PRACTICE 5.2 – The Power Rule for Derivatives

* Full, worked solutions can be found in the folder linked on the Course Website ☺

Exercise 5C

Find the derivative of each function:

1 $f(x) = x^7$

2 $f(x) = x^{18}$

3 $f(x) = x^{\frac{-1}{2}}$

4 $f(x) = \sqrt[5]{x}$

5 $f(x) = \dfrac{1}{\sqrt{x}}$

6 $f(x) = \sqrt[4]{x^3}$

Exercise 5D

1 Find the gradient function for each of the following functions.

a $y = x^4 - \dfrac{1}{2}x^2$

b $f(x) = 5x(x^2 - 1)$

c $f(x) = 6x^4 - 3x^2 - 10$

d $s = 2t^2 + 3t$

e $v = -9.8t + 4.9$

f $c = 24x + 10$

2 Find $f'(x)$ for each function.

a $f(x) = 6\sqrt{x}$

b $f(x) = 5\sqrt[5]{x^3}$

c $f(x) = \dfrac{2}{x} - 3\sqrt{x}$

3 Differentiate with respect to x.

a $f(x) = \dfrac{3}{2x^2}$

b $f(x) = \dfrac{3}{(2x)^2}$

c $f(x) = 4\pi x^3 + 3\pi$

d $f(x) = (x + 1)^2$

e $f(x) = \dfrac{x^3 + x - 3}{x}$

f $f(x) = (2x-1)(x^2+3)$

4 Find $\dfrac{dy}{dx}$ for each function.

a $y = 1 + x\sqrt{x}$

b $y = \dfrac{7}{x^2} - \dfrac{1}{\sqrt{x}}$

c $y = \sqrt[3]{x} + \sqrt[4]{x}$

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Want More Practice?

Try some optional problems. Answers available on the last page.

EXERCISE 15A

1 Find $f'(x)$ given that $f(x)$ is:

- **a** x^3
- **b** $2x^3$
- **c** $7x^2$
- **d** $6\sqrt{x}$
- **e** $3\sqrt[3]{x}$
- **f** x^2+x
- **g** $4-2x^2$
- **h** x^2+3x-5
- **i** $\frac{1}{2}x^4-6x^2$
- **j** $\dfrac{3x-6}{x}$
- **k** $\dfrac{2x-3}{x^2}$
- **l** $\dfrac{x^3+5}{x}$
- **m** $\dfrac{x^3+x-3}{x}$
- **n** $\dfrac{1}{\sqrt{x}}$
- **o** $(2x-1)^2$
- **p** $(x+2)^3$

2 Find $\dfrac{dy}{dx}$ for:

- **a** $y=2.5x^3-1.4x^2-1.3$
- **b** $y=\pi x^2$
- **c** $y=\dfrac{1}{5x^2}$
- **d** $y=100x$
- **e** $y=10(x+1)$
- **f** $y=4\pi x^3$

3 Differentiate with respect to x:

- **a** $6x+2$
- **b** $x\sqrt{x}$
- **c** $(5-x)^2$
- **d** $\dfrac{6x^2-9x^4}{3x}$
- **e** $(x+1)(x-2)$
- **f** $\dfrac{1}{x^2}+6\sqrt{x}$
- **g** $4x-\dfrac{1}{4x}$
- **h** $x(x+1)(2x-5)$

4 Find the gradient of the tangent to:

- **a** $y=x^2$ at $x=2$
- **b** $y=\dfrac{8}{x^2}$ at the point $(9, \frac{8}{81})$
- **c** $y=2x^2-3x+7$ at $x=-1$
- **d** $y=\dfrac{2x^2-5}{x}$ at the point $(2, \frac{3}{2})$
- **e** $y=\dfrac{x^2-4}{x^2}$ at the point $(4, \frac{3}{4})$
- **f** $y=\dfrac{x^3-4x-8}{x^2}$ at $x=-1$

Check your answers using technology.

5 Suppose $f(x)=x^2+(b+1)x+2c$, $f(2)=4$, and $f'(-1)=2$.
Find the constants b and c.

6 Find the gradient function of $f(x)$ where $f(x)$ is:

a $4\sqrt{x}+x$ **b** $\sqrt[3]{x}$ **c** $-\frac{2}{\sqrt{x}}$ **d** $2x-\sqrt{x}$

e $\frac{4}{\sqrt{x}}-5$ **f** $3x^2-x\sqrt{x}$ **g** $\frac{5}{x^2\sqrt{x}}$ **h** $2x-\frac{3}{x\sqrt{x}}$

7 **a** If $y=4x-\frac{3}{x}$, find $\frac{dy}{dx}$ and interpret its meaning.

b The position of a car moving along a straight road is given by $S=2t^2+4t$ metres where t is the time in seconds. Find $\frac{dS}{dt}$ and interpret its meaning.

c The cost of producing x toasters each week is given by $C=1785+3x+0.002x^2$ dollars. Find $\frac{dC}{dx}$ and interpret its meaning.

ANSWERS

1 **a** $3x^2$ **b** $6x^2$ **c** $14x$ **d** $\frac{3}{\sqrt{x}}$

e $\frac{1}{\sqrt[3]{x^2}}$ **f** $2x+1$ **g** $-4x$ **h** $2x+3$

i $2x^3-12x$ **j** $\frac{6}{x^2}$ **k** $-\frac{2}{x^2}+\frac{6}{x^3}$

l $2x-\frac{5}{x^2}$ **m** $2x+\frac{3}{x^2}$ **n** $-\frac{1}{2x\sqrt{x}}$

o $8x-4$ **p** $3x^2+12x+12$

2 **a** $7.5x^2-2.8x$ **b** $2\pi x$ **c** $-\frac{2}{5x^3}$

d 100 **e** 10 **f** $12\pi x^2$

3 **a** 6 **b** $\frac{3\sqrt{x}}{2}$ **c** $2x-10$

d $2-9x^2$ **e** $2x-1$ **f** $-\frac{2}{x^3}+\frac{3}{\sqrt{x}}$

g $4+\frac{1}{4x^2}$ **h** $6x^2-6x-5$

4 **a** 4 **b** $-\frac{16}{729}$ **c** -7 **d** $\frac{13}{4}$ **e** $\frac{1}{8}$ **f** -11

5 $b = 3,\ c = -4$

6 **a** $\dfrac{2}{\sqrt{x}} + 1$ **b** $\dfrac{1}{3\sqrt[3]{x^2}}$ **c** $\dfrac{1}{x\sqrt{x}}$

d $2 - \dfrac{1}{2\sqrt{x}}$ **e** $-\dfrac{2}{x\sqrt{x}}$ **f** $6x - \frac{3}{2}\sqrt{x}$

g $\dfrac{-25}{2x^3\sqrt{x}}$ **h** $2 + \dfrac{9}{2x^2\sqrt{x}}$

7 **a** $\dfrac{dy}{dx} = 4 + \dfrac{3}{x^2}$, $\dfrac{dy}{dx}$ is the gradient function of $y = 4x - \dfrac{3}{x}$ from which the gradient at any point can be found.

b $\dfrac{dS}{dt} = 4t + 4\ \ \mathrm{m\,s^{-1}}$, $\dfrac{dS}{dt}$ is the instantaneous rate of change in position at the time t, or the velocity function.

c $\dfrac{dC}{dx} = 3 + 0.004x$ \$ per toaster, $\dfrac{dC}{dx}$ is the instantaneous rate of change in cost as the number of toasters changes.