Systems Evaluation Midterm - Judah Anttila - v1

Honor Statement:

On my honor as a student, I promise that I have neither given nor received any assistance on this exam. I recognize that any violation of my promise is a violation of OU's academic integrity system.

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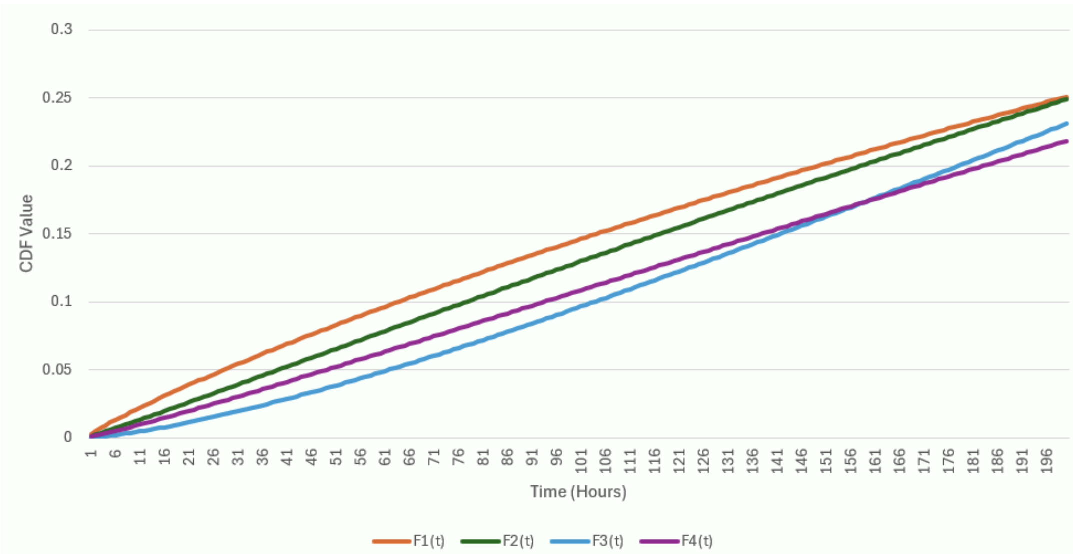
Problem 1

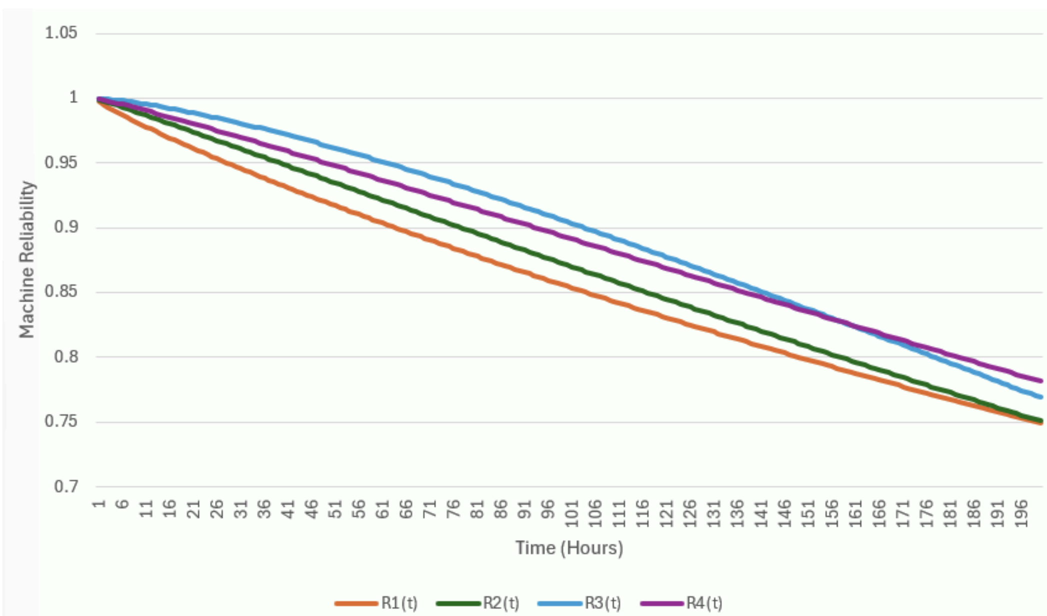
a. **Plot the cdf of time to failure for each machine (four curves on one graph) for 200 hours.**

