Project 3 - Explorations & Applications in Space

Select *ONE* of the project options below.

You may work individually or with a partner.

0) Platonic Solids

Instructions (not yet complete)

- [ ] Complete all five platonic solids out of origami. You may want to use a glue stick or tape for the icosahedron and octahedron.Add a picture of your origami platonic solids
  Here is the link to the instructions (start on page 32): https://1drv.ms/b/s!AmpbeQojORfugZRUkoXu0KFJktVuIg
- [ ] Complete the platonic solids table
- [ ] Answer the questions below
- [ ] Proof that only three polygons tile the plane
- [ ] Proof that there are only five platonic solids

Complete the following table for the platonic solids:

Name	Number of Faces	Number of Edges	Number of Vertices	Number of faces meeting at each vertex	Angle sum at each vertex
Tetrahedron	4				
Hexahedron (cube)					

1. What must be true about the angle sum at each vertex of a platonic solid? Why?
2. What will happen if the angle sum at a vertex is equal to 360 degrees? Greater than 360 degrees?
3. What is the minimum number of shapes that can meet at a vertex in a polyhedron/platonic solid? Why is this the minimum?

4. How many platonic solids are there? There are an infinite number of regular polygons, why do you think there are not also an infinite number of regular polyhedra (platonic solids)?
5. Explain why there can only be the number of platonic solids that you stated in (4). Give an argument for why there cannot be more.
6. What is the relationship between the number of faces, edges and vertices in a platonic solid? Give an algebraic (equation) representation.

1) Surveying

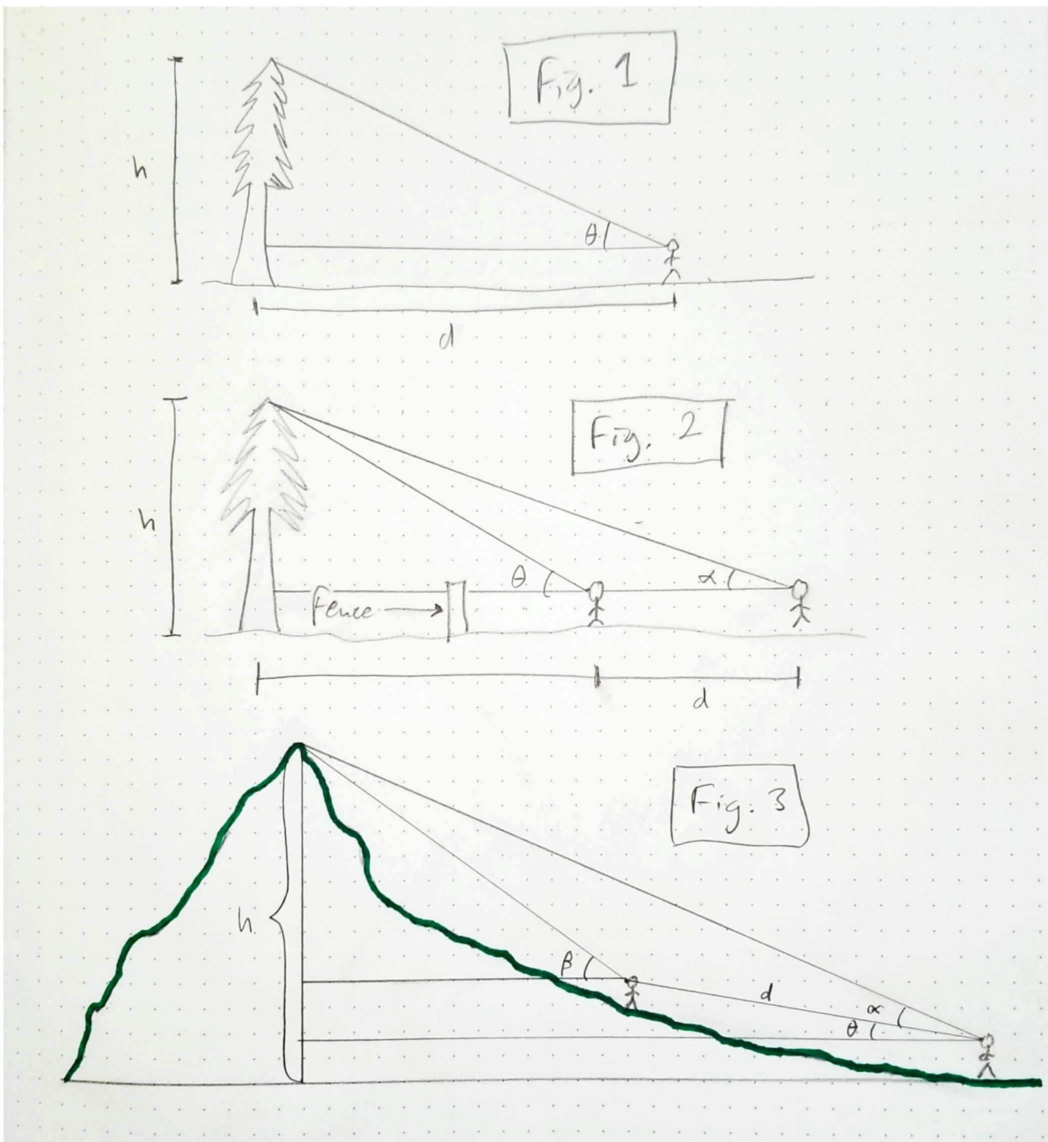
In this project you will apply trigonometry to measure the heights and then consider different types of trigonometry. You'll use a clinometer to measure two different objects. The first will be a measurement of an object like a tree that you cannot access the base of where there is (approximately) flat space around. The second is more challenging, where you both cannot access the base and there is sloped space around the object (typical of a mountain).

