Maine Learning Results: Mathematics 9-12 (2020)

Geometry Standards

*** denotes an essential standard/indicator**

HSA.SSE.A.1: Interpret expressions that represent a quantity in terms of its context. ★

Developing/Proficient/Fluent SSE.A.1a: Interpret parts of an expression, such as terms, factors, and coefficients. ★

Developing/Proficient/Fluent SSE.A.1b: Interpret multi-part expressions by viewing one or more of their parts as a single entity. For example, view P(1+r)^n as the product of P and a factor not depending on P and interpret the parts. ★

Developing HSA.SSE.A.2: Use the structure of an expression to identify ways to rewrite it. For example, see x^4 - y^4 as (x^2)^2 - (y^2)^2 , allowing for it to be recognized as a difference of squares that can be factored as (x^2 - y^2)(x^2 + y^2).

HSA.SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★

HSA.SSE.B.3a: Rewrite a quadratic expression (such as by factoring) to reveal the zeros of the function it defines. ★

HSA.SSE.B.3b: Rewrite a quadratic expression (such as by completing the square) to reveal the maximum or minimum value of the function it defines. ★

HSA.SSE.B.3c: Use the properties of exponents to transform expressions for exponential functions. For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. ★

HSA.APR.A.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under certain operations.

Developing HSA.APR.A.1a: Perform operations on polynomial expressions (addition, subtraction, multiplication, and division), and compare the system of polynomials to the system of integers.

Developing HSA.APR.A.1b: Factor and/or expand polynomial expressions, identify and combine like terms, and apply the Distributive Property.

Introductory/Developing (+) HSA.APR.D.7: Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Developing/Proficient HSA.CED.A.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. ★

Developing HSA.CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★

Developing HSA.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods such as lobsters, blueberries, and potatoes. ★

Developing HSA.CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R. ★

Proficient/Fluent HSA.REI.A.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify or refute a solution method.

Developing/Proficient HSA.REI.A.2: Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise

Developing/Proficient HSA.REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

Introductory/Developing/Proficient HSA.REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
Introductory/Developing/Fluent HSA.REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
Introductory (honors only) HSA.REI.C.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the point(s) of intersection between the line y = -3x and the circle x^2 + y^2 = 3.
Introductory/Developing/Proficient HSA.REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Show that any point on the graph of an equation in two variables is a solution to the equation.
Developing/Proficient HSA.REI.D.11: Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. ★
Introductory/Developing HSA.REI.D.12: Graph the solutions of a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set of a system of linear inequalities in two variables as the intersection of the corresponding half-planes
Developing/Proficient/Fluent HSF.IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
Developing/Proficient/Fluent HSF.IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
Introductory (honors only) HSF.IF.A.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.
Introductory/Developing HSF.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features may include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative and absolute maximums and minimums; symmetries; end behavior; and periodicity. ★
Introductory/Developing HSF.IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. ★
Proficient/Fluent HSF.IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. ★
HSF.IF.C.7: Graph functions expressed symbolically and as well as show and describe key features of the graph, by hand in simple cases and using technology for more complicated cases. ★

> **Developing/Proficient HSF.IF.C.7a:** Graph linear and quadratic functions and show intercepts, maxima, and minima.
>
> **Introductory (honors only) HSF.IF.C.7b: i)** Graph square root and piecewise-defined functions, (including step functions and absolute value functions), as well as show and describe key features of the graph.

HSF.IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

> **Introductory (honors only) HSF.IF.C.8a:** Use the process of factoring and completing the square in a quadratic function to show zeros, maximum and minimum values, and symmetry of the graph, and interpret these in terms of a context.

Introductory HSF.IF.C.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
HSF.BF.A.1: Write a function that describes a relationship between two quantities. ★

Introductory HSF.BF.A.1a: Determine an explicit expression, a recursive process, or steps for calculation from a context.

Introductory/Developing(+) HSF.BF.A.1c: Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Introductory (honors only) HSF.BF.B.3: Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate 20 an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Okay as written.

HSF.BF.B.4: Find inverse functions.

Introductory HSF.BF.B.4a: Solve an equation of the form f(x) = c (where c represents the output value of the function) for a simple function f that has an inverse and write an expression for the inverse. For example, if f(x) =2 x^3 , then solving f(x) = c leads to x = (c/2)^⅓, which is the general formula for finding an input from a specific output, c, for this function.

HSF.LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions. ★

Introductory (honors only) HSF.LE.A.1a: Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

Developing HSF.LE.A.1b: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Introductory (honors only) HSF.LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). ★

Introductory/Developing HSF.LE.A.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. ★

Introductory (honors only) HSF.LE.B.5: Interpret the parameters in a linear or exponential function in terms of a context. ★

Developing (honors only) HSF.TF.A.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

Developing (honors only) HSF.TF.A.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Introductory/Developing (+) HSF.TF.A.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for x, π + x, and 2π - x in terms of their values for x, where x is any real number.

