Example. 2.1. *Three forces F_1, F_2 and F_3 are acting on a body as shown in Fig. 2.7 and the body is in equilibrium. If the magnitude of force F_3 is 400 N, find the magnitude of forces F_1 and F_2.*

Solution: Given:

Force, $F_3 = 400$ N.

As the body is in equilibrium, the resultant force in x-direction should be zero and also the resultant force in y-direction should be zero.

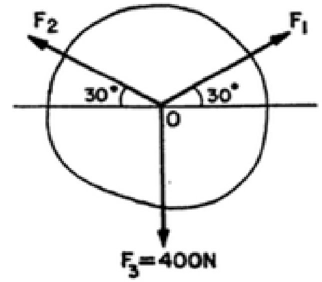


Fig. 2.7

For, $\sum F_x = 0$, we get

$$F_1 \cos 30^o - F_2 \cos 30^o = 0$$

$$Or\ F_1 - F_2 = 0$$

$$Or\ F_1 = F_2$$

For, $\sum F_y = 0$, we get

$$F_1 \sin 30^o + F_2 \sin 30^o - 400 = 0$$

$$\therefore F_1 = F_2 = 400\ N$$

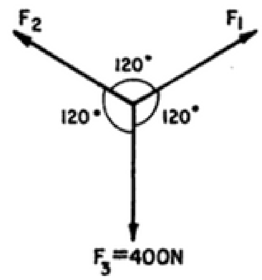


Fig. 2.8

2nd Method

If three forces are acting on a body at a point and the body is in equilibrium, Lami's theorem can be applied.

Using Lami's theorem,

$$\frac{F_1}{\sin sin\ 120} = \frac{F_2}{\sin sin\ 120} = \frac{400}{\sin sin\ 120}$$

$$\therefore F_1 = F_2 = 400\ N$$

Example. 2.2. *A lamp weighing 5 N is suspended from the ceiling by a chain. It is pulled by a horizontal cord until the chain makes an angle of 60°with the ceiling as shown in Fig. 2.9 find the tensions in the chain and the cord by applying Lami's theorem.*

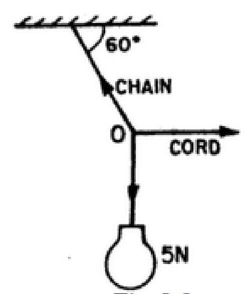


Fig. 2.9

Solution: Given:

Weight of lamp = 5 N

Angle made by chain with ceiling = 60°

Let T_1 = tension in the cord

T_2 = tension in the chain

Now from the geometry, it is obvious that angle between T_1 and lamp will be 90°, between lamp and T_2 150° and between T_2 and T_1 120°

Applying Lami's theorem, we get

$$\frac{T_1}{\sin sin\ 150} = \frac{T_2}{\sin sin\ 90} = \frac{5}{\sin sin\ 120}$$

$$\therefore T_1 = 5 \times \frac{\sin sin\ 150}{\sin sin\ 120} = 2.887\ N$$

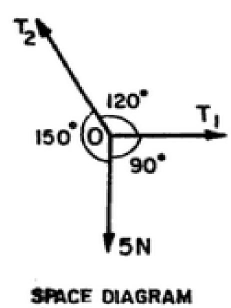


Fig. 2.10

$$\therefore T_2 = 5 \times \frac{\sin sin\ 90}{\sin sin\ 120} = 5.774\ N$$

Example 2.3. *A body weighing 2000 N is suspended with a chain AB 2 m long. It is pulled by a horizontal force of 320 N as shown in Fig. 2.11. Find the force in the chain and the lateral displacement of the body.*

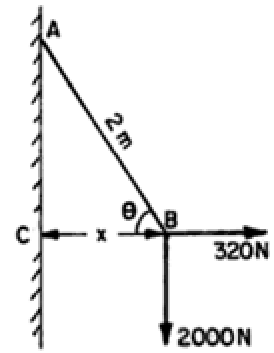


Fig. 2.11

Solution. Given:

Weight suspended at B = 2000 N

Length AB = 2 m

Horizontal force at B = 320 N

Let F = force in chain AB

θ = angle made by AB with horizontal.

The point B is in equilibrium under the action of three forces. Hence using Lami's theorem, we get

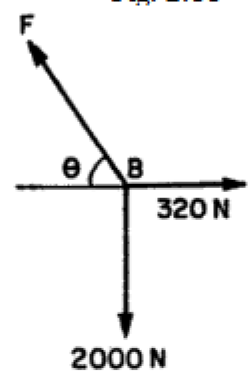


Fig. 2.12

$$\frac{F}{\sin sin\ 90} = \frac{2000}{\sin sin\ (180-\theta)} = \frac{320}{\sin sin\ (90+\theta)}$$

$$F \sin sin\ \theta = 2000 \qquad \ldots\ldots\ldots\ldots\ldots (i)$$

$$F \cos cos\ \theta = 320 \qquad \ldots\ldots\ldots\ldots\ldots (ii)$$

Dividing equation (*i*) by equation (*ii*), we get

$$\tan tan\ \theta = \frac{2000}{320} = 6.25$$

$$\therefore \theta = tan^{-1}(6025) = 80.9^o$$

Substituting the value of θ in equation (*i*), we get

$$F \sin sin\ 80.9 = 2000$$

$$\therefore F = 2025.5\ N$$

From the Fig. $\cos cos\ \theta = \frac{x}{2}$

$$\therefore x = 2 \cos cos\ 80.9 = 0.3163\ m.$$

Example 2.4. *A ball of weight 120 N rests in a right-angled groove, as shown in Fig. 2.13. The sides of the groove are inclined at an angle of 30°and 60°to the horizontal. If all the surfaces are smooth, then determine the reactions R_A and R_C at the points of contact.*

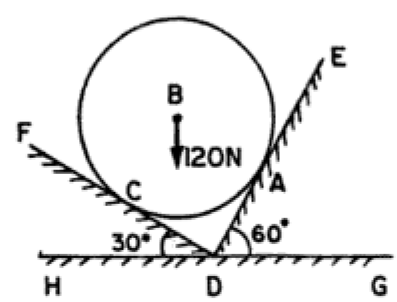


Fig. 2.13

Solution. Given:

Weight of the ball, W = 120 N

Angle of groove = 90°

Angle made by side FD with horizontal = 30°

Angle made by side ED with horizontal = 60°

Consider the equilibrium of the ball.

For the equilibrium of the ball,

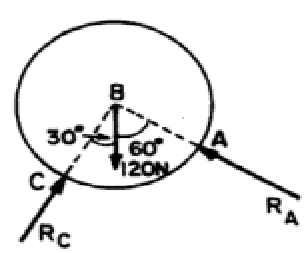


Fig. 2.14

$$\sum F_x = 0 \text{ and } \sum F_y = 0$$

For$\sum F_x = 0$, we have

$$R_C \sin sin\, 30 - R_A \sin sin\, 60 = 0$$

$$\therefore R_C = R_A \times \frac{\sin sin\, 60}{\sin sin\, 30} = 1.732\, R_A$$

For $\sum F_y = 0$, we have

$$120 - R_A\, cos60 - R_C \cos cos\, 30 = 0$$

$$120 = R_A \times 0.5 + (1.732 R_A) \times 0.866$$

$$\therefore R_A = \frac{120}{2} = 60\, N$$

$$\therefore R_C = 1.732 \times 60 = 103.92\, N$$

Example 2.5. *Determine the reactions at A and B for the Fig. 2.15*

Solution. Given:

Weight of ball, W = 100 N

Consider the equilibrium of the ball.

