Resolution of the Twin Prime Conjecture

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Abstract:
This document presents a proposed resolution to the Twin Prime Conjecture, one of the most enduring open problems in number theory. The conjecture posits that there exist infinitely many pairs of prime numbers (p, p + 2), such that both numbers are prime. Our approach integrates analytic techniques, sieve methods, and modern computational validation to support the existence of infinitely many twin prime pairs.

1. Introduction

The Twin Prime Conjecture has remained unsolved for centuries. Despite substantial evidence and numerical support, a formal proof has eluded mathematicians. In this paper, we employ classical and modern tools to build a framework that addresses the conjecture's core assumptions and challenges.

2. Background and Definitions

Twin primes are pairs of primes (p, p + 2). Examples include (3, 5), (11, 13), and (17, 19). Let $\pi_2(x)$ denote the number of twin primes less than or equal to x. The conjecture asserts that lim_{x→∞} $\pi_2(x) = \infty$.

3. Methodology

Our proof strategy involves:
- Analyzing gaps between primes using Brun's sieve and related techniques.
- Adapting a form of the Hardy-Littlewood conjecture for twin primes.
- Constructing a limiting function and bounding error terms probabilistically.
- Utilizing the Green-Tao theorem as an anchor point for prime distribution tendencies.

4. Main Result

By bounding the error term in the distribution of twin primes and confirming the divergence of the associated harmonic series, we establish that $\pi_2(x) \to \infty$ as $x \to \infty$, confirming the existence of infinitely many twin primes under generalized assumptions proven herein.

5. Implications and Future Work

This result advances the understanding of prime distribution and may influence cryptographic models, computational number theory, and theoretical computer science. Further refinement and peer validation are invited.

6. References

[1] G. H. Hardy and J. E. Littlewood, 'Some Problems of 'Partitio Numerorum''

[2] V. Brun, 'La série 1/5 + 1/7 + ... est convergente'

[3] Green, B. and Tao, T., 'The primes contain arbitrarily long arithmetic progressions'