Figure 1 - CDF of Time to Failure for Each Machine



b. **Plot $R_i(t)$ for 200 hours (four curves on one graph).**

Figure 2 - Reliability of Each Machine over Time



Note. The y axis starts at .7 for a more detailed understanding of the curves.

c. **Calculate reliability for each machine, R_i, evaluated at two work weeks.**

Table 1 - Reliability of Each Machine at Two Work Weeks

R1(80 hrs)	R2(80 hrs)	R3(80 hrs)	R4(80 hrs)
0.8790499	0.8968289	0.9291198	0.9151443

Problem 2

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a. **What is the optimal allocation of redundancy if factory floor total reliability is the only Objective?**

If total reliability is the only objective, the optimal allocation of redundancy is to have **3** of Machine 1, **3** of Machine 2, **2** of Machine 3, and **2** of Machine 4. This results in a maximum system reliability of **0.98498**. Accordingly, 0.98498 be used as the target t_1 in goal programming parts (c), (d), and (e).

Table 2 - Optimal Redundancy for (a)

name	n1	n2	n3	n4
value	3	3	2	2

Figure 3 - Mathematical Formulation for Part (a)

Objective Function: Maximize System Reliability

$$Max\left(\prod_{i=1}^{i=4}\left[1 - (1 - R_i)^{n_i}\right]\right)$$

Decision Variables: Allocation of Redundancy by Machine

$$n_1, n_2, n_3, n_4$$

Parameters: Single Machine Reliability at 80 Work Hours

$$R_1 = 0.87905, R_2 = 0.89683, R_3 = 0.92912, R_4 = 0.91514,$$

Constraints

$$\sum_{i=1}^{i=4} (n_i) = 10$$

$$n_i \geq 1 \quad \forall\, i \in \{1, 2, 3, 4\}$$

$$n_i \leq 5 \quad \forall\, i \in \{1, 2, 3, 4\}$$

$$n_i \in \{\aleph\} \quad \forall\, i \in \{1, 2, 3, 4\}$$

b. **What is the optimal allocation of redundancy if total acquisition cost is the only objective?**

If the total acquisition cost is the only objective, the optimal allocation of redundancy is to buy **5** of Machine 1, **3** of Machine 2, **1** of Machine 3, and **1** of Machine 4. This results in a minimum total acquisition cost of **$ 2,766,146**. Accordingly, $2,766,146 be used as the target t_2 in goal programming parts (c), (d), and (e).

Table 3 - Optimal Redundancy for (b)

name	n1	n2	n3	n4
value	5	3	1	1

Figure 4 - Mathematical Formulation for Part (b)

$$Objective\ Function:\ Minimize\ Total\ Acquisition\ Cost$$

$$Min\left(\sum_{i=1}^{i=4} [n_i \cdot (a_i e^{\frac{b_i}{1-R_i}})] \right)$$

$$Decision\ Variables:\ Allocation\ of\ Redundancy\ by\ Machine$$

$$n_1, n_2, n_3, n_4$$

$$Parameters:$$

$$R_1 = 0.87905, R_2 = 0.89683, R_3 = 0.92912, R_4 = 0.91514,$$

$$a_1 = 1.9, a_2 = 2.9, a_3 = 1.2, a_4 = 0.9$$

$$b_1 = 1.4, b_2 = 1.2, b_3 = 0.9, b_4 = 1.1$$

$$Constraints$$

$$\sum_{i=1}^{i=4} (n_i) = 10$$

$$n_i \geq 1 \quad \forall\, i \in \{1, 2, 3, 4\}$$

$$n_i \leq 5 \quad \forall\, i \in \{1, 2, 3, 4\}$$
$$n_i \in \{\aleph\} \quad \forall\, i \in \{1, 2, 3, 4\}$$

c. **Consider the individual optimal objective function values from a and b as targets in a goal programming formulation. Consider the situation where total reliability and total acquisition cost are equally important. What is the optimal allocation of redundancy?**

If total reliability and total acquisition cost are weighted as equally important, then the optimal allocation of redundancy is to have **4** of Machine 1, **2** of Machine 2, **2** of Machine 3, and **2** of Machine 4. This results in the minimum weighted scaled deviations from the targets being **0.9749**. This is summarized in Table 4.