The reactions R_A and R_B will pass through the center of the ball. The three forces are acting on the ball and ball is in equilibrium hence Lami's theorem can be applied. The three forces meet at a point. Using Lami's theorem, we get

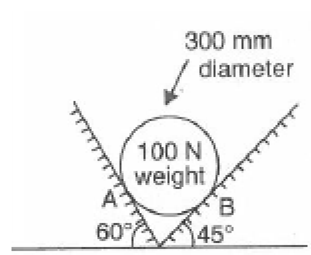


Fig. 2.15

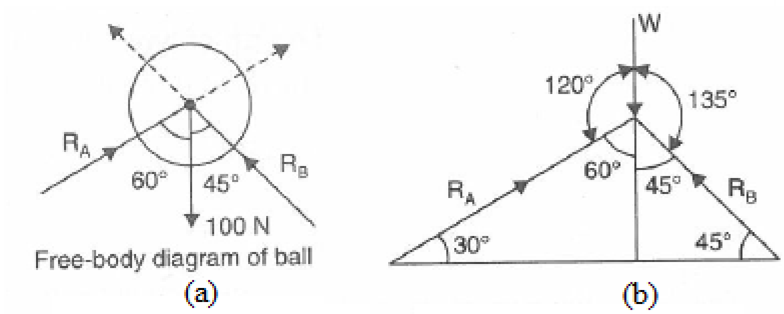


Fig. 2.16

$$\frac{W}{\sin sin\, 105} = \frac{R_A}{\sin sin\, 135} = \frac{R_B}{\sin sin\, 120}$$

$$\therefore R_A = 100 \times \frac{\sin sin\, 135}{\sin sin\, 105} = 73.2\, N$$

$$\therefore R_B = 100\times \frac{\sin sin\, 120}{\sin sin\, 105} = 89.65\, N$$

Example 2.6. *A circular roller of radius 5 cm and of weight 100 N rests on a smooth horizontal surface and is held in position by an inclined bar AB of length 10 cm as shown in Fig. 2.17 A horizontal force of 200 N is acting at B. Find the tension in the bar AB and the vertical reaction at C.*

Solution. Given:
Weight, W = 100 N
Radius *i.e.,* BC = 5 cm
Length of bar, AB = 10 cm
Horizontal force at B = 200 N

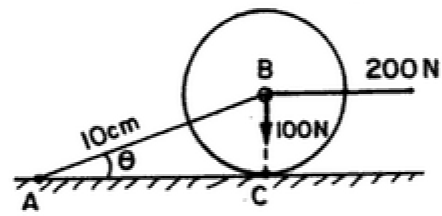


Fig. 2.17

In $\Delta\, ABC$, $\sin sin\, \theta = \frac{BC}{AB} = 0.5$

$$\therefore \theta = sin^{-1}(0.5) = 30^{o}$$

Let T = tension in the string AB
Consider the equilibrium of the roller.
Consider the equilibrium of the ball.
For the equilibrium of the roller,

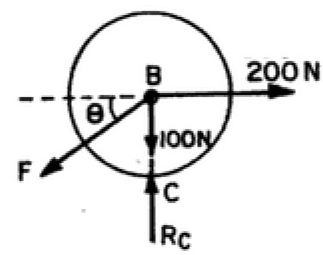


Fig. 2.18

$$\sum F_x = 0 \text{ and } \sum F_y = 0$$

For $\sum F_x = 0$, we have

$$T \cos cos\, \theta - 200 = 0$$

$$\therefore T = \frac{200}{\cos cos\, 30} = 230.94\, N$$

For $\sum F_y = 0$, we have

$$R_C - W - T \sin sin\, \theta = 0$$

$$\therefore R_C = 100 + 230.94\times \sin sin\, 30 = 215.47\, N$$

Example. 2.7. *Two identical rollers, each of weight W = 1000 N, are supported by an inclined plane and a vertical wall as shown in Fig. 2.19 Find the reactions at the points of supports A,B, and C. Assume all the surfaces to be smooth.*

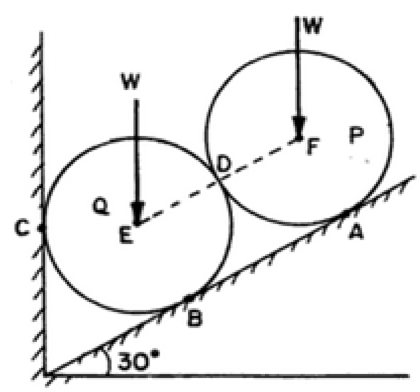
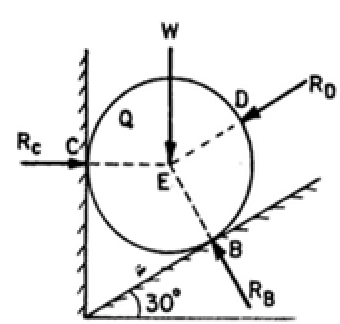


Fig. 2.19

Solution. Given:
Weight of each roller = 1000 N
Radius of each roller is same. Hence line EF will be parallel to AB.
Equilibrium of roller P:



For $\sum F_x = 0$, we have

$$R_D \sin sin\, 60 - R_A \sin sin\, 30 = 0$$

$$\therefore R_D = R_A \frac{\sin sin\, 30}{\sin sin\, 60} = 0.577 R_A \quad \ldots\ldots\ldots\ldots (i)$$

For $\sum F_y = 0$, we have

$$R_D \cos cos\, 60 + R_A \cos cos\, 30 - 1000 = 0$$

$$0.577 R_A \times 0.5 + R_A \times 0.866 = 1000$$

$$\therefore R_A = \frac{1000}{1.545} = 866.17 N$$

Substituting the value of R_A on equation (i), we get

$$\therefore R_D = 0.577 \times 866.17 = 499.78\ N$$

Equilibrium of roller Q:

For $\sum F_x = 0$, we have

$$R_B \sin sin\, 30 + R_D \sin sin\, 60 - R_C = 0$$

$$R_B \times 0.5 + 449.78 \times 0.866 - R_C = 0$$

$$R_C = 0.5\, R_B + 432.8 \quad \ldots\ldots\ldots\ldots (ii)$$

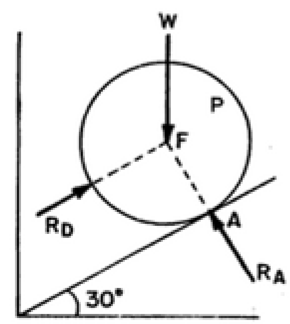


Fig. 2.21

For $\sum F_y = 0$, we have

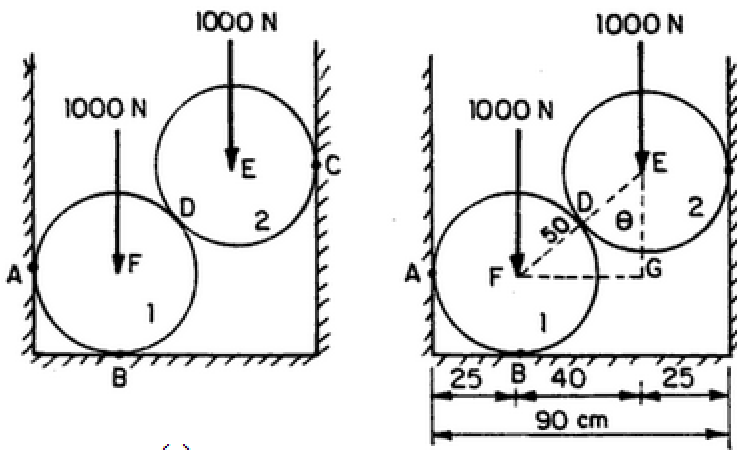
$$R_B \cos cos\, 30 - 1000 - R_D \cos cos\, 60 = 0$$

$$\therefore R_B = \frac{1249.89}{0.866} = 1443.3\ N$$

Substituting the value of R_B in equation (ii), we get

$$R_C = 0.5 \times 1443.3 + 432.8 = 1154.45\ N$$

Example. 2.8. *Two spheres, each of weight 1000 N and of radius 25 cm rest in a horizontal channel of width 90 cm as shown in Fig. 2.22. Find the reactions on the points of contact A, B, and C.*



(a) (b)

Fig. 2.22

Solution. Given:

Weight of each sphere, W = 1000 N

Radius of each sphere, R = 25 cm

∴ AF = BF =FD = DE = CE = 25 cm

Width of horizontal channel = 90 cm

Join the center E to center F.

Now EF = 25 + 25 = 50 cm, FG = 40 cm

In $\Delta\, EFG,\ EG\ =\ \sqrt{EF^2 - FG^2}\ =\ \sqrt{50^2 - 40^2}\ =\ 30\ cm$

$\therefore \cos cos\,\theta\ =\ \frac{EG}{EF}\ =\ \frac{30}{50}\ =\ \frac{3}{5}\ and\ \sin sin\,\theta\ =\ \frac{FG}{EF}\ =\ \frac{40}{50}\ =\ \frac{4}{5}$

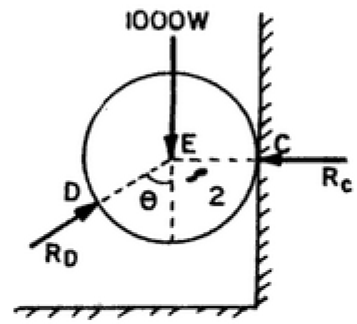


Fig. 2.23

Equilibrium of sphere No.2:

For $\sum F_x\ =\ 0$, we have

$$R_D \sin sin\,\theta\ =\ R_C \qquad \ldots\ldots\ldots (i)$$

For $\sum F_y\ =\ 0$, we have

$R_D \cos cos\,\theta\ =\ 1000$

$$\therefore R_D\ =\ \frac{1000}{cos cos\,\theta}\ =\ 1000\times\frac{5}{3}\ =\ \frac{5000}{3}\ N$$