Once you complete these measurements, you need to do additional calculations to determine how much your calculations are off by, when taking into account the curvature of the earth. Finally, you'll do a little research on what trigonometry looks like in hyperbolic and spherical space. You do not need to do the calculations, just read a bit about it and discuss how these other trigs compare to the "standard" trig.

Instructions:

- [ ] **Abstract:** Give an overview of your project, including a summary of what you did (1-2 paragraphs)
- [ ] Download a clinometer app. I use an Android app called "Bubble Level"
- [ ] Part 1: Find the height of an object that you cannot access the base of (fig.2)
    - [ ] Draw a diagram
    - [ ] Show your calculations
    - [ ] Include photographs to show your setup and the object that you measured
- [ ] Part 2: Find the height of a mountain, or similar geographic feature which has ***sloped*** ground below and around it (fig. 3)
    - [ ] Draw a diagram
    - [ ] Show your calculations
    - [ ] Include photographs to show your setup and the object that you measured
- [ ] Part 3: Determine how much your answer for the mountain height (Part 2) is off by when taking into account the curvature of the earth.
    - [ ] Include your diagrams and calculations

- ☐ Part 4: Do some reading about hyperbolic and spherical trig
  - ☐ Write a paragraph summarizing what you learned about hyperbolic and spherical trigonometry. What did you find interesting/surprising?
  - ☐ Write a paragraph discussing the similarities and differences between these trigs and the standard trig. In what contexts would each of them be useful?
- ☐ Optional: Write a couple proofs



For each part, do not forget to take into account the eye height of the observer!!!

Part 1

Find the height of a tree, or some other tall object which you are not able to access the base of (refer to Fig. 2). The idea here is that you can measure something even if you can't access it directly--perhaps it is surrounded by a fence which keeps you at a distance. You will need to find the two indicated angles using your clinometer, and the indicated length 'd'. The law of sines may be helpful here, though you can solve it using only sine, cosine and tangent, along with some algebra.

Part 2

Find the height of a mountain or other object, like a hill or very large rock, that has sloped ground below and around it (refer to Fig. 3). The idea here is that you can measure something even if it is not surrounded by flat ground. You will need to find the three indicated angles using your clinometer, and the indicated length 'd'. The law of sines is helpful here as well.

Just to be clear, these are the four measurements you need for your calculations:

1. From the first position, the angle of elevation to the second position
2. From the first position, the angle of elevation to the top of the object
3. From the second position, the angle of elevation to the top of the object
4. The distance from the first position to the second position

Part 3

For the mountain height, figure out approximately how much your answer is off by taking into account the curvature of the earth.

Proofs

If you want credit in the proof LO, you may write up two proofs (based on our axiomatic system) for things like the law of sines, cosines, basic proportionality theorem, etc. They should be theorems that relate to trig. Talk about which bricks the project depends on, and which the proofs depend on.