Table 4 - Optimal Redundancy for (c)

name	n1	n2	n3	n4
value	4	2	2	2

Figure 5 - Goal Programming Formulation for Part (c)

Objective Function: Minimize The Weighted Scaled Deviations

$$min(\sum_{j=1}^{j=2} (\frac{1}{t_j} \cdot (w_j^- d_j^- + w_j^+ d_j^+)))$$

Decision Variables: Allocation of Redundancy by Machine

$$n_1, n_2, n_3, n_4$$

Slack Decision Variables

$$d_1^+, d_1^-, d_2^+, d_2^-$$

Goal Weights: Both goals equally important

$$w_1^+ = w_1^- = w_2^+ = w_2^- = 1$$

Parameters

$$R_1 = 0.87905, R_2 = 0.89683, R_3 = 0.92912, R_4 = 0.91514,$$
$$a_1 = 1.9, a_2 = 2.9, a_3 = 1.2, a_4 = 0.9$$
$$b_1 = 1.4, b_2 = 1.2, b_3 = 0.9, b_4 = 1.1$$

Goal Constraints and Targets

$$(\prod_{i=1}^{i=4}[1 - (1 - R_i)^{n_i}]) + d_1^- - d_1^+ = t_1 = 0.98498$$

$$(\sum_{i=1}^{i=4}[n_i \cdot (a_i e^{\frac{b_i}{1-R_i}})]) + d_2^- - d_2^+ = t_2 = \$2,766,146$$

$$Rigid\ Constraints$$

$$\sum_{i=1}^{i=4}(n_i) = 10$$

$$n_i \geq 1 \quad \forall\, i \in \{1, 2, 3, 4\}$$

$$n_i \leq 5 \quad \forall\, i \in \{1, 2, 3, 4\}$$

$$n_i \in \{\aleph\} \quad \forall\, i \in \{1, 2, 3, 4\}$$

$$d_j^+, d_j^- \geq 0 \quad \forall\, j \in \{1, 2\}$$

d. Consider the situation where total reliability is twice as important as total acquisition cost. What is the optimal allocation of redundancy?

If total reliability is weighted as twice as important as total acquisition cost, then the optimal allocation of redundancy does not change. It remains optimal to have **4** of Machine 1, **2** of Machine 2, **2** of Machine 3, and **2** of Machine 4. This results in the minimum weighted scaled deviations from the targets being **0.10551**. This is summarized in Table 5.

Table 5 - Optimal Redundancy for (d)

name	n1	n2	n3	n4
value	4	2	2	2

Figure 6 - Goal Programming Formulation for Part (d)

$$Objective\ Function:\ Minimize\ The\ Weighted\ Scaled\ Deviations$$

$$min(\sum_{j=1}^{j=2}(\frac{1}{t_j} \cdot (w_j^- d_j^- + w_j^+ d_j^+)))$$

$$Decision\ Variables:\ Allocation\ of\ Redundancy\ by\ Machine$$

$$n_1, n_2, n_3, n_4$$

$$Slack\ Decision\ Variables$$

$$d_1^+, d_1^-, d_2^+, d_2^-$$

$$Goal\ Weights:\ System\ Reliability\ is\ Twice\ as\ Important\ as\ Total\ Acquisition\ Cost$$

$$w_1^+ = w_1^- = 2 \; ; \; w_2^+ = w_2^- = 1$$

$$Parameters$$

$$R_1 = 0.87905, R_2 = 0.89683, R_3 = 0.92912, R_4 = 0.91514,$$

$$a_1 = 1.9\,, a_2 = 2.9\,, a_3 = 1.2\,, a_4 = 0.9$$

$$b_1 = 1.4\,, b_2 = 1.2\,, b_3 = 0.9\,, b_4 = 1.1$$

$$Goal\ Constraints\ and\ Targets$$

$$\left(\prod_{i=1}^{i=4}[1 - (1 - R_i)^{n_i}]\right) + d_1^- - d_1^+ = t_1 = 0.98498$$