Substituting the value of R_D in equation (*i*),

$$\therefore R_C\ =\ \frac{5000}{3}\times\frac{4}{5}\ =\ 1333.33\ N$$

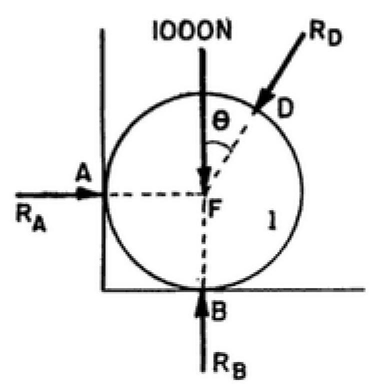


Fig. 2.24

Equilibrium of Sphere No.1:

For $\sum F_x\ =\ 0$, we have

$$R_A = R_D \sin sin\, \theta$$

$$\therefore R_A = 1333.33\ N$$

For $\sum F_y = 0$, we have\

$$R_B - 1000 - R_D \cos cos\, \theta = 0$$

$$\therefore R_B = 1000 + \frac{5000}{3} \times \frac{3}{5} = 2000\ N.$$

Example. 2.9. *Two smooth circular cylinders, each of weight W = 1000 N and radius 15 cm, are connected at their centers by a string AB of length 40 cm and rest upon a horizontal plane, supporting above them is a third cylinder of weight 2000 N and radius 15 cm as shown in Fig. 2.25. Find the force S in the string AB and the pressure produced on the floor at the points of contact D and E.*

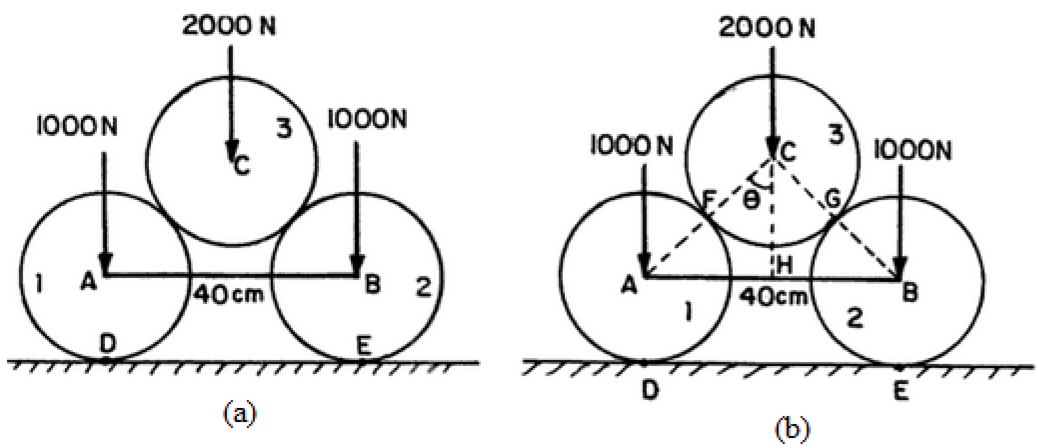


Fig. 2.25

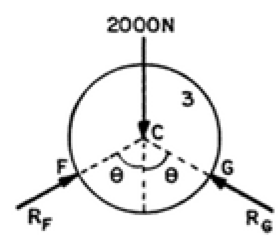


Fig. 2.26

Solution. Given:
Weight of cylinders 1 and 2 = 1000 N
Weight of cylinder 3 = 2000 N
Radius of each cylinder = 15 cm
Length of string AB = 40 cm
From Fig. AC = AF + FC = 15 + 15 = 30 cm

$AH = \frac{1}{2} \times AB = \frac{1}{2} \times 40 = 20$ cm

From $\Delta\, ACH$, $\sin sin\, \theta = \frac{AH}{AC} = \frac{20}{30} = 0.667$

$$\therefore \theta = sin^{-1}(0.667) = 41.836^o$$

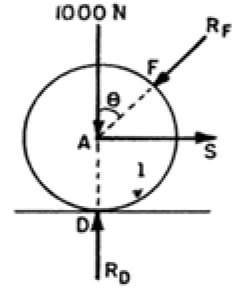


Fig. 2.27

Equilibrium of cylinder 3:
Resolving the forces horizontally,

$$R_F \sin sin\,\theta - R_G \sin sin\,\theta = 0$$

$$\therefore R_F = R_G$$

Resolving the forces vertically,

$$R_F \cos cos\,\theta + R_G \cos cos\,\theta = 2000$$

$$R_F = \frac{2000}{2\times cos 41.836} = 1342.179\ N$$

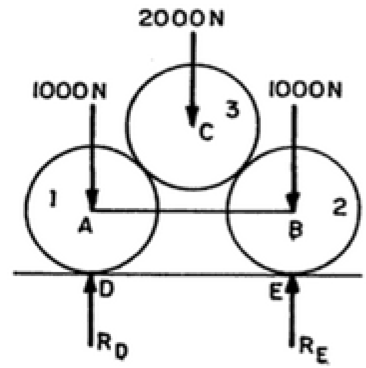


Fig. 2.28

Equilibrium of cylinder 1:
Resolving the forces horizontally,

$$S - R_F \sin sin\,\theta = 0$$

$$\therefore S = 1342.179 \times \sin sin\,41.836 = 895.2\ N$$

Resolving the forces vertically,

$$R_D - 1000 - R_F \cos cos\,\theta = 0$$

$$\therefore R_D = 1000 + 1342.179 \times \cos cos\,41.836 = 1999.99 = 2000\ N$$

Equilibrium of cylinders 1, 2 and 3 taken together:
Resolving the forces vertically,

$$R_D + R_E - 1000 - 2000 - 1000 = 0$$

$$\therefore R_E = 4000 - 2000 = 2000\ N$$

Example. 2.10. *A roller of radius 40 cm, weighing 3000 N is to be pulled over a rectangular block of height 20 cm as shown in Fig. 2.29 by a horizontal force applied at the end of a string wound round the circumference of the roller. Find the magnitude of the horizontal force which will just turn the roller over the corner of the rectangular block. Also determine the magnitude and direction of reactions at A and B. All surfaces may be taken as smooth.*

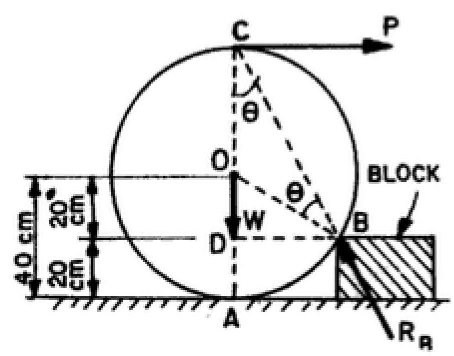


Fig. 2.29

Solution. Given:
Radius of the roller = 40 cm
Weight W = 3000 N
Height of block = 20 cm
When the roller is about to turn over the corner of the rectangular block, the roller lifts at the point A and then there will be no contact between the roller and the point A. hence reaction R_A at point A will become zero.

For equilibrium, the forces P, W and R_B should pass through a common point. As the force P and weight W is passing through point C, hence the reaction R_B must also pass through the point C. Therefore, the line BC gives the direction of the reaction R_B.

$$From\ \Delta BCD,\ BO = Radius = 40\ cm,$$

$$OD = OA - AD = 40 - 20 = 20\ cm$$

$$\therefore BD = \sqrt{BO^2 - OD^2} = \sqrt{40^2 - 20^2} = 34.64\ cm$$

$$\tan tan\ \theta = \frac{BD}{CD} = \frac{34.64}{CO+OD} = \frac{34.64}{40+20} = 0.5773$$

$$\therefore \theta = tan^{-1}(0.5773) = 29.999^o = 30^o$$

Resolving the forces horizontally,

$$P - R_B \sin sin\ \theta = 0$$

$$P = R_B \sin sin\ 30 = 0.5\ R_B$$

……….. (*i*)

Resolving forces vertically,

$$W - R_B \cos cos\ \theta = 0$$

$$\therefore R_B = \frac{3000}{\cos cos\ 30} = 3464.2\ N$$

Substituting the value of R_B in equation (*i*),

$$\therefore P = 0.5\times 3464.2 = 1732.1\ N$$

2nd method

This problem can also be solved by taking moments of all the three forces about the point B

$$P\times CD = W\times BD$$

$$P\times 60 = 3000\times 34.64$$

$$\therefore P = \frac{3000\times 34.64}{60} = 1732\ N$$

Example.2.11. *If in the above problem, the force P is applied horizontally at the center of the roller, what would be the magnitude of this force? Also determine the least force and its line of action at the roller center, for turning the roller over the rectangular block.*

Solution. Given:

Radius of roller = 40 cm

Weight W = 3000 N

Height of block = 20 cm

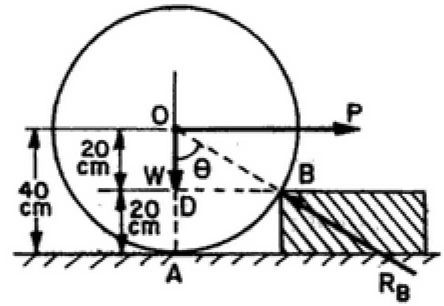


Fig. 2.30

When the roller is about to turn over the corner of the rectangular block, the roller lifts at the point A and then there will be no contact between the roller and the point A. hence reaction R_A at point A will become zero.

