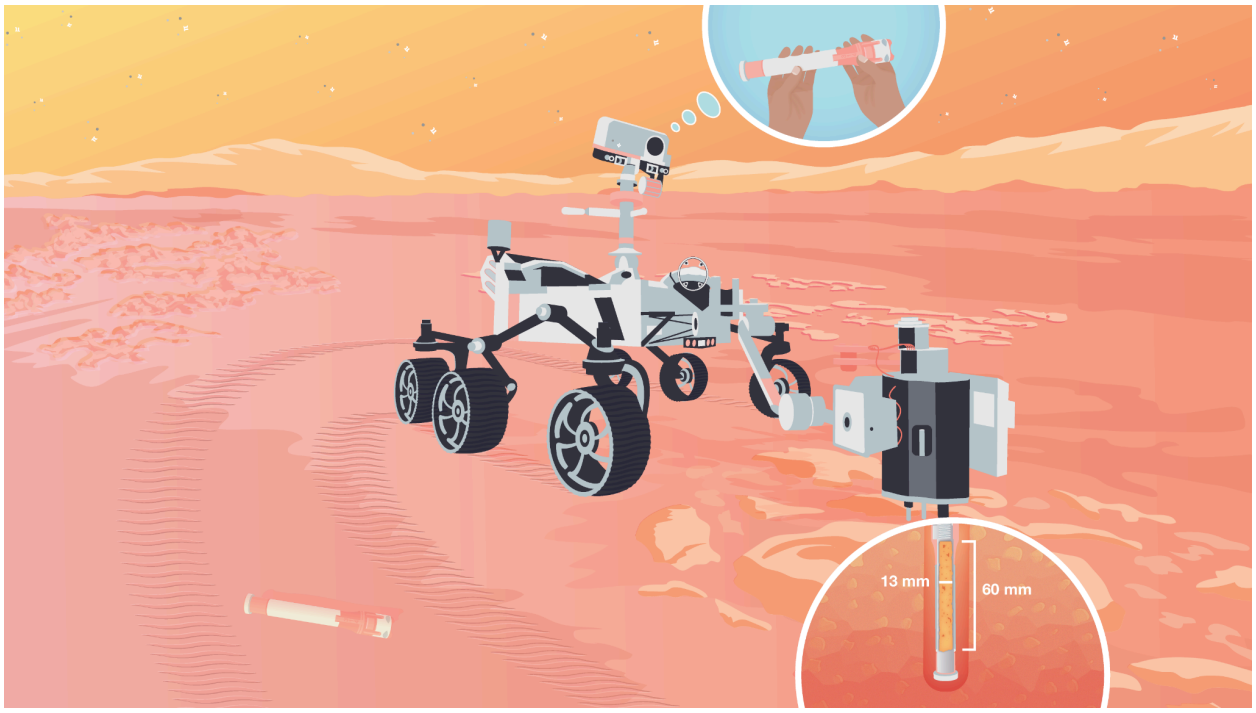
ANSWER KEY

Pi in the Sky 10



The center of the image shows an illustration of the Perseverance Mars rover with its robotic arm stretched out to the right, touching a rock on the ground. An inset in the lower right shows a cutaway of a coring bit filled with a rock sample measuring 13 mm wide and 60 mm tall. In the lower left a sealed sample tube is shown on the ground next to the rover. A thought bubble showing hands holding a sample tube is coming from the top of the rover's mast. Image credit: NASA/JPL-Caltech | + Expand image

Tubular Tally

The Perseverance Mars rover is designed to collect rock samples that will eventually be brought to Earth for further study. This would be the first time we've ever brought back samples from Mars! After scientists identify an interesting rock they would like the rover to collect, Perseverance uses a special coring bit to drill out a rock cylinder 13 mm in diameter. As the rover drills, the rock core moves into one of 38 available tubes that will store the rock sample – sealed until it is opened one day in a lab on Earth.