$$\left(\sum_{i=1}^{i=4}[n_i \cdot (a_i e^{\frac{b_i}{1-R_i}})]\right) + d_2^- - d_2^+ = t_2 = \$2,766,146$$

$$Rigid\ Constraints$$

$$\sum_{i=1}^{i=4}(n_i) = 10$$

$$n_i \geq 1 \quad \forall\, i \in \{1, 2, 3, 4\}$$

$$n_i \leq 5 \quad \forall\, i \in \{1, 2, 3, 4\}$$

$$n_i \in \{\aleph\} \quad \forall\, i \in \{1, 2, 3, 4\}$$

$$d_j^+, d_j^- \geq 0 \;\; \forall\, j \in \{1, 2\}$$

e. Consider the situation where total acquisition cost is twice as important as total reliability. What is the optimal allocation of redundancy?

If total acquisition cost is weighted as twice as important as the total reliability, then the optimal allocation of redundancy changes. Now, the optimal allocation of redundancy is to have **5** of Machine 1, **2** of Machine 2, **1** of Machine 3, and **2** of Machine 4. This results in the minimum weighted scaled deviations from the targets being **0.11499**. Results are shown in Table 6.

Table 6 - Optimal Redundancy for (e)

name	n1	n2	n3	n4
value	5	2	1	2

Figure 7 - Goal Programming Formulation for Part (e)

$$Objective\ Function\!:\ Minimize\ The\ Weighted\ Scaled\ Deviations$$

$$min\left(\sum_{j=1}^{j=2}\left(\frac{1}{t_j} \cdot (w_j^- d_j^- + w_j^+ d_j^+)\right)\right)$$

$$Decision\ Variables:\ Allocation\ of\ Redundancy\ by\ Machine$$
$$n_1, n_2, n_3, n_4$$
$$Slack\ \ Decision\ Variables$$
$$d_1^+, d_1^-, d_2^+, d_2^-$$

$$Goal\ Weights:\ Total\ Acquisition\ Cost\ is\ Twice\ as\ Important\ as\ System\ Reliability$$
$$w_1^+ = w_1^- = 1\ ;\ w_2^+ = w_2^- = 2$$

$$Parameters$$
$$R_1 = 0.87905, R_2 = 0.89683, R_3 = 0.92912, R_4 = 0.91514,$$
$$a_1 = 1.9\ , a_2 = 2.9\ , a_3 = 1.2\ , a_4 = 0.9$$
$$b_1 = 1.4\ , b_2 = 1.2\ , b_3 = 0.9\ , b_4 = 1.1$$

$$Goal\ Constraints\ and\ Targets$$
$$\left(\prod_{i=1}^{i=4}[1 - (1 - R_i)^{n_i}]\right) + d_1^- - d_1^+ = t_1 = 0.98498$$
$$\left(\sum_{i=1}^{i=4}[n_i \cdot (a_i e^{\frac{b_i}{1-R_i}})]\right) + d_2^- - d_2^+ = t_2 = \$2,766,146$$

$$Rigid\ Constraints$$
$$\sum_{i=1}^{i=4}(n_i) = 10$$
$$n_i \geq 1 \quad \forall\ i \in \{1, 2, 3, 4\}$$
$$n_i \leq 5 \quad \forall\ i \in \{1, 2, 3, 4\}$$
$$n_i \in \{\aleph\} \quad \forall\ i \in \{1, 2, 3, 4\}$$
$$d_j^+, d_j^- \geq 0\ \ \forall\ j \in \{1, 2\}$$

f. Comment on all results, discussing how the allocation changes (if any) across the different perspectives in parts a through e.

A summary of the results is shown in Table 7.

Table 7 - Summary of Redundancy By Objective

Objective	Optimal Allocation of Redundancy			
	n1	n2	n3	n4
(a)	3	3	2	2
(b)	5	3	1	1
(c)	4	2	2	2
(d)	4	2	2	2
(e)	5	2	1	2

From this, we get the intuition that machine 1 produces the most "bang for the buck" in terms of cost to reliability ratio for our goals. This is followed by machines 2, 4 and 3. This is corroborated by dividing the cost to the reliability for each of the machines, as shown in Table 8.