For equilibrium, the forces P, W and R_B should pass through a common point. As the force P and weight W is passing through point O, hence the reaction R_B must also pass through the point O. Therefore, the line BO gives the direction of the reaction R_B.

From $\Delta\, BOD$ $\cos cos\, \theta\, =\, \frac{OD}{BO} = \frac{20}{40} =\, 0.5$

$$\therefore \theta\, =\, cos^{-1}(0.5) = 60^{o}$$

Resolving the forces vertically

$$R_B \cos cos\, \theta\, -\, W\, =\, 0$$

$$\therefore R_B\, =\, \frac{W}{\cos cos\, \theta} = \frac{3000}{\cos cos\, 60}\, =\, 6000\, N$$

Resolving the forces horizontally

$$P\, -\, R_B \sin sin\, \theta\, =\, 0$$

$$\therefore P\, =\, R_B \sin sin\, \theta\, =\, 6000\times \sin sin\, 60\, =\, 5196\, N$$

2nd method

The force P can also be calculated by the method of moments. Taking moments of all the forces about the point B, we get

$$P\times OD\, -\, W\times BD\, =\, 0$$

$$\therefore P\, =\, \frac{W\times BD}{OD} = \frac{3000\times 34.64}{20} = 5196\, N$$

Least force and its line of action:

Let P_{min} = Least force applied.

A = Angle of the least force

From $\Delta\, OBC,\, BC\, =\, BO \sin sin\, \alpha$

Taking moments of all forces about point B, we get

$$P_{min}\times BC\, -\, W\times BD\, =\, 0$$

$$\therefore P_{min} = \frac{W\times BD}{BC} = \frac{3000\times 34.64}{BO \sin sin\, \alpha} = \frac{3000\times 34.64}{40\, \sin sin\, \alpha}$$

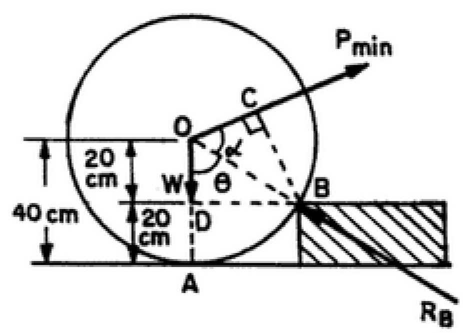


Fig. 2.31

The force P will be minimum when sin α is maximum. But sin α will be maximum, when α = 90° or sin α =1. Substituting the value of α, we get minimum force.

$$\therefore P_{min} = \frac{3000\times 34.64}{40\times 1} = 2598\, N.$$

Example.2.12. *A rigid bar AB with balls of weights W_1=75 N and W_2 = 125 N at ends is supported inside a circular ring as shown in Fig. 2.32 Radius of the ring and AB are such that radii AC and BC make right angle at the centre of the ring C. Neglecting friction and weight of AB, ascertain equilibrium configuration defined by angle [(α – β)/2] that AB makes with the horizontal. Find the contact reaction at A and B, and axial force in rod AB.*

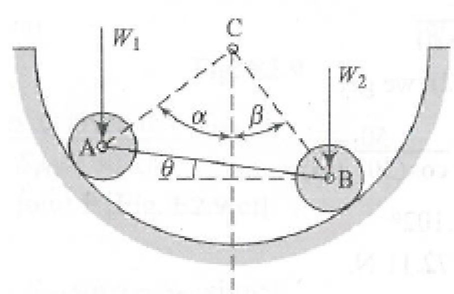


Fig. 2.32

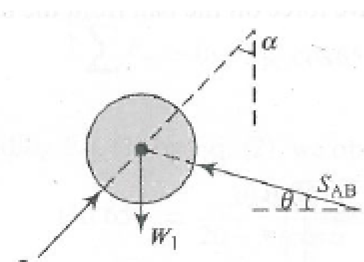


Fig. 2.33

Solution.

From the condition of inclination of AB,

$\theta = \frac{\alpha-\beta}{2}$ *or* $\theta + \beta = \alpha - \theta$

Consider Equilibrium of left ball

Resolving forces horizontally,

$$T \cos cos\, \theta = R_1 \sin sin\, \alpha$$

$$\therefore R_1 = T \frac{\cos cos\, \theta}{\sin sin\, \alpha}$$

Resolving the forces vertically,

$$R_1 \cos cos\, \alpha + T \sin sin\, \theta = W_1$$

$$T \frac{\cos cos\, \theta}{\sin sin\, \alpha} \cos cos\, \alpha + T\, sin\theta = 75$$

$$T \frac{\cos cos\, (\alpha-\theta)}{\sin sin\, \alpha} = 75 \qquad \ldots (i)$$

Consider equilibrium of right ball

Resolving forces horizontally,

$$T \cos cos\, \theta = R_2 \sin sin\, \beta$$

$$\therefore R_2 = T \frac{\cos cos\, \theta}{\sin sin\, \beta}$$

Resolving the forces vertically,

$$R_2 \cos cos\, \beta - T \sin sin\, \theta = W_2$$

$$T \frac{\cos cos\, \theta}{\sin sin\, \beta} \cos cos\, \beta - T\, sin\theta = 125$$

$$T \frac{\cos cos\, (\theta+\beta)}{\sin sin\, \beta} = 125 \qquad \ldots (ii)$$

Equation (ii) divided by equation (i)

$$\frac{\cos cos\, (\theta+\beta)}{\sin sin\, \beta} \times \frac{\sin sin\, \alpha}{\cos cos\, (\alpha-\theta)} = \frac{125}{75}$$

$$\frac{\sin sin\ \alpha}{\sin sin\ \beta} = \frac{5}{3}\ or\ \frac{\sin sin\ \alpha}{\sin sin\ (90-\alpha)} = \frac{5}{3}$$

$$\therefore \tan tan\ \alpha = \frac{5}{3}$$

$$\therefore \alpha = 59.04^o$$

$$\beta = 90 - 59.04 = 30.96^o$$

$$\theta = 14.04^o$$

By solving, we get

$$T = 90.95\ N;\ R_1 = 102.89\ N;\ R_2 = 171.51\ N$$

Example.2.13. *A prismatic bar of length 2.1 m and weight 365 N rests within a hemispherical bowl of radius 0.7 m, as shown in Fig. 2.35. Determine the angle α for the position of equilibrium.*

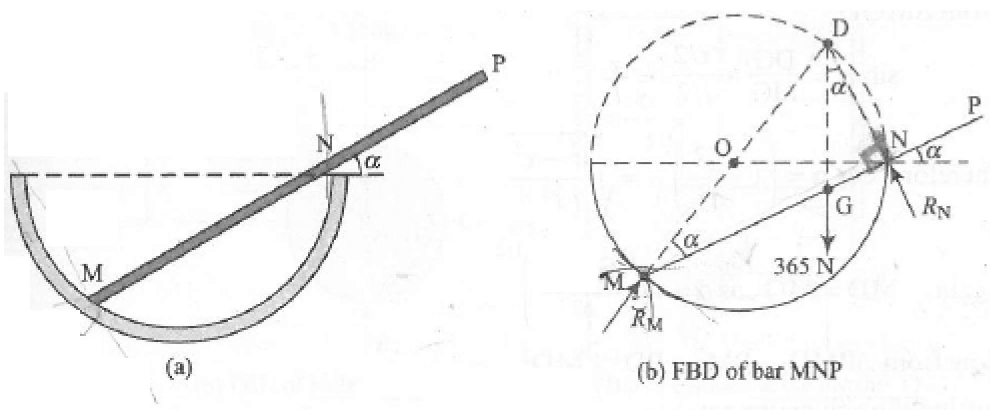


Fig. 2.35

Solution.

The bar will remain in equilibrium under the action of reactions at M and N, and self weight of the bar acting at G. hence, these three forces must be concurrent.