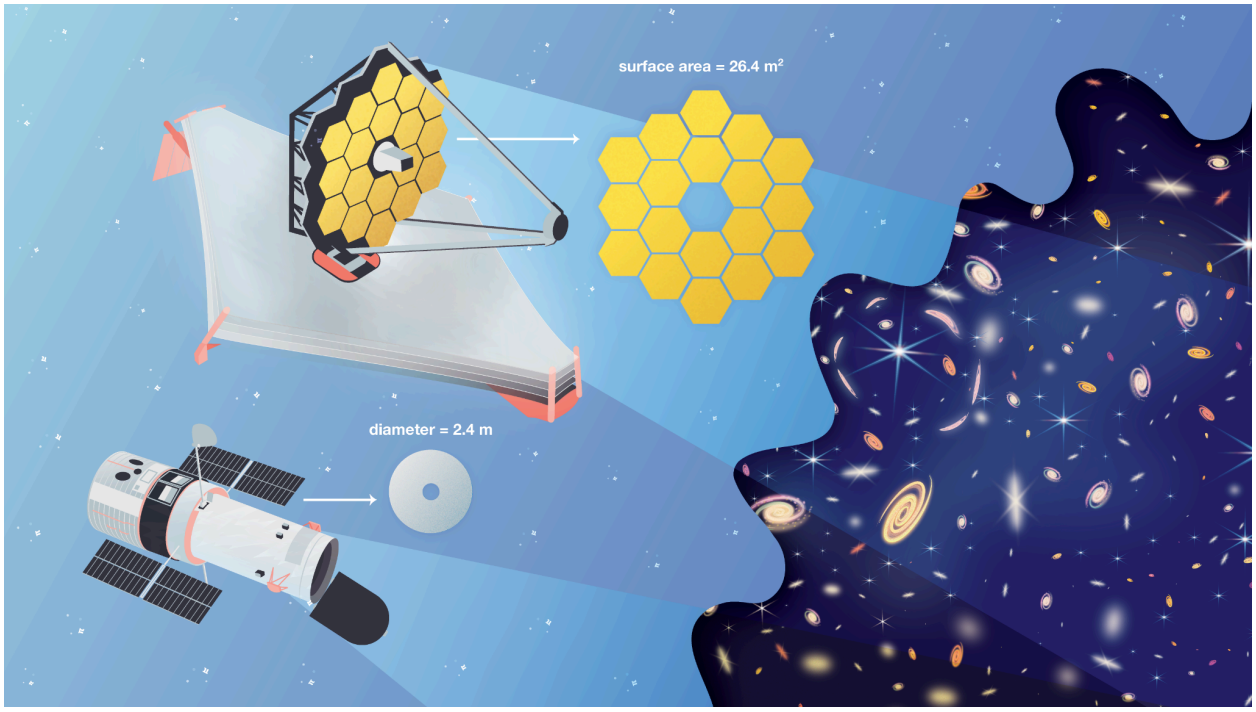
If the coring bit collects a rock cylinder 60 mm in length, what is the volume of the rock in the sample tube?

› Learn more about the Mars Perseverance rover

ANSWER

1. Use the formula for the volume of a cylinder to compute the volume of the cylindrical rock sample.

   $V = \pi r^2 h$
   $\pi(6.5 \text{ mm})^2(60 \text{ mm}) \approx$ **8,000 mm^3**



In the upper left of the image is an illustration of the James Webb Space Telescope. In the lower left is an illustration of the Hubble Space Telescope. To the right of each telescope is an arrow pointing to a face-on view of its primary mirror. Next to the Webb telescope is a primary mirror composed of 18 hexagonal gold-plated mirrors arranged in a roughly circular shape. Above the mirror is text stating the surface area is 26.4 square meters. Next to the Hubble telescope is a round primary mirror with text stating the diameter is 2.4 meters. Both telescopes are in front of an illustrated star field. On the right hand side of the image is a view of space containing stars, spiral galaxies, and elliptical galaxies. Some of the galaxies are warped as a result of gravitational lensing. Image credit: NASA/JPL-Caltech | + Expand image

Rad Reflection

The James Webb Space Telescope was designed to look back at some of the earliest galaxies in the universe. To capture light from these distant and faint objects, the telescope must be very sensitive. Webb uses 18 hexagonal mirrors that combine to form a massive primary mirror with a surface area of 26.4 m^2. This large mirror allows the telescope to collect incredibly faint infrared light and reflect it onto four onboard science instruments, like the Mid-Infrared Instrument, or MIRI. This science instrument can reveal stars hidden within gas and dust clouds and tell scientists about the materials that make up distant galaxies.