Table 8 - Cost to Reliability Ratios

Machine	M1	M2	M3	M4
$ / Reliability	$ 229,990	$ 363,946	$ 422,239	$ 419,365

Additionally, In all cases where goal programming was used, we undershoot on the reliability and overshoot on the cost in comparison to our targets. This makes sense considering our targets were found in (a) and (b) when reliability and cost were the only objectives, respectively.

The analysis above is only for 80 work weeks. For further analysis, sensitivity analysis could be conducted by varying the times from 0 hours to 100,000 hours, conducting parts (a) through (e) again, and finding the mode for each machine's allocation for the problems. This would allow us to make a more robust decision that would hold better across all times, rather than just base everything off of the machines' reliability and acquisition cost at two work weeks.

Problem 3

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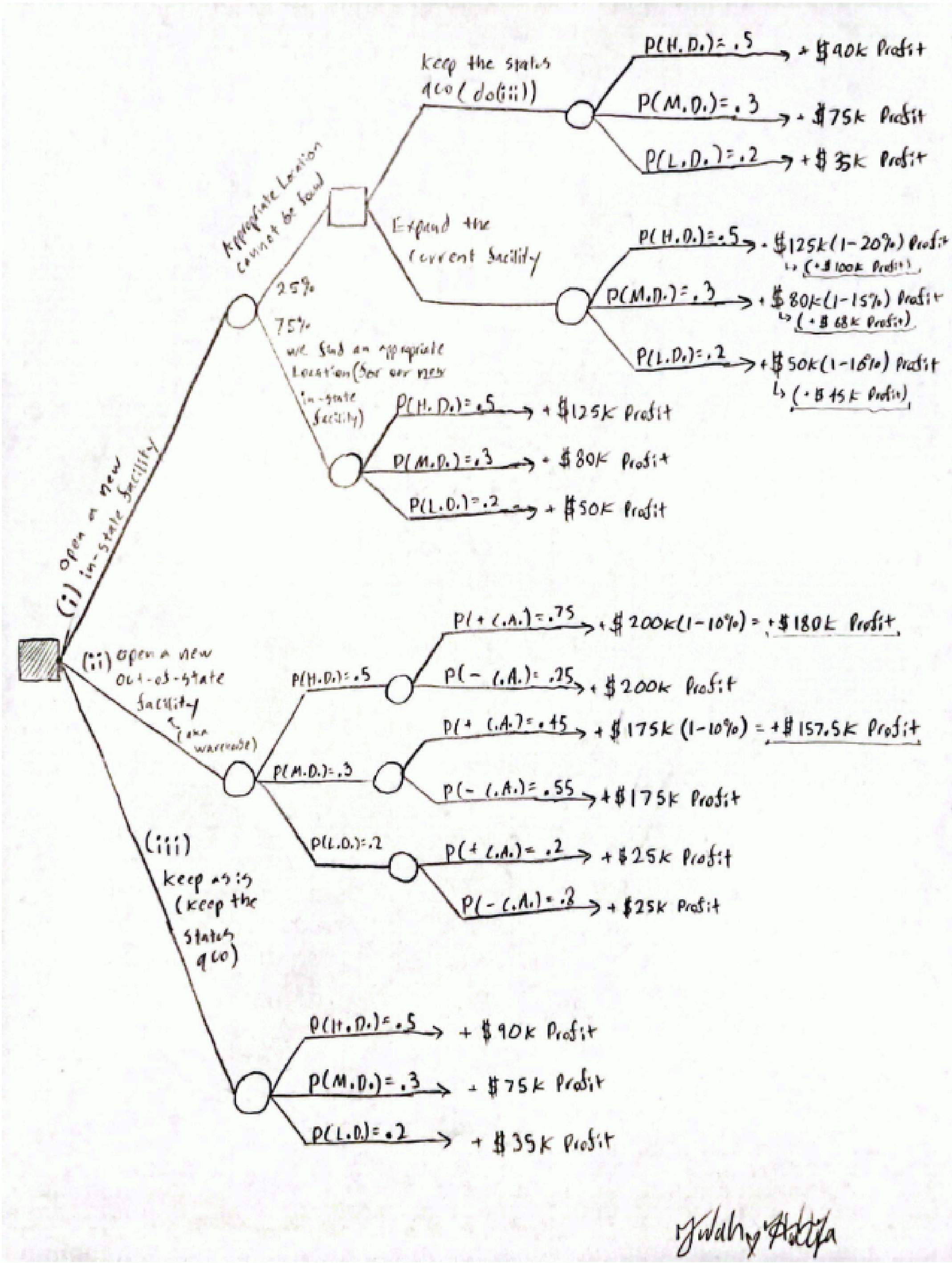
a. **Provide a graphical representation of the decision problem.**