Introductory/Developing (+) HSF.TF.B.7: Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★

Introductory (honors only) (+) HSF.TF.C.8: Prove the Pythagorean identity sin2 (θ) + cos2 (θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.

Introductory (honors only) (+) HSF.TF.C.9: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

***Developing/Proficient HSG.CO.A.1:** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

***Developing/Proficient HSG.CO.A.2:** Represent transformations in the plane using, e.g., transparencies and/or geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

***Developing/Proficient HSG.CO.A.3:** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

***Developing/Proficient HSG.CO.A.4:** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
***Developing/Proficient HSG.CO.A.5:** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
***Introductory/Proficient HSG.CO.B.6:** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
***Introductory/Proficient HSG.CO.B.7:** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
Introductory/Developing HSG.CO.B.8: Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions
***Developing/Proficient HSG.CO.C.9:** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent, and conversely prove lines are parallel; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
***Developing/Proficient HSG.CO.C.10:** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent, and conversely prove a triangle is isosceles; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
***Introductory/Proficient HSG.CO.C.11:** Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
***Introductory/Proficient HSG.CO.D.12:** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
***Introductory/Proficient HSG.CO.D.13:** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
HSG.SRT.A.1: Verify experimentally the properties of dilations given by a center and a scale factor:

- **Introductory/Developing HSG.SRT.A.1a:** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
- **Introductory/Developing HSG.SRT.A.1b:** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Developing/Proficient HSG.SRT.A.2: Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
Developing/Proficient HSG.SRT.A.3: Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
***Developing/Proficient HSG.SRT.B.4:** Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
***Developing/Proficient HSG.SRT.B.5:** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
Introductory/Proficient HSG.SRT.C.6: Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
Introductory/Proficient HSG.SRT.C.7: Explain and use the relationship between the sine and cosine of complementary angles.
Developing/Proficient HSG.SRT.C.8: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. For example, find the current height of the tallest pine tree in Maine using the angle of elevation and the distance from the tree. ★

Introductory (honors only) (+) HSG.SRT.D.9: Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
Introductory (honors only) (+) HSG.SRT.D.10: Prove the Laws of Sines and Cosines and use them to solve problems.
Introductory (+) HSG.SRT.D.11: Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Introductory/Developing HSG.C.A.1: Prove that all circles are similar.
***Introductory/Developing/Proficient HSG.C.A.2:** Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
Introductory/Proficient HSG.C.A.3: Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.
Introductory/Developing (+) HSG.C.A.4: Construct a tangent line from a point outside a given circle to the circle.
Introductory/Developing HSG.C.B.5: Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
Introductory (honors only) HSG.GPE.A.1: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
Introductory (honors only) HSG.GPE.A.2: Derive the equation of a parabola given a focus and directrix.
Introductory (honors only) (+) HSG.GPE.A.3: Derive the equations of ellipses and hyperbolas given the foci and directrix, using the fact that the sum or difference of distances from the foci is constant.
Introductory/Developing HSG.GPE.B.4: Use coordinates to prove simple geometric theorems algebraically including the distance formula and its relationship to the Pythagorean Theorem. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2).
***Introductory/Developing/Proficient HSG.GPE.B.5:** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
Introductory/Developing HSG.GPE.B.6: Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
***Introductory/Developing/Fluent HSG.GPE.B.7:** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. ★
Introductory/Developing/Proficient HSG.GMD.A.1: Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and/or informal limit arguments.
Introductory (honors only) (+) HSG.GMD.A.2: Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
***Developing/Proficient HSG.GMD.A.3:** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. ★
***Introductory/Developing/Proficient HSG.GMD.B.4:** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
Developing/Proficient HSG.MG.A.1: Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). ★
Introductory/Developing HSG.MG.A.2: Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). ★
Introductory HSG.MG.A.3: Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). ★

Developing/Proficient HSN.Q.A.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. Example: Marlena made a scale drawing of the sand volleyball court at her summer camp. The drawing of

the volleyball court is 6 cm long by 3 cm wide. The actual volleyball court is 18 meters long. What scale did Marlena use for the drawing? ★
Developing/**Proficient** **HSN.Q.A.2:** Define appropriate quantities for the purpose of descriptive modeling. Example: If a town in Aroostook county with a population of 1254 people is projected to double in size every 105 years, what will the population be 315 years from now? ★
Developing/**Proficient** **HSN.Q.A.3:** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. Example: The label on a ½ - liter bottle of flavored water bottled in Maine indicates that one serving of 8 ounce contains 60 calories. The label also says that the full bottle contains 130 calories. Is this the actual amount or the estimated amount of calories in this bottle? How would you explain any discrepancy? ★