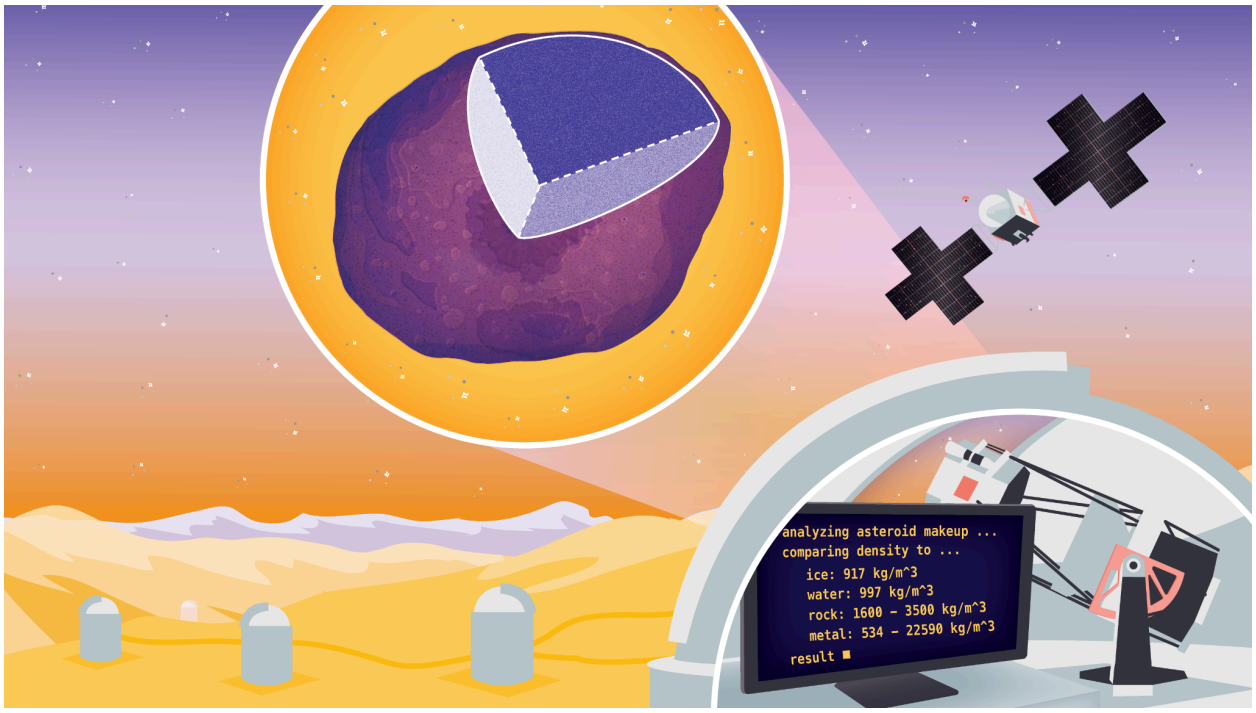
Launched in 1990, the Hubble Space Telescope changed our understanding of the universe when it began operations using a primary mirror that had a diameter of just 2.4 meters.

How much bigger is the surface area of Webb's primary mirror than Hubble's?