Figure 8 - Facility Location Decision Tree



b. **Evaluate the three alternatives (i) opening in-state, (ii) opening out-of-state, and (iii) keeping the status quo.**

After evaluating the expected values of each of the alternatives, we come up with Table 9.

Table 9 - Expected Profit by Alternative

Alternative	Expected Profit
(i)	$ 92,225
(ii)	**$ 147,638**
(iii)	$ 74,500

Based on the expected value, it will be most profitable to open a new out-of-state facility warehouse.

Problem 4

a. **Calculate weights for each supplier selection criterion for each stakeholder individually. Determine if the assessments are consistent, though use them regardless.**

AHP was used to find out what each stakeholder would weight each of the three supplier selection criteria (C1, C2, C3) . Each stakeholder's consistency index (CI) was also found using AHP. The results from each analysis are summarized in Table 10.

Table 10 - Each Stakeholder's Elicited Supplier Weighs and Consistency

Stakeholder:	F	P	S	A1	A2
C1	0.647947	0.20141207	0.122182	0.652991	0.320238
C2	0.122182	0.70706033	0.229871	0.250997	0.557143
C3	0.229871	0.0915276	0.647947	0.096011	0.122619
CI	0.001848	0.0479017	0.001848	0.009174	0.009162

Because each stakeholder's consistency indexes are less than .1, each stakeholder's weightings can be assumed to be consistent.

b. **Calculate an overall set of weights for the supplier selection criterion using the weighted arithmetic mean across the five stakeholders. Interpret these weights.**

To find how much to weight each supplier's perspective, AHP was also used. The results are shown in the "Weight" row of Table 11.

Table 11 - The Importance of Each Stakeholder in the Decision

Weight	0.50523	0.28233533	0.11525	0.04859	0.04859
Stakeholder:	F	P	S	A1	A2
C1	0.64795	0.20141207	0.12218	0.65299	0.32024
C2	0.12218	0.70706033	0.22987	0.251	0.55714
C3	0.22987	0.0915276	0.64795	0.09601	0.12262

Then, a simple weighted arithmetic mean across the five stakeholders was done to find the overall supplier weightings of C1, C2, and C3. The results are shown in Table 12.

Table 12 - The Aggregated Supplier Weights

For	Overall Weight is
C1 net	0.445601982
C2 net	0.327120802
C3 net	0.227277215

Interpreting this analysis, we find that when all stakeholders' perspectives are accounted for, average lifecycle cost, worst case lifecycle cost, and maintainability are weighted as **~44.6%** important, **~32.7%** important, and **~22.7%** important, respectively. This result is valid considering all steps along the way were shown to be consistent.

Problem 5

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a. **Find the maximum likelihood estimates of annual operating cost distributions for each machine for each supplier.**

The annual operating cost distributions were assumed to be normally distributed. The method of maximum likelihood was used to obtain best-fit parameters for the Mean and Standard Deviation. These results are shown in Table 13.

Table 13 - Best-Fit Mean and Standard Deviation Parameters

Supplier's Machine #	S1, M1	S1, M2	S1, M3	S1, M4	S2, M1	S2, M2	S2, M3	S2, M4	S3, M1	S3, M2	S3, M3	S3, M4
best fit mew	13372.7	14363.5	63242.4	14020.7	17453.2	17492.4	18850.2	15968.6	15981.8	15678.4	191717.8	15189.2
best fit sigma	3162.2	4763.7	188142.0	3035.8	1548.2	1307.5	564.3	1369.6	2515.8	2632.7	530256.4	2297.3

b. Calculate the present value of average lifecycle cost of the factory floor setup for each supplier.