From $\Delta\ MDN$,

$$MN = MD \cos cos\ \alpha = 2\times0.7 \cos cos\ \alpha$$

$$DN = MD \sin sin\ \alpha = 2\times0.7 \sin sin\ \alpha$$

$$GN = MN - NG = 1.4 \cos cos\ \alpha - \frac{2.1}{2}$$

From $\Delta\ GDN$,

$$\tan tan\ \alpha = \frac{GN}{DN} = \frac{1.4\cos cos\ \alpha - 1.05}{1.4\sin sin\ \alpha}$$

$$1.4\,sin^2\alpha = 1.4\,cos^2\alpha - 1.05\cos cos\,\alpha$$

$$1.4 - 1.4cos^2\alpha = 1.4\,cos^2\alpha - 1.05\cos cos\,\alpha$$

$$cos^2\alpha - 0.375\cos cos\,\alpha - 0.5 = 0$$

$$\cos cos\,\alpha = 0.919$$

$$\therefore \alpha = 23.21^o$$

Problems on connected bodies

Example. 2.14. *A system of connected flexible cables shown in Fig. 2.36 is supporting two vertical forces 200 N and 250 N at points B and D. Determine the forces in various segments of the cable.*

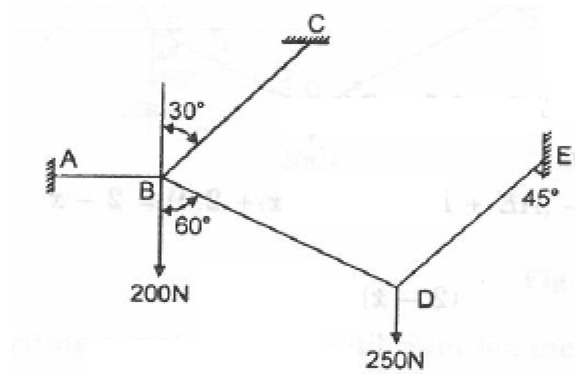


Fig. 2.36

Solution. Given:

Applying Lami's theorem to the system of forces at point D,

$$\frac{T_1}{\sin sin\,120} = \frac{T_2}{\sin sin\,135} = \frac{250}{\sin sin\,105}$$

$$\therefore T_1 = 250 \times \frac{\sin sin\,120}{\sin sin\,125} = 224.14\,N$$

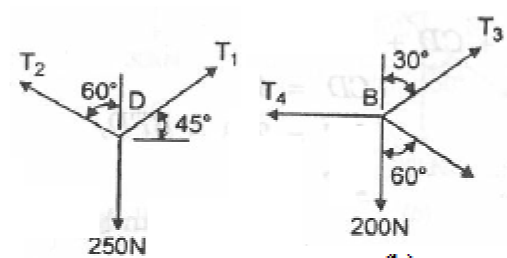
$$\therefore T_2 = 250 \times \frac{\sin sin\,135}{\sin sin\,125} = 183.01\,N$$

Consider the system of forces acting at B

Resolving forces vertically,

$$T_3\cos cos\,30 - 200 - T_2\cos cos\,60 = 0$$

$$\therefore T_3 = \frac{200+183.01\cos cos\,60}{\cos cos\,30} = 336.6\,N$$



(a) (b)

Fig. 2.37

Resolving forces horizontally,

$$T_4 - T_2\sin sin\,60 - T_3\sin sin\,30 = 0$$

$$\therefore T_4 = 183.01\times \sin sin\,60 + 336.6 \times \sin sin\,30 = 326.79\,N$$

Example. 2.15. *A wire rope is fixed at two points A and D as shown in Fig. 2.38 Two weights 20 kN and 30kN are attached to it at B and C, respectively. The weights rest with portions AB and BC inclined at 30°and 50° respectively to the vertical. Find the tension in the wire in segments AB, BC and CD and also the inclination of the segments CD to vertical.*

Solution.

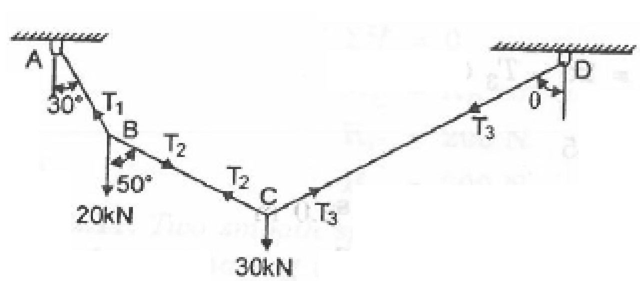


Fig. 2.38

Applying Lami's theorem for the system of forces at B,

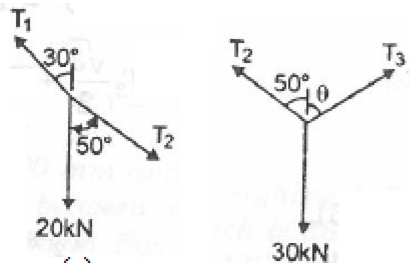
$$\frac{T_1}{\sin sin\, 50} = \frac{T_2}{\sin sin\, 150} = \frac{20}{\sin sin\, 160}$$

$$\therefore T_1 = 20 \times \frac{\sin sin\, 500}{\sin sin\, 160} = 44.79\ kN$$

$\therefore T_2 = 20 \times \frac{\sin sin\, 150}{\sin sin\, 160} = 29.24\ kN$

Consider the system of forces acting at C

Resolving forces horizontally



(a) (b)

Fig. 2.39

$T_3 \sin sin\, \theta = T_2 \sin sin\, 50 = 29.24 \times \sin sin\, 50 = 22.4\ kN$ ……….. (*i*)

Resolving forces vertically,

$T_3 \cos cos\, \theta = 30 - T_2 \cos cos\, 50 = 11.2\ kN$ ………. (*ii*)

Dividing equation (*i*) by equation (*ii*)

$$\tan tan\, \theta = 1.998$$

$$\therefore \theta = 63.422^o$$

$$T_3 = 25.045\ kN$$

Example.2.16. *A wire is fixed at two points A and D as shown in Fig. 2.40. Two weights 20 kN and 25 kN are supported at B and C, respectively. When equilibrium is reached it is found that inclination of AB is 30°and that of CD is 60°to the vertical. Determine the tension in the segments AB, BC, and CD of the rope and also the inclination of BC to the vertical.*

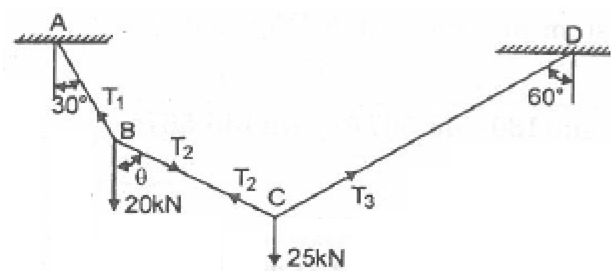


Fig. 2.40

Solution.

Consider the system of forces acting at B

Resolving the forces horizontally

$$T_2 \sin sin\, \theta = T_1 \sin sin\, 30$$

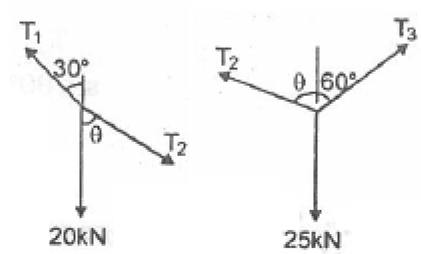
……………. (*i*)

Resolving th1e forces vertically

$$T_2 \cos cos\, \theta = T_1 \cos cos\, 30 - 20$$

.……………. (*ii*)

Consider the system of forces acting at C



(a) (b)

Fig. 2.41

Resolving the forces horizontally

$$T_2 \sin sin\, \theta \ = \ T_3 \sin sin\, 60 \qquad \ldots\ldots\ldots\ldots\ldots (iii)$$

Resolving th1e forces vertically

$$T_2 \cos cos\, \theta \ = \ 25 \ - \ T_3 \cos cos\, 60 \qquad \ldots\ldots\ldots\ldots\ldots (iv)$$

From (*i*) and (*iii*)

$$T_1 \sin sin\, 30 \ = T_3 \sin sin\, 60$$

$$\therefore T_1 \ = \ \sqrt{3} T_3$$

From (*ii*) and (*iv*)

$$T_1 \cos cos\, 30 \ - \ 20 \ = \ 25 \ - \ T_3 \cos cos\, 60$$

$$\sqrt{3} T_3 \frac{\sqrt{3}}{2} + T_3 \frac{1}{2} = \ 45$$

$$\therefore T_3 \ = \ 22.5\ kN$$

$$T_1 \ = \ \sqrt{3} \times 22.5 \ = \ 38.97\ kN$$

From equation (*i*), $T_2 \sin sin\, \theta \ = \ 38.97 \times \ \sin sin\, 30 \ = \ 19.48$

From equation (*ii*), $T_2 \cos cos\, \theta \ = \ 38.97 \times \ \cos cos\, 30 \ - \ 20 \ = \ 13.75$

$$\therefore \tan tan\, \theta \ = \ 1.4167$$

$$\theta \ = \ tan^{-1}(1.4167) = 54.78^o$$

$$\therefore T_2 \ = \ 23.844\ kN$$

Example. 2.17. *A rope AB, 4.5 m long is connected at two points A and B at the same level 4 m apart. A load of 1500 N is suspended from a point C on the rope 1.5 m from A as shown in Fig. 2.42. What load connected at point D on the rope, 1 m from B will be necessary to keep the position CD level?*

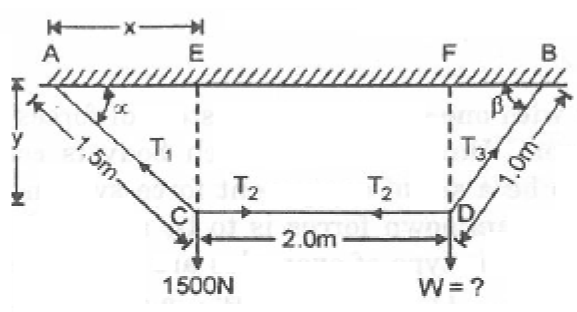


Fig. 2.42

Solution.