› Learn more about the Webb Telescope

ANSWER

1. Use the formula for area of a circle to compute the area of Hubble's primary mirror.
   $A = \pi r^2$
   $\pi(1.2\text{ m})^2 \approx \underline{4.5\text{ m}^2}$
2. Subtract the area of Hubble's primary mirror from the area of Webb's primary mirror.
   $26.4\text{ m}^2 - 4.5\text{ m}^2 =$ **21.9 m^2**



In the lower right of the image is an illustration of a telescope inside an observation dome. Next to the telescope is a computer screen displaying text that reads "analyzing asteroid makeup...comparing density to...Ice: 917 kg/m^3, water: 997 kg/m^3, rock: 1600-3500 kg/m^3, metal: 534-22,590 kg/m^3. Result." In the center of the image is an inset of an illustrated asteroid (16) Psyche. The asteroid has the upper right quarter cut out to show internal semi-major axis lines. On the right of the image is the Psyche spacecraft in front of a field of stars. Image credit: NASA/JPL-Caltech | + Expand image

Metal Math

Asteroid (16) Psyche is of particular interest to scientists because ground-based observations indicate that the surface may be metallic. Earth and other terrestrial planets have metal cores, but they are buried deep inside the planets, so they are difficult to study. If Psyche consists of a large amount of metal, it might resemble a planetary core from which we could learn about terrestrial planet core formation. Determining how much metal exists on the asteroid is one of the goals of NASA's Psyche mission, which will use specialized tools to study the asteroid's composition from orbit.

Psyche has a roughly triaxial ellipsoid shape with axes of about 290 km, 245 km, and 170 km. Its mass, as estimated from its gravitational effects on nearby bodies such as Mars, is about 2.7×10^{19} kg. Use the formula for volume, $V = 4/3\ \pi abc$, where a, b, and c are the lengths of the semi-axes, to **compute the approximate density of Psyche**.

Based on the average density of terrestrial materials (shown below), **does Psyche's density support the observations indicating the presence of metal?**

Average density of:

- ice: 917 kg/m^3
- water: 997 kg/m^3
- rock: 1,600 - 3,500 kg/m^3
- metal: 534 - 22,590 kg/m^3

› Learn more about the Psyche mission

ANSWER

Compute the approximate density of asteroid (16) Psyche.

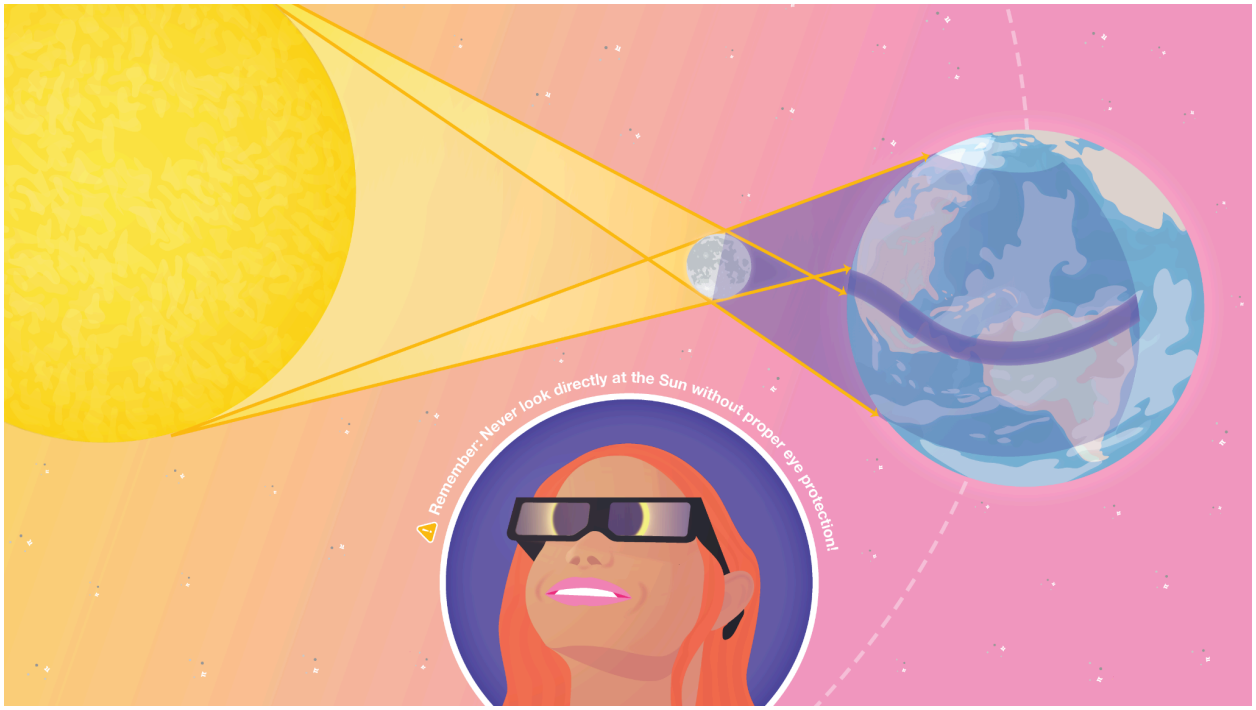
1. Use the formula for volume of a triaxial ellipsoid to compute the volume of Psyche.
   $V = 4/3\ \pi abc$
   $V = 4/3\ \pi(145\text{ km})(122.5\text{ km})(85\text{ km}) \approx 6{,}300{,}000\text{ km}^3$