Present value of the average lifecycle cost was calculated by keeping the acquisition costs for new machines–equivalent to what is shown in Table 8–at present day, and transposing the average annual operating costs to the present day using the P/A rule. Results are shown in Table 14.

Table 14 - Present Value of System Cost (Average)

Supplier	Present Value of System Average Lifecycle Cost
S1	$ 5,353,513.76
S2	$ 4,855,913.59
S3	$ 7,445,435.96

On Average, supplier two is the best.

c. Calculate the present value of the worst case lifecycle cost of the factory floor setup for each supplier, where the worst case is considered to be the lifecycle cost value associated with 5% in the worst case tail.

A procedure identical to (b) was conducted, the only difference was adjusting each of the Machine annual costs to reflect their worst case annual cost, then summing those up for the overall system worst case annual cost. After transposing those to the present value, the worst case scenario results are shown in Table 15.

Table 15 - Present Value of System Cost (Worst Case)

Supplier	Present Value of System Worst Case Lifecycle Cost
S1	$ 8,954,852.55
S2	$ 4,953,723.91
S3	$ 17,123,128.35

Supplier two still remains by far the best. This is due to the low variability in each of its annual machine operating costs.

d. **Rank the suppliers according to the three criteria individually. Then provide the ranking of suppliers using TOPSIS. Scale benefits and costs according to Eqs. (3) and (4), respectively.**

Starting from Table 16, we rank the suppliers.

Table 16 - Supplier Performance by Criteria

For	C1 net	C2 net	C3 net
S1	$ 5,353,514	$ 8,954,853	36.2
S2	$ 4,855,914	$ 4,953,724	31.3
S3	$ 7,445,436	$ 17,123,128	34.8

Ranking the suppliers according to the three criteria individually, we get what is shown in Table 17.

Table 17 - Quick Ranking

	Best	Middling	Worst
C1	S2	S1	S3
C2	S2	S1	S3
C3	S2	S3	S1

To make this more rigorous and quantifiable, TOPSIS is used. The results are shown in Table 18. The higher the TOPSIS score, the better.

Table 18 - TOPSIS Ranking

Supplier	TOPSIS Score	Ranking
S1	0.782	2
S2	1	1
S3	0.590	3

According to TOPSIS, supplier two (S2) should be picked.

e. **Comment on everything, including the selection of a supplier.**

Supplier two (S2) produces the overall cheapest machines, with the lowest annual cost variability, and are also the quickest to repair on average. As such, supplier two (S2) is strictly better than all other options.

Problem 6

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a. Construct the payoff matrix associated with each combination of alternative and uncertain demand scenario.

The payoff matrix in Table 19 was constructed from the following formula for monthly profit.

Figure 9 - Formula for Monthly Widget Profit

$$M = \text{The number of widgets we make in 1 month (the alternative)}$$
$$S = \text{The number of widgets we sell in 1 month (the demand)}$$
$$Profit(M, S) = \$150,000 \cdot S - \$120,000 \cdot (M - S)$$
$$M \geq S$$