Ignore the pics below for now

NE of Grand Forks MN


62
66
62
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21
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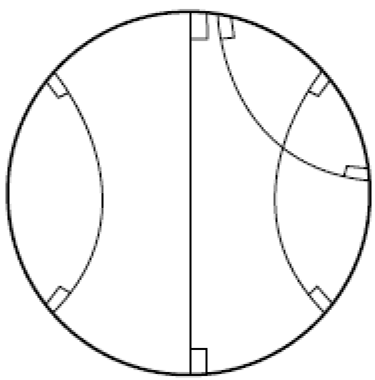
2) Poincare Disk Model

Hyperbolic space is difficult to grasp conceptually, since it seems so different from our experience of reality. One way to address this is by working with different models of hyperbolic geometry. One of the most popular is the Poincare disk model. For this project you will research and work with the Poincare disk model for hyperbolic geometry. Here's a link to an interactive version that you can use:
[Hyperbolic Canvas (Poincaré Disk)]



Instructions:

- ☐ **Abstract:** Give an overview of your project, including a summary of what you did (1-2 paragraphs)
- ☐ **Research** the Poincare disk model for hyperbolic geometry. Then write a paragraph explaining the model in your own words. Be sure to address the questions below.
  - ☐ How big is the model? Where on the hyperbolic plane is the circular boundary?
  - ☐ How are lines defined in the model?
  - ☐ How are circles defined in the model?
  - ☐ Does the model preserve angles?
  - ☐ Does the model preserve distances?
  - ☐ What happens when you tile the space? How do the shapes near the boundary differ from the shapes in the middle?
  - ☐ Explain how the parallel postulate plays out in the model. How is it that multiple lines through a point can be parallel to a given line in the model?
  - ☐ Discuss triangle angle sums in the model.
- ☐ **Demonstrate** at least four theorems/facts using the model. These can include facts about parallel lines, angle sum of a triangle, exterior angles, would-be rectangles, Pythagorean Theorem etc. At least one must be true in Euclidean geometry. At least two must NOT be true in Euclidean geometry. Include a screenshot and brief description for all four, making sure to clarify which theorem/fact you are demonstrating. You do ***not***

need to prove them, just demonstrate a case (you will need to have GeoGebra print the values of angles and/or sides for some or all of these).