Drop perpendiculars CE and DF on AB.

Let CE = *y*, and AE = *x*

From $\Delta\ AEC$

$$x^2 + y^2 = 1.5^2 = \ 2.25 \qquad \ldots\ldots\ldots\ (i)$$

Now, AB = 4 m

AC + CD + BD = 4.5 m

CD = 4.5 – 1.5 – 1.0 = 2 m

∴ EF = 2 m

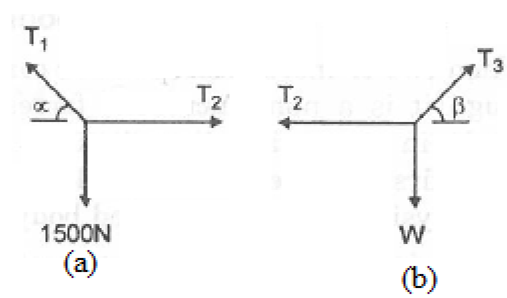


Fig. 2.43

$$BF = AB - (AE+EF) = 4 - (x+2) = 2 - x$$

From $\Delta\, BFD$

$$BF^2 + DF^2 = 1^2$$

$$(2 - x)^2 + y^2 = 1 \qquad \ldots\ldots\ldots\ldots (ii)$$

From (*i*) and (*ii*)

$$x^2 - (2 - x)^2 = 1.25$$

$$x^2 - 4 + 4x - x^2 = 1.25$$

$$\therefore x = 1.3125\, m$$

$$\alpha = cos^{-1}\left(\frac{1.3125}{1.5}\right) = 28.955^o$$

$$\beta = cos^{-1}\left(\frac{2-1.3125}{1.5}\right) = 46.567^o$$

Applying Lami's theorem to the system of forces acting at point C

$$\frac{T_1}{\text{sin}sin\ 90} = \frac{T_2}{\text{sin}sin\ 118.955} = \frac{1500}{\text{sin}sin\ (180-28.955)}$$

$$\therefore T_1 = 3098.39\, N$$

$$T_2 = 2711.09\, N$$

Applying Lami's theorem to the system of forces, at B

$$\frac{T_3}{\text{sin}sin\ 90} = \frac{T_2}{\text{sin}sin\ 136.567} = \frac{W}{\text{sin}sin\ (180-46.567)}$$

$$\therefore T_3 = 3993.28\, N$$

$$W = 2863.53\, N$$

Example. 2.18. *A simply supported beam of span 6 m carries point loads of 3 kN and 6 kN at a distance of 2 m and 4 m from the left end A as shown in Fig 2.22. Find the reactions at A and B.*

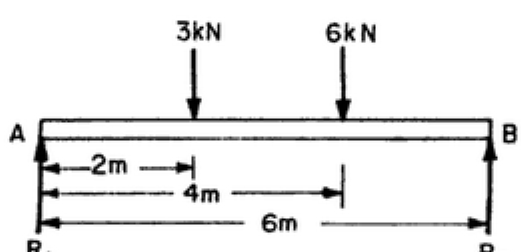


Fig. 2.22

Solution. Given:

Span of beam = 6m

Let R_A = Reaction at A

R_B = Reaction at B

As the beam is in equilibrium, the moments of all the forces about any point should be zero.

Now taking the moment of all forces about A, and equating the resultant moment to zero, we get

$$R_B \times 6 - 3\times 2 - 6\times 4 = 0$$

$$6R_B = 30$$

$$\therefore R_B = \frac{30}{6} = 5\,kN$$

Also, for equilibrium, $\sum F_y = 0$

$$R_A + R_B = 3 + 6 = 9$$

$$\therefore R_A = 9 - 5 = 4\,kN$$

Example. 2.19. *A simply supported beam AB of length 9 m, carries a uniformly distributed load of 10 kN/m for a distance of 6 m from the left end. Calculate the reactions at A and B.*

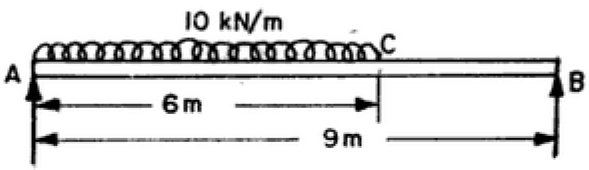


Fig. 2.23

Solution. Given:
Length of beam = 9 m
Rate of U.D.L. = 10 kN/m
Length of U.D.L. = 6 m
Total load due to U.D.L. = (Length of U.D.L.) × (Rate of U.D.L.)

$$= 6\times 10 = 60\,kN$$

This load of 60 kN will be acting at the middle point of AC i.e., at a distance of 6/2 = 3 m from A.
Let R_A = Reaction at A
R_B = Reaction at B
Taking the moments of all forces about point A, and equating the resultant moment to zero, we get

$$R_B \times 9 - 60\times 3 = 0$$

$$9\,R_B = 180$$

$$\therefore R_B = \frac{180}{9}\,20\,kN$$

Also for equilibrium, $\sum F_y = 0$

$$R_A + R_B = 60$$

$$\therefore R_A = 60 - 20 = 40\,kN$$

Example. 2.20 *A simply supported beam of length 10 m, carries the uniformly distributed load and two point loads as shown in Fig. 2.24. Calculate the reactions at A and B.*

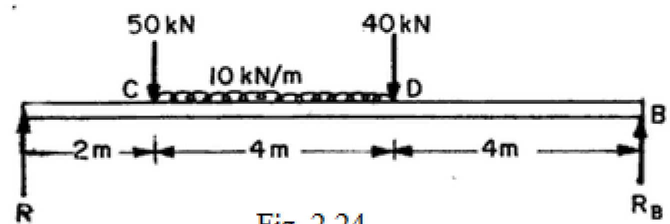


Fig. 2.24

Solution. Given:

Length of beam = 10 m

Rate of U.D.L. = 10 kN/m

Length of U.D.L. = 4 m

Total load due to U.D.L. = (Length of U.D.L.) × (Rate of U.D.L.)

$$= 4\times10 = 40\ kN$$

This load of 40 kN will be acting at the middle point of CD i.e., at a distance of 4/2 = 2 m from C. (Or at a distance of 2 + 2 = 4 m from point A)

Let R_A = Reaction at A

R_B = Reaction at B

Taking the moments of all forces about point A, and equating the resultant moment to zero, we get

$$R_B\times10 - 50\times2 - 40\times(2+4) - 40\times4 = 0$$

$$10\,R_B = 100 + 240 + 160 = 500$$

$$\therefore R_B = \frac{500}{10} = 50\ kN$$

Also for equilibrium, $\sum F_y = 0$

$$R_A + R_B = 50 + 40 + 40 = 130$$

$$\therefore R_A = 130 - 50 = 80\ kN$$

Example. 2.21. *A simply supported beam of span 9 m carries a uniformly varying load from zero at end A to 900 N/m at end B. Calculate the reactions at the two ends of the support.*

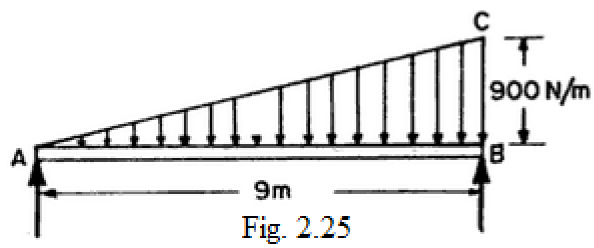


Fig. 2.25

Solution. Given:

Span of beam = 9m

Load at end A = 0

Load at end B = 900 N/m

Total load on the beam = Area of ABC = $\frac{AB\times BC}{2}$ = $\frac{9\times 90}{2}$ = 4050 N

This load will be acting at the C.G. of the Δ ABC, i.e., at a distance of $\frac{2}{3}\times AB = \frac{2}{3}\times9$ = 6 m from end A.