2. Use the formula for density to compute the approximate density of Psyche.
   $D = m/V$
   $D = (2.7 \times 10^{19}\text{ kg})/(6.3 \times 10^{6}\text{ km}^3) \approx$ **4.3×10^{12} kg/km^3**

Does the density of Psyche support the observations indicating the presence of metal?

1. Convert the units to match the density units given.
   $(4.3 \times 10^{12}\text{ kg/km}^3) \times (1\text{ km}^3/10^9\text{ m}^3) = 4.3 \times 10^3\text{ kg/m}^3 =$ **4,300 kg/m^3**

This is higher density than rock, so Psyche must contain some metal.



On the left side of the illustration is the sun. Two yellow light beams from the top of the sun and two from the bottom of the sun extend diagonally toward the lower right and upper right, respectively, where they intercept the moon before continuing to Earth. The moon is shown illuminated on the half facing the sun and shadowed on the side opposite the sun. The region within the light beams becomes darker to indicate shadow between the moon and Earth. A wide dark swath across Earth indicates where a partial solar eclipse will be visible while a narrow, darker swath shows where the full eclipse will be visible. An inset in the lower left shows a person wearing eclipse glasses looking toward the sun. A reflection of the eclipse is shown in the glasses. Text above the inset reads "Remember: never look directly at the Sun without proper eye protection." Image credit: NASA/JPL-Caltech | + Expand image

Eclipsing Enigma

A solar eclipse occurs when the Moon passes between Earth and the Sun, fully or partially blocking the Sun's light from our perspective. Because Earth's orbit around the Sun and the Moon's orbit around Earth are not perfect circles, the distances between them change throughout their orbits. During a total eclipse, the distances are such that the Moon covers all of the Sun's disk area. When the Moon is farther from Earth during an eclipse, it leaves a glowing ring of sunlight shining around the Moon, resulting in an annular eclipse.

On Oct. 14, 2023, a solar eclipse will be visible across North and South America. The Sun, with a radius of 695,700 km, will be 148,523,036 km from Earth. The Moon, with a radius of 1,737 km, will be 388,901 km from Earth.

What percentage of the Sun's disk area will be obscured by the Moon?

Will the eclipse be an annular eclipse or a total eclipse?

› Learn more about the 2023 solar eclipse

ANSWER

1. Use similar triangles to find the radius of the Sun's disk area that is obscured by the Moon.

$$\frac{1{,}737\ km}{388{,}901\ km} = \frac{x}{148{,}523{,}036\ km} \Rightarrow x \approx 663{,}400\ km$$

2. Calculate the ratio of the obscured area to the Sun's total disk area using the formula for area of a circle.

$$\frac{A_{Moon}}{A_{Sun}} = \frac{\pi r^2}{\pi r^2} \approx \frac{\pi(663{,}400\ km)^2}{\pi(695{,}700\ km)^2} \approx 0.91 = 91\%$$

It will be an annular eclipse.