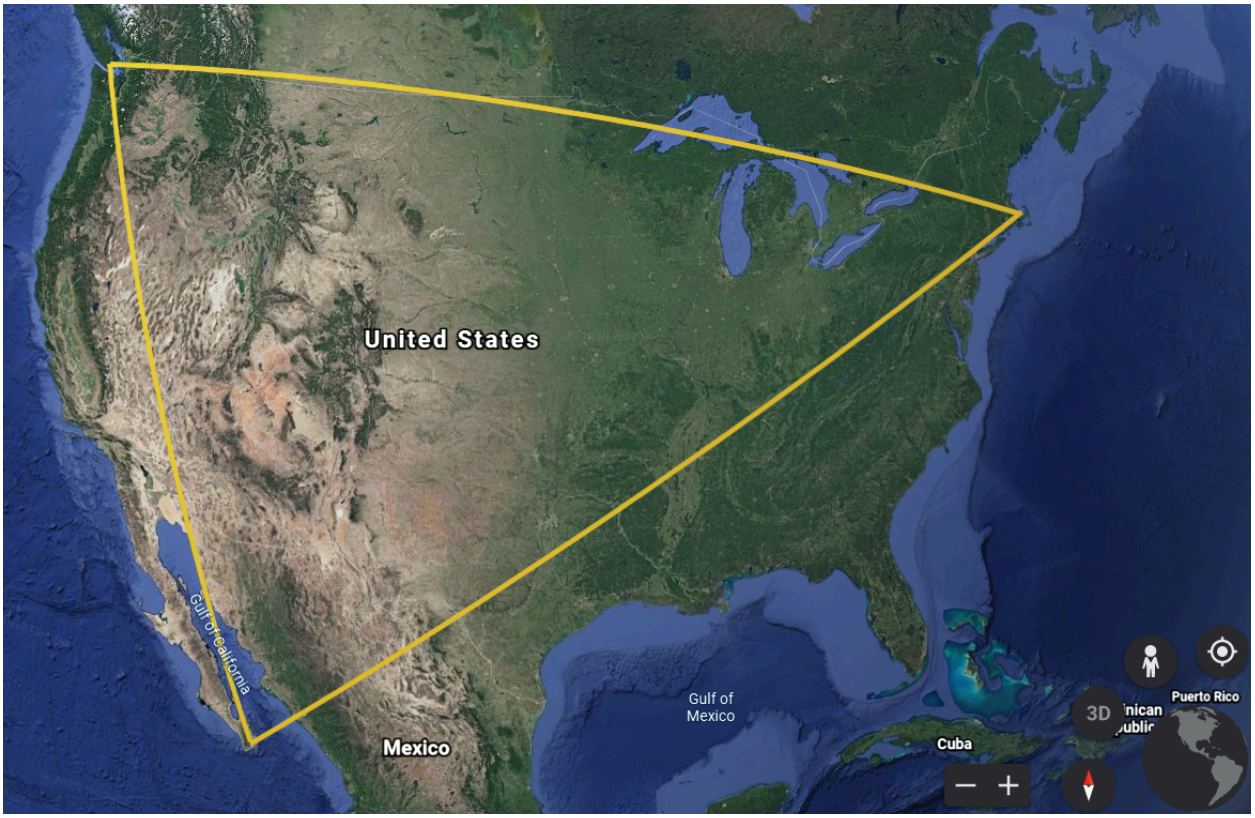
  - [ ] Be sure you have screenshots for each of the four!
- [ ] ~~**Transform**. Do the following two transformations on the Poincare disk, and include screenshots of each.~~
  - [ ] ~~Reflection~~
  - [ ] ~~Slide~~
  - [ ] ~~Which of the two cannot work in the way that it works in Euclidean space? Explain why.~~
    ~~Note: This is where the parallel postulate shows up in vsauce's proof for the angle sum of a triangle.~~
- [ ] Download HyperRogue and spend a little time playing it.
  - [ ] Write about how this experience gave you some insight into the Poincare model and hyperbolic geometry generally.
- [ ] Complete proofs for two theorems in hyperbolic geometry for credit in the proof LO. Note that you may need to use the hyperbolic version of the parallel postulate and you can use the fact that in hyperbolic space the angle sum of a triangle is less than 180 degrees (we have sufficiently demonstrated this in class). Choose from the following:
  - [ ] Prove that if
  - [ ] Prove that the summit angles of a Khayyam quadrilateral are acute
  - [ ] Prove that rectangles (and squares) do not exist
  - [ ] Prove that two parallel lines do not remain the same distance apart

3) Spherical Trigonometry (challenging)

https://www.loom.com/share/9cf734e4c724452680f52f1580392f07

Even though it took over 2000 years for mathematicians to accept that the parallel postulate may not be true in all contexts, mathematicians had, in fact, been doing non-Euclidean geometry in some form for a very long time. The ancient Greeks (as well as other societies) understood that the earth is approximately spherical (they even knew how big it was!), and if they used Euclidean geometry to calculate paths for their ships, they would not end up in the right place. Euclid even proved theorems about certain properties of spheres toward the end of his book, The Elements. Hence, the study of spherical geometry and trigonometry is thousands of years old, notwithstanding the fact that it is rarely taught in schools (even colleges) today. In this project you will do some research on and solve problems with spherical trigonometry.

In the diagram below, I used Google Earth to make a triangle connecting Vancouver Island (V), Baja (B) and Cape Cod (C).



Instructions:

- ☐ **Abstract:** Give an overview of your project, including a summary of what you did (1-2 paragraphs)
- ☐ **Research:** Do some research on spherical trigonometry and its history. Then write a summary of what you learned, addressing the questions below:
  - ☐ Who were some of the people/societies that developed the study of spherical trigonometry, and why was it important to them?
  - ☐ What are some of the assumptions underlying the study of spherical geometry/trigonometry?
  - ☐ What are a couple important theorems/facts about spherical trigonometry? Why are they useful?
  - ☐ Discuss the similarities and differences between spherical trigonometry and Euclidean trigonometry. Be sure to discuss why the trig we learned in class does not work on a sphere and weave your understanding of axiomatic systems and the parallel postulate into this discussion.
- ☐ **Problems:** Pose and solve four problems involving spherical trigonometry, as specified below.