Table 19 - Payoff Matrix with Probabilities

Probability	0.02	0.06	0.09	0.11	0.13	0.15	0.18	0.11	0.07	0.05	0.03
Sold / Made	5	6	7	8	9	10	11	12	13	14	15
1											
2											
3											
4											
5	$ 750,000										
6	$ 630,000	$ 900,000									
7	$ 510,000	$ 780,000	$ 1,050,000								
8	$ 390,000	$ 660,000	$ 930,000	$ 1,200,000							
9	$ 270,000	$ 540,000	$ 810,000	$ 1,080,000	$ 1,350,000						
10	$ 150,000	$ 420,000	$ 690,000	$ 960,000	$ 1,230,000	$ 1,500,000					
11	$ 30,000	$ 300,000	$ 570,000	$ 840,000	$ 1,110,000	$ 1,380,000	$ 1,650,000				
12	$ (90,000)	$ 180,000	$ 450,000	$ 720,000	$ 990,000	$ 1,260,000	$ 1,530,000	$ 1,800,000			
13	$ (210,000)	$ 60,000	$ 330,000	$ 600,000	$ 870,000	$ 1,140,000	$ 1,410,000	$ 1,680,000	$ 1,950,000		
14	$ (330,000)	$ (60,000)	$ 210,000	$ 480,000	$ 750,000	$ 1,020,000	$ 1,290,000	$ 1,560,000	$ 1,830,000	$ 2,100,000	
15	$ (450,000)	$ (180,000)	$ 90,000	$ 360,000	$ 630,000	$ 900,000	$ 1,170,000	$ 1,440,000	$ 1,710,000	$ 1,980,000	$ 2,250,000
16	$ (570,000)	$ (300,000)	$ (30,000)	$ 240,000	$ 510,000	$ 780,000	$ 1,050,000	$ 1,320,000	$ 1,590,000	$ 1,860,000	$ 2,130,000
17	$ (690,000)	$ (420,000)	$ (150,000)	$ 120,000	$ 390,000	$ 660,000	$ 930,000	$ 1,200,000	$ 1,470,000	$ 1,740,000	$ 2,010,000
18	$ (810,000)	$ (540,000)	$ (270,000)	$ -	$ 270,000	$ 540,000	$ 810,000	$ 1,080,000	$ 1,350,000	$ 1,620,000	$ 1,890,000
19	$ (930,000)	$ (660,000)	$ (390,000)	$ (120,000)	$ 150,000	$ 420,000	$ 690,000	$ 960,000	$ 1,230,000	$ 1,500,000	$ 1,770,000
20	$ (1,050,000)	$ (780,000)	$ (510,000)	$ (240,000)	$ 30,000	$ 300,000	$ 570,000	$ 840,000	$ 1,110,000	$ 1,380,000	$ 1,650,000

b. What monthly production decision should be made according to a pessimistic decision maker?

The worst case is that only 5 are sold. So, out of our "5" column, picking the best scenario out of the worst case would entail choosing to **make only 5 widgets per month**.

c. **What monthly production decision should be made according to an optimistic decision maker?**

The best case is that 15 are sold. So, out of our "15" column, picking the best scenario out of the worst case would entail choosing to **make 15 widgets per month**.

d. **What monthly production decision should be made if the decision-maker uses solely the expected value?**

However, we know the probabilities, so decision making under uncertainty is not needed. Let us use these probabilities to calculate the expected values of each production alternative. This is shown in Table 20.

Table 20 - Expected Monthly Profit by Alternative

Alternative	Expected Value of Profit
1	
2	
3	
4	
5	$ 750,000
6	$ 894,600
7	$ 1,023,000
8	$ 1,127,100
9	$ 1,201,500
10	**$ 1,240,800**
11	$ 1,239,600
12	$ 1,189,800
13	$ 1,110,300
14	$ 1,011,900
15	$ 900,000
16	$ 780,000
17	$ 660,000
18	$ 540,000
19	$ 420,000
20	$ 300,000

As seen, we should make **10** widgets per month to maximize average expected value of profit. But the consequences of overproducing by 1 are virtually inconsequential. Thus, we should err on the side of overproducing rather than underproducing.

e. **We assumed that customers wouldn't "balk" (i.e., wouldn't get upset if Haimes, Inc. had already sold its monthly widget production and thus the customer's demand goes unmet). How would the problem change if we wanted to consider balking (don't do any calculations but discuss how the structure of the problem would change)?**

If customers balked, then everywhere that is in red in Table 19 would have a penalty cost in it reflective of the opportunity cost. Because of these new values, we would have to re-run our pessimistic, optimistic, and expected value analysis to reflect this change.

Problem 7

Be reflective of all the tools that you've integrated to solve this series of fairly simple problems. Summarize in three sentences. Then write a 5-7-5 haiku devoted to TOPSIS.

Summary:

We used the Weibull distribution to model machine reliability, goal programming to find the best allocation of redundancy, and a decision tree to find where to build a new facility. Then we used AHP to create weights to balance stakeholder perspectives on machine supplier criteria. Then, we used the method of maximum likelihood to fit normal distributions to annual costs, Engineering Economics to transpose these annual future costs into the present, decided which supplier to get the new machines from, and used an empirical distribution of demand to optimize our expected value of profits.

Haiku:

TOPSIS, can't you see?
You and AHP blend so
commensurately