (see radiation measurement and scintillation counter).

Gamma rays can also interact with atomic nuclei. In the process of pair production, a gamma-ray photon with an energy exceeding twice the rest mass energy of the electron (greater than 1.02 MeV), when passing close to a nucleus, is directly converted into an electron-positron pair (see photograph). At even higher energies (greater than 10 MeV), a gamma ray can be directly absorbed by a nucleus, causing the ejection of nuclear particles (see photodisintegration) or the splitting of the nucleus in a process known as photofission.



Continuous spectra of electromagnetic radiation

Such spectra are emitted by any warm substance. Heat is the irregular motion of electrons, atoms, and molecules; the higher the temperature, the more rapid the motion. Since electrons are much lighter than atoms, irregular thermal motion produces irregular oscillatory charge motion, which reflects a continuous spectrum of frequencies. Each oscillation at a particular frequency can be considered a tiny “antenna” that emits and receives electromagnetic radiation. As a piece of iron is heated to increasingly high temperatures, it first glows red, then yellow, and finally white. In short, all the colours of the visible spectrum are represented. Even before the iron begins to glow red, one can feel the emission of infrared waves by the heat sensation on the skin. A white-hot piece of iron also emits ultraviolet radiation, which can be detected by a photographic film.

Not all materials heated to the same temperature emit the same amount and spectral distribution of electromagnetic waves. For example, a piece of glass heated next to iron looks nearly colourless, but it feels hotter to the skin (it emits more infrared rays) than does the iron. This observation illustrates the rule of reciprocity: a body radiates strongly at those frequencies that it is able to absorb, because for both processes it needs the tiny antennas of that range of frequencies. Glass is transparent in the visible range of light because it lacks possible electronic absorption at these particular frequencies. As a consequence, glass cannot glow red because it cannot absorb red. On the other hand, glass is a better emitter/absorber in the infrared than iron or any other metal that strongly reflects such lower-frequency electromagnetic waves. This selective emissivity and absorptivity is important for understanding the greenhouse effect (see below The greenhouse effect of the atmosphere) and many other phenomena in nature. The tungsten filament of a lightbulb has a temperature of

2,500 K (4,040 °F) and emits large amounts of visible light but relatively little infrared because metals, as mentioned above, have small emissivities in the infrared range. This is of course fortunate, since one wants light from a lightbulb but not much heat. The light emitted by a candle originates from very hot carbon soot particles in the flame, which strongly absorb and thus emit visible light. By contrast, the gas flame of a kitchen range is pale, even though it is hotter than a candle flame, because of the absence of soot. Light from the stars originates from the high temperature of the gases at their surface. A wide spectrum of radiation is emitted from the Sun's surface, the temperature of which is about 5,800 K. The radiation output is 60 million watts for every square metre of solar surface, which is equivalent to the amount produced by an average-size commercial power-generating station that can supply electric power for about 30,000 households.

The spectral composition of a heated body depends on the materials of which the body consists. That is not the case for an ideal radiator or absorber. Such an ideal object absorbs and thus emits radiation of all frequencies equally and fully. A radiator/absorber of this kind is called a blackbody, and its radiation spectrum is referred to as blackbody radiation, which depends on only one parameter, its temperature. Scientists devise and study such ideal objects because their properties can be known exactly. This information can then be used to determine and understand why real objects, such as a piece of iron or glass, a cloud, or a star, behave differently.

Q.5 What is nature of light? How light is an electromagnetic wave? Explain its characteristics. Also elaborate speed of light in matter.

Light is a transverse, electromagnetic wave that can be seen by the typical human. The wave nature of light was first illustrated through experiments on diffraction and interference. Like all electromagnetic waves, light can travel through a vacuum. The transverse nature of light can be demonstrated through polarization.

- In 1678, Christiaan Huygens (1629–1695) published *Traité de la Lumière*, where he argued in favor of the wave nature of light. Huygens stated that an expanding sphere of light behaves as if each point on the wave front were a new source of radiation of the same frequency and phase.
- Thomas Young (1773–1829) and Augustin-Jean Fresnel (1788–1827) disproved Newton's corpuscular theory.

Sources

Light is produced by one of two methods...

- Incandescence is the emission of light from "hot" matter ($T \gtrsim 800$ K).
- Luminescence is the emission of light when excited electrons fall to lower energy levels (in matter that may or may not be "hot").

Speed

Just notes so far. The speed of light in a vacuum is represented by the letter c from the Latin *celeritas* — swiftness. Measurements of the speed of light.

- The speed of light in a vacuum is a universal constant in all reference frames.
- The speed of light in a vacuum is fixed at 299,792,458 m/s by the current definition of the meter.

- The speed of light in a medium is always slower than the speed of light in a vacuum.
- The speed of light depends upon the medium through which it travels. The speed of anything with mass is always less than the speed of light in a vacuum.

Other characteristics

The amplitude of a light wave is related to its intensity.

- Intensity is the absolute measure of a light wave's power density.
- Brightness is the relative intensity as perceived by the average human eye.

The frequency of a light wave is related to its color.

- Color is such a complex topic that it has its own section in this book.
- Monochromatic light is described by only one frequency.
 - o Laser light is effectively monochromatic.
 - o There are six simple, named colors in English (and many other languages) each associated with a band of monochromatic light. In order of increasing frequency they are red, orange, yellow, green, blue, and violet.
 - o Light is sometimes also known as visible light to contrast it from "ultraviolet light" and "infrared light"
 - o Other forms of electromagnetic radiation that are not visible to humans are sometimes also known informally as "light"
- Polychromatic light is described by many different frequencies.
 - o Nearly every light source is polychromatic.
 - o White light is polychromatic.
- A graph of relative intensity vs. frequency is called a spectrum (plural: spectra).
Although frequently associated with light, the term can be applied to any wave phenomena.
- A continuous spectrum is one in which every frequency is present within some range.
 - o Blackbody radiators emit a continuous spectrum.
- A discrete spectrum is one in which only a well-defined set of isolated frequencies are present.
(A discrete spectrum is a finite collection of monochromatic light waves.)
 - o The excited electrons in a gas emit a discrete spectrum.
- The wavelength of a light wave is inversely proportional to its frequency.
- Light is often described by its wavelength in a vacuum.
- Light ranges in wavelength from 400 nm on the violet end to 700 nm on the red end of the visible spectrum.
- Phase differences between light waves can produce visible interference effects.
(There are several sections in this book on interference phenomena and light.)
- Leftovers about animals.

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- Falcon can see a 10 cm. object from a distance of 1.5 km.
- Fly's Eye has a flicker fusion rate of 300/s. Humans have a flicker fusion rate of only 60/s in bright light and 24/s in dim light. The flicker fusion rate is the frequency with which the "flicker" of an image cannot be distinguished as an individual event. Like the frame of a movie... if you slowed it down, you would see individual frames. Speed it up and you see a constantly moving image. Octopus' eye has a flicker fusion frequency of 70/s in bright light.
- Penguin has a flat cornea that allows for clear vision underwater. Penguins can also see into the ultraviolet range of the electromagnetic spectrum.
- Sparrow Retina has 400,000 photoreceptors per square. mm.
- Reindeer can see ultraviolet wavelengths, which may help them view contrasts in their mostly white environment.