- ☐ *Note: you will need to use the radius of the sphere (the radius of the earth) to solve these problems. Also, you will need to use the law of sines/cosines or a triangle solver to do some of the calculations.*
    - ☐ Make a large triangle on the earth (perimeter of over 5000km). Using the three side lengths, find the measures of the three angles. Add them up. How is this different from the angles of a triangle in Euclidean space?
      3,055 | 2,925 | 1,539
    - ☐ Make a large triangle on the earth (perimeter of over 5000km). Using two side lengths, and the measure of the angle between them, find the length of the third side. In the diagram, this could be the lengths from B to C and from B to V, along with the angle C and V make at B. Find out what the length would have been in Euclidean space. What is the difference between the two?
      3,055 | 2,925 | 31 degrees
    - ☐ Make a large triangle on the earth (perimeter of over 5000km). Compute the area. Then find out what the area would have been in Euclidean space. What is the difference between the two?
    - ☐ Make a large *right* triangle on the earth (perimeter of over 5000km). Using the lengths of the two sides adjacent to the right angle, apply the spherical version of the Pythagorean Theorem to compute the length of the third side. Then find out what the length would have been in Euclidean space. What is the difference between the two?
- ☐ Proofs: Demonstrate two theorems in spherical geometry
    - ☐ Demonstrate that the exterior angle theorem (not the Euclidean one) is false in spherical geometry
    - ☐ Demonstrate that squares and rectangles don't exist in spherical geometry

This site may be helpful:
https://www.intmath.com/vectors/3d-earth-geometry.php

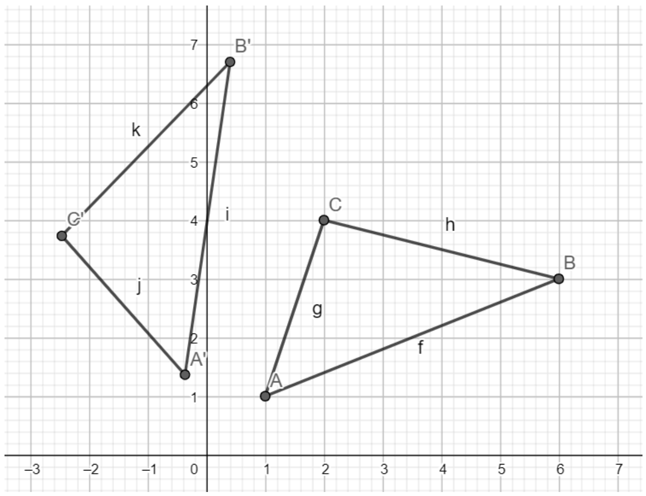
4) Transformations with Matrices

https://www.loom.com/share/b894f48398bf470f92433c1067b71284

Geometric transformations are basically just ways of moving an object and/or changing its size. They include:

- Rotation - rotating an object by some angle around some central point
- Reflection - reflecting an object over a line

- Glide - moving an object a certain distance in such a way that it keeps its orientation
- Dilation - making an object larger or smaller, while maintaining its overall shape



Rotation example

These can all be done with a compass and straightedge, but it is fairly cumbersome. Using a computer, they can also be done with matrices, which is how it is done in computer graphics (e.g. video games) and engineering (e.g. CAD/architectural drawings).

In this project, you will explore how matrices can be used to perform geometric transformations on the plane.

Here is a link to a GeoGebra file that can help get you started:
[Transformations with Matrices]

Instructions:

- ☐ **Abstract:** Give an overview of your project, including a summary of what you did (1-2 paragraphs)
- ☐ Do two transformations by hand on paper with a compass and a straightedge.
    - ☐ Reflection
        - ☐ Draw an xy-coordinate plane. Then draw a triangle on the plane and label the three vertices
        - ☐ Reflect the triangle over the x-axis or the y-axis using only a compass and straightedge
        - ☐ Write the coordinates of the new reflected triangle
    - ☐ Rotation

- ☐ Draw an xy-coordinate plane. Then draw a triangle on the plane and label the three vertices
- ☐ Rotate the triangle by an angle of 60 degrees around the origin using only a compass and straightedge
- ☐ Estimate and write the coordinates of the new rotated triangle

- ☐ Research matrix multiplication (there are many websites/videos on this). Then write a couple sentences explaining how matrix multiplication works. Show one example of multiplying two 2x2 matrices by hand.
- ☐ Research the methods for using matrices to preform different transformations (glide, reflection, rotation and dilation). For each of the four transformations, make a unique triangle and then apply that transformation using matrices. Take a screenshot showing the original figure and the transformed figure.