Let R_A = Reaction at A

R_B = Reaction at B

Taking the moments of all forces about point A, and equating the resultant moment to zero, we get

$$R_B \times 9 - 4050 \times 6$$

$$\therefore R_B = \frac{4050 \times 6}{9} = 2700\,N$$

Also for equilibrium, $\sum F_y = 0$

$$R_A + R_B = 4050$$

$$\therefore R_A = 4050 - 2700 = 1350\,N$$

Example. 2.22. *A simply supported beam of length 5 m carries a uniformly increasing load of 800 N/m at one end to 1600 N/m at the other end. Calculate the reactions at both ends.*

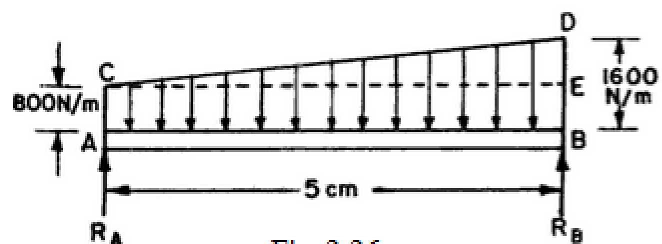


Fig. 2.26

Solution. Given:

Length of beam = 5 m

Load at A = 800 N/m

Load at B = 1600 N/m

Total load on the beam = Area of load diagram ABCD

= Area of Rectangle ABEC + Area of Triangle CED

$= 5 \times 800 + \frac{1}{2} \times 5 \times 800$

= 4000 + 2000 = 6000 N

The C.G. of the rectangle ABEC will be at a distance of $\frac{5}{2}$ = 2.5 m from A, whereas the C.G. of triangle CED will be at a distance of $\frac{2}{3} \times 5$ = 3.33 m from A.

Let R_A = Reaction at A

R_B = Reaction at B

Taking the moments of all forces about point A, and equating the resultant moment to zero, we get

$$R_B \times 5 - (5 \times 800) \times 2.5 - \left(\frac{1}{2} \times 5 \times 800\right) \times \left(\frac{2}{3} \times 5\right) = 0$$

$$5R_B = 10000 + 6666.66 = 16666.66$$

$$\therefore R_B = \frac{16666.66}{5}\ 3333.33\,N$$

Also for equilibrium, $\sum F_y = 0$

$$R_A + R_B = 6000$$

$$\therefore R_A = 6000 - 3333.33 = 2666.67\ N$$

Example. 2.23. *A beam AB of span 8 m, overhanging on both sides, is loaded as shown in Fig. 2.27. Calculate the reactions at both ends.*

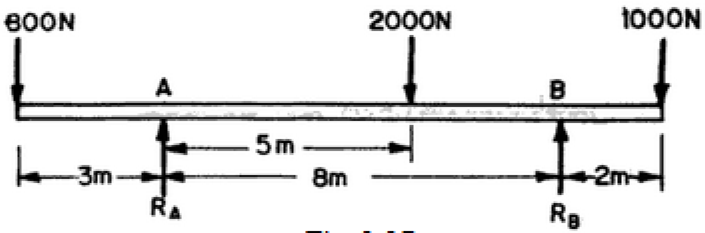


Fig. 2.27

Solution. Given:
Span of beam = 8 m
Let R_A = Reaction at A
R_B = Reaction at B
Taking the moments of all forces about point A, and equating the resultant moment to zero, we get

$$R_B \times 8 + 800 \times 3 - 2000 \times 5 - 1000 \times (8 + 2) = 0$$

$$8R_B = 20000 - 2400 = 17600$$

$$R_B = \frac{17600}{8} = 2200\ N$$

Also for equilibrium, $\sum F_y = 0$

$$R_A + R_B = 3800$$

$$\therefore R_A = 3800 - 2200 = 1600\ N$$

Example. 2.24. *A beam of span 4 m, overhanging on one side upto a length of 2m, carries a uniformly distributed load of 2 kN/m over the entire length of 6 m and a point load of 2 kN/m as shown in Fig. 2.28. Calculate the reactions at A and B.*

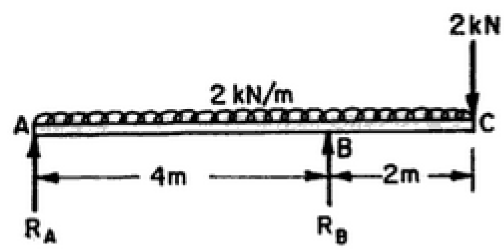


Fig. 2.28

Solution. Given:
Span of beam = 4 m
Total length = 6 m
Rate of U.D.L. = 2 kN/m
Total load due to U.D.L. = 2 × 6 = 12 kN
The load of 12 kN will act at the middle point of AC, i.e., at a distance of 3 m from A.
Let R_A = Reaction at A
R_B = Reaction at B
Taking the moments of all forces about point A, and equating the resultant moment to zero, we get

$$R_B \times 4 - 12 \times 3 - 2 \times (4 + 2) = 0$$

$$4\,R_B = 36 + 12 = 48$$

$$\therefore R_B = \frac{48}{4} = 12\,kN$$

Also for equilibrium, $\sum F_y = 0$

$$R_A + R_B = 12 + 2 = 14$$

$$\therefore R_A = 14 - 12 = 2\,kN$$

Example. 2.25. *A beam AB 1.7 m long is loaded as shown in Fig. 2.29.Ddetermine the reactions at A and B.*

Solution. Given:

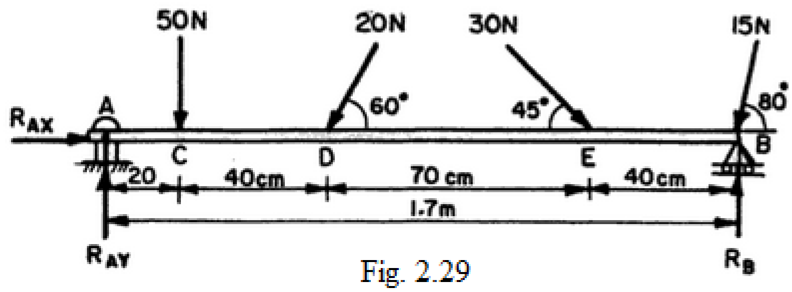


Fig. 2.29

Length of beam = 1.7 m

Let R_A = Reaction at A

R_B = Reaction at B

Since the beam is supported on rollers at B, therefore the reaction R_B will be vertical.

The beam is hinged at A, and is carrying inclined load, therefore the reaction R_A will be inclined. This means R_A will have two components, i.e., vertical component and horizontal component.

Let R_{AX} = Horizontal component of reaction R_A

R_{AY} = Vertical component of reaction R_A.

First resolve all the inclined loads into their vertical and horizontal components.

i) Vertical component of load at D = 20 sin 60 = 17.32 N
and its horizontal component = 20 cos 60 = 10 N ←

ii) Vertical component of load at E = 30 sin 45 = 21.21 N
And its horizontal component = 30 cos 45 = 21.21N →

iii) Vertical component of load at B = 15 sin 80 = 14.77 N
And its horizontal component = 15 cos 80 = 2.6 N ←

From condition of equilibrium, $\sum F_x = 0$

$$R_{AX} - 10 + 21.21 - 2.6 = 0$$

$$\therefore R_{AX} = -8.61\,N$$

-ve sign shows that the assumed direction of R_{AX} is wrong. Correct direction will be opposite to the assumed direction. Assumed direction of R_{AX} is towards right. Hence correct direction of R_{AX} will be towards left.

$\therefore R_{AX} = 8.61\, N \leftarrow$

To find R_B, take moments of all forces about A.

