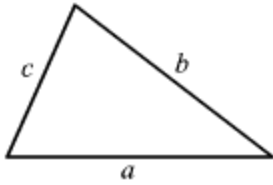


Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.



In the figure, the following inequalities hold.

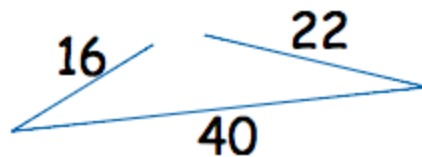
$$a + b > c$$

$$a + c > b$$

$$b + c > a$$

Why? Well imagine one side is not shorter:

- If a side is longer, then the other two sides don't meet:



- If a side is equal to the other two sides it is not a triangle (just a straight line back and forth).



Example 1:

Check whether it is possible to have a triangle with the given side lengths.

7, 9, 13

Add any two sides and see if it is greater than the other side.

The sum of 7 and 9 is 16 and 16 is greater than 13.

The sum of 9 and 13 is 21 and 21 is greater than 7.

The sum of 7 and 13 is 20 and 20 is greater than 9.

This set of side lengths satisfies the Triangle Inequality Theorem.

These lengths do form a triangle.

Example 2:

Check whether the given side lengths form a triangle.

4, 8, 15

Check whether the sides satisfy the Triangle Inequality Theorem.

Add any two sides and see if it is greater than the other side.

The sum of 4 and 8 is 12 and 12 is less than 15.

This set of side lengths does **NOT** satisfy Triangle Inequality Theorem.

These lengths do **NOT** form a triangle.

Example 3:

The triangle inequality theorem states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. The lengths of two sides of a triangle are 16 ft and 21 ft. Find the possible lengths of the third side?

- The third side must have a length greater than *blank* ft and less than *blank* ft

Say the three sides of the triangle are a, b and c. Here, $a = 16$ and $b = 21$. We must find the possible values of c.

- Since $a + c > b$, we can input:
 - $16 + c > 21$ (**subtract the sides)
 - $c > 5$.
 - So the lower bound of c is everything above 5.
- We also know that $a + b > c$. Inputting:
 - $16 + 21 > c$ (**add the sides)
 - $37 > c$.
 - So c must be less than, but not including 37.
- In conclusion, the side length c must satisfy the following inequality:
 - $5 < c < 37$

- The third side must have a length greater than 5 ft and less than 37 ft