  Note: In the provided file, you will modify the matrix F to make different figures. The matrix T has the values of the transformed figure. The matrix R is used to rotate, the matrix G is used to glide, and the matrix Q can be used for both reflections and dilations.

  - ☐ Include screenshots of all four transformations (individually, do not do compound transformations at this point). Give a brief description indicating what each one is. (30 degree rotation, reflection over the x-axis, etc.)
  - ☐ Write your answers to the following questions:
    - ☐ Which of the four does not involve matrix multiplication? Explain why you think this is.
    - ☐ Which of the four does not preserve equality (the shape changes in size)? Explain.
- ☐ Next, do two examples where you take a figure and apply multiple transformations. Include a screenshot for each and indicate what the transformations were.
- ☐ Optional: if you want credit in the proof LO, complete a couple coordinate geometry proofs (see option 5 for more details).
- ☐ Finally write 1-2 paragraphs explaining what you learned and why transformation matrices are used by engineers and for computer graphics, rather than the Greek approach using a compass and straightedge. What advantages does the matrix approach have?

5) Euclid vs. Descartes Smackdown

Descartes was famously critical of Euclid and the other Greek geometers for their, in his opinion, unnecessarily difficult and cumbersome approach to proofs. Descartes had the huge advantage that other mathematicians had developed a version of the coordinate plane by his time, and he solved/proved geometry problems using this coordinate plane. In this project, you will look at both Euclid's and Descartes' approach to proofs and constructions, and discuss which one you prefer.

Instructions:

- [ ] **Abstract:** Give an overview of your project, including a summary of what you did (1-2 paragraphs)
- [ ] Complete two ***proofs*** in both styles
    - [ ] Start by researching coordinate geometry proofs (Note: For these, the setup is often the most difficult part. Your coordinate geometry proofs need to involve variables, not fixed numbers.)
    - [ ] *This video may be helpful: [How to Write Coordinate Proofs | Geometry How to Proof Help]*
    - [ ] Select two proofs to complete in both styles. Below are some recommendations for things to prove using both methods:
        - [ ] The diagonals of a square are equal and perpendicular
        - [ ] The diagonals of a rectangle are equal and bisect each other
        - [ ] The median of a triangle is parallel to the base and half the length of the base
        - [ ] A rhombus is a parallelogram (opposite sides are parallel)
        - [ ] Thales' theorem
    - [ ] You may use proofs you find online as a guide. For the Greek-style proofs, adapt them to our axiomatic system as needed.
- [ ] Complete two ***constructions*** in both styles.
- [ ] *As an example, here is a desmos file with an algebraic rhombus construction:* [Coordinate Geometry Rhombus with Lines]
    - [ ] Constructions on paper with a compass and a straightedge
        - [ ] Complete two constructions
        - [ ] Label each construction
    - [ ] Algebraic constructions using desmos
    - [ ] *Don't copy the rhombus above, but you can do an equilateral triangle. Also, you do not need to use variables that allow the construction to animate. Your construction can be fixed.*
        - [ ] Complete two constructions
        - [ ] Answer the following two questions
            - [ ] What algebraic tool replaces the compass on the coordinate plane?

- [ ] What algebraic tool replaces the straightedge on the coordinate plane?
- [ ] ***Discuss*** the following:
    - [ ] What are the advantages and disadvantages of each type of proof? Which do you prefer? Explain why.
    - [ ] What are the advantages and disadvantages of each type of construction? Which do you prefer? Explain why.

—-Other ideas—-

Project

Projections: What are the different ways that spherical and hyperbolic surfaces are projected onto Euclidean space? Which preserve distance? Angles? Other things? What are the advantages and disadvantages of each?

The Wyoming Problem

Hyperbolic trigonometry?

Tensors?

Analyze the hyperbolic soccerball with hyperbolic trig, and see how much off the angle sums the class got were. Would need to find a metric for the level of Gaussian curvature based on sizes of the polygons and the angle sum at each vertex. Also, max size of a triangle on the hyperbolic soccerball.

Hyperbolic geometry - compare the cost of going through the middle of a circle (like a mountain) or around it on hyperbolic vs. Euclidean geometry (pi is not a thing in hyperbolic space). What would the best roadmap be for a city in hyperbolic space?