For equilibrium, $\sum M_A = 0$

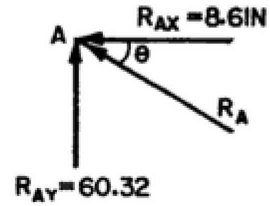


Fig. 2.30

$$50 \times 20 + (20 \sin sin\, 60) \times (20 + 40) + (30 \times sin 45) \times (20 + 40 + 70) + (15\, sin 80) \times (170) - 170$$

$$\therefore R_B = \frac{7307.9}{170} = 42.89\, N$$

To find R_{AY}, apply condition of equilibrium, $\sum F_y = 0$

$$R_{AY} + R_B = 50 + 20 \sin sin\, 60 + 30 \sin sin\, 45 + 15 \sin sin\, 80$$

$$\therefore R_{AY} = 60.32\, N\uparrow$$

$\therefore$ Reaction at A, $R_A = \sqrt{R_{AX}^{\ 2} + R_{AY}^{\ 2}}$

$$= \sqrt{8.61^2 + 60.32^2} = 60.92\, N$$

The angle made by R_A with x- direction is given by

$$\tan tan\, \theta = \frac{R_{AY}}{R_{AX}} = \frac{60.32}{8.61} = 7.006$$

$$\therefore \theta = tan^{-1}(7.006) = 81.87^o$$

Example. 2.26. *A beam AB 6 m long is loaded as shown in Fig. 2.31. Determine the reactions at A and B.*

Solution. Given:

Length of beam = 6 m

Let R_A = Reaction at A

R_B = Reaction at B

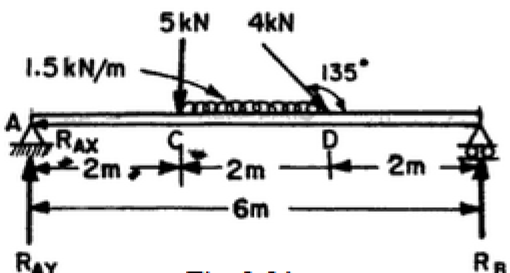


Fig. 2.31

Since the beam is supported on rollers at B, therefore the reaction R_B will be vertical.

The beam is hinged at A, and is carrying inclined load, therefore the reaction R_A will be inclined. This means R_A will have two components, i.e., vertical component and horizontal component.

Let R_{AX} = Horizontal component of reaction R_A

R_{AY} = Vertical component of reaction R_A.

Horizontal component of 4 kN at D = 4 cos 45 = 2.828 kN→

And its vertical component = 4 sin 45 = 2.828 kN↓

For Equilibrium, $\sum F_x = 0$

$$- R_{AX} + 2.828 = 0$$

$$\therefore R_{AX} = 2.828\, kN$$

To find R_B, take moments of all forces about A.

For equilibrium, $\sum M_A = 0$

$$R_B \times 6 - 5\times 2 - (2\times 1.5)\left(2 + \frac{2}{2}\right) - (4 \sin sin\, 45)(2 + 2) = 0$$

$$6R_B = 30.312$$

$$\therefore R_B = \frac{30.312}{6} = 5.052\, kN$$

To find R_{AY}, apply condition of equilibrium, $\sum F_y = 0$

$$R_{AY} + R_B - 5 - (1.5\times 2) - 4 sin 45 = 0$$

$$\therefore R_{AY} = 5.776\, kN$$

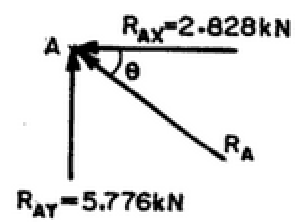


Fig. 2.32

$\therefore$ Reaction at A, $R_A = \sqrt{R_{AX}^2 + R_{AY}^2}$

$$= \sqrt{2.828^2 + 5.776^2} = 6.43\, kN$$

The angle made by R_A with x- direction is given by

$$\tan tan\, \theta = \frac{R_{AY}}{R_{AX}} = \frac{5.776}{2.828} = 2.0424$$

$$\therefore \theta = tan^{-1}(2.0424) = 63.9^o$$

Example. 2.27. *A beam AB 10 m long is hinged at A and supported on rollers over a smooth surface inclined at 30°to the horizontal at B. The beam is loaded as shown in Fig. 2.33. Determine reactions at A and B.*

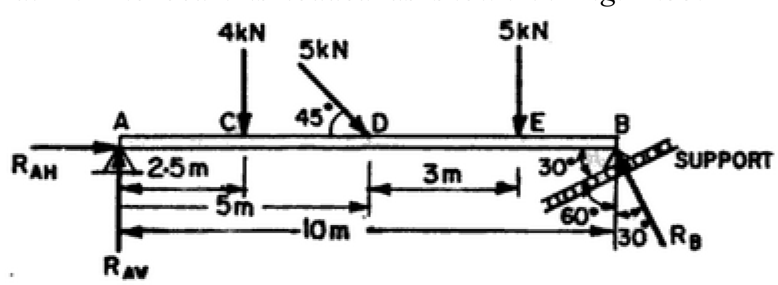


Fig. 2.33

Solution. Given:

Length of beam = 10 m

Let R_A = Reaction at A

R_B = Reaction at B

The reaction R_B will be normal to the support as the beam at B is supported on the rollers. But the support at B is making an angle of 30° with the horizontal or 60° with the vertical. Hence the reaction R_B is making an angle of 30° with the vertical.

The vertical component of $R_B = R_B \cos 30°$

And horizontal component of $R_B = R_B \sin 30°$

Resolving the load of 5 kN acting at D into horizontal and vertical components, we get

Vertical component of 5 kN = 5 sin 45 = 3.535 kN

Horizontal component of 5 kN = 5 cos 45 = 3.535 kN

The beam is hinged at A, and is carrying inclined load, therefore the reaction R_A will be inclined. This means R_A will have two components, i.e., vertical component and horizontal component.

Let R_{AX} = Horizontal component of reaction R_A

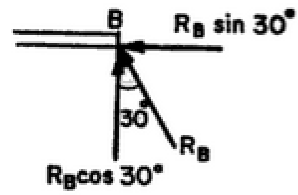
R_{AY} = Vertical component of reaction R_A.

For equilibrium of the beam, the moments of all forces about any point should be zero.

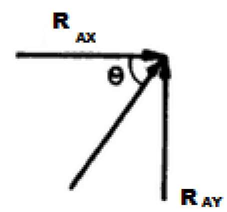
Taking moments about point A,

$$(R_B \cos cos\, 30) \times 10 - 4 \times 2.5 - (5 \sin sin\, 45) \times 5 - 5 \times 8 = 0$$

$$\therefore R_B = 7.81\, kN$$



(a)



(b)

Fig. 2.34

For equilibrium, $\sum F_x = 0$

$$R_{AX} + 5 \cos cos\, 45 \; - R_B \sin sin\, 30 \; = 0$$

$$\therefore R_{AX} = 0.37\, kN$$

For equilibrium, $\sum F_y = 0$

$$R_{AY} + \; R_B \cos cos\, 30 - 4 - 5 \sin sin\, 45 - 5 = 0$$

$$\therefore R_{AY} = 5.77\, kN$$

$\therefore$ Reaction at A, $R_A = \sqrt{R_{AX}^{\;2} + R_{AY}^{\;2}}$

$$= \sqrt{0.37^2 + 5.77^2}\, 5.78\, kN$$

The angle made by R_A with x- direction is given by

$$\tan tan\, \theta = \frac{R_{AY}}{R_{AX}} = \frac{5.77}{0.37} = 15.59$$

$$\therefore \theta = tan^{-1}(15.59) = 86.33^o$$

Example. 2.28. *A simply supported beam AB of 7 m span is subjected to: i) 4 kN m clockwise couple at 2 m from A, ii) 8 kN m anti-clockwise couple at 5 m from A and iii) a triangular load with zero intensity at 2 m from A increasing to 4 kN per m at a point 5 m from A. determine reactions at A and B.*

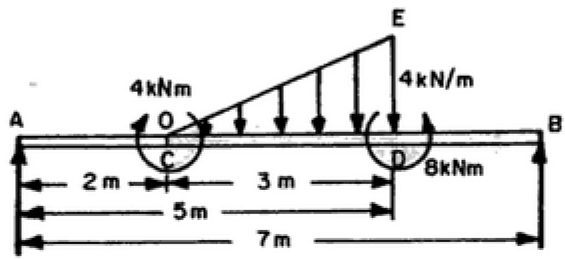


Fig. 2.35

Solution. Given:
Span of beam = 7 m
Couple at C = 4 kN m (clockwise)
Couple at D = 8 kN m (anti-clockwise)
Triangular load from C to D with:
Vertical load at C = 0
Vertical load at D = 4 kN/m
∴ Totral load on beam = Area of triangle *CDE* = $\frac{CD \times DE}{2} = \frac{3\times4}{2}$ = 6 kN
This load will be acting at the C.G. of the triangle *CDE* i.e., at a distance of $\frac{2}{3} \times CD = \frac{2}{3} \times 3$ = 2 m from C or 2+2 = 4 m from end A.
Let R_A = Reaction at A
R_B = Reaction at B
Taking the moments of all forces about point A, and equating the resultant moment to zero, we get

$$-R_B\times7 + 4 - 8 + 6\times4 = 0$$

$$R_B = \frac{20}{7}\, kN$$

Also for equilibrium, $\sum F_y = 0$

$$R_A + R_B = 6$$

∴ Also for equilibrium, $\sum F_y = 0$

$$R_A = \frac{22}{7}\, kN$$