Organizing Around Landmarks of Ten: Building Number Fluency for Addition & Subtraction

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Multi-digit numbers for very young children are, largely, piles of ones. Initially, determining a total amount of items requires *direct modeling* where both sets of numbers are counted out and then, starting from one, all the items are counted. Eventually being able to *count on* saves students time. Moving to the next developmental level of *decomposing numbers to work in chunks* (Deriving and flexible strategies) requires significant scaffolding for students to, a) recognize that it is possible, and b) become adept at doing it. If counting on becomes habitual, it is often difficult to break that habit in the upper grades. Supporting students in developing the knowledge and skill of organizing around landmarks of ten not only increases the efficiency by which to add and subtract multi-digit numbers but underlying algebraic properties are developed as well. This article looks at one particular number activity that can be used in Number Talks, small group station activities, intervention support settings, etc. The activities are apt for all grade levels. It is merely a question of the size and domain of numbers within which the students are working.

Decomposing Number - This is an important number concept students need to come to understand and be fluent in when operating on number. At kindergarten and first grade, this is as simple as coming to know all the different combinations to make 6 such as 6 + 0, 5 + 1, 4 + 2, 3 + 3. With each of these combinations, students also need to know that 6 remains the total regardless of how it is broken into various whole number combinations. In the upper grades, knowing $\frac{6}{3}$ can be decomposed in $\frac{3}{3} + \frac{3}{3}$ to determine its equivalence to 2 or that $\frac{2}{4}$ can be decomposed into $\frac{1}{4} + \frac{1}{4}$ in order to find the solution to $2\frac{1}{4} - \frac{3}{4}$ is a key element to becoming fluent with adding, subtracting, multiplying, and dividing numbers. In the context of the activity that is the focus in the article, it is the combinations of 10 that are critical.

As a Strategy for Addition and Subtraction - When I want to add the numbers 7 + 5 or subtract 13 - 6 and I don't have recall of those combinations yet, many students fall back counting on or back by ones to determine the solution. While for a first or early second grader this may be developmentally normal, it becomes extremely inefficient once students begin operating with multi-digit numbers. This habit in fourth, fifth, and sixth grades truly makes mathematics laborious for students as they progress in more complex mathematical concepts and procedures. *Decomposing the numbers to organize around a landmark of ten* increases the efficiency of the computation. Example:

> Instead of solving 7 + 5 as 7 + 1 + 1 + 1 + 1 + 1 as a counter by ones would do, a student adept at decomposing number to organize around landmarks of 10 do 7 + 5 = 7 + 3 + 2 = 10 + 2
>
> This strategy extends to when wanting to add:

295 + 7	[295 + 5 + 2]
3.98 + 2.05	[3.98 + .02 + 2.03]
-25 + -27	[-25 + -25 + -2]
$2\frac{5}{8} + \frac{7}{8}$	[$2\frac{5}{8} + \frac{3}{8} + \frac{4}{8}$]

The same applies when looking to efficiently subtract one number from another especially when it is necessary to move into the decade below. Example:

13 - 6 = 13 - 3 - 3 = 10 - 3

453 - 75 = 453 - 53 - 22 = 400 - 22

$2\frac{5}{8} - \frac{7}{8} = 2\frac{5}{8} - \frac{5}{8} - \frac{2}{8} = 2 - \frac{2}{8}$

Activities to Build Organizing Around Landmarks of 10

Get to 10, Get to 20, Get to the Next 10, Get to 100

Kindergarten and First-grade students should ideally come away with knowing their combinations of 10 as fluently as possible.

First Graders by the end of the year should use what they know about getting to 10 to not only add over 10, e.g. 7 + 5 = 7 + 3 + 2 = 10 + 2, but also use getting to 10 to get to 20

Second graders should perfect their fluency adding over 10 [7 + 5] and extend that to going over any decade, e.g. 57 + 5 = 57 + 3 + 2 = 60 + 2. This means using what they know about getting to 10 to *get to the NEXT ten*. Additionally, second graders should work on fluidly getting to 100 and from any double-digit number to another, e.g. How much to get from 32 to 55?

Third through fifth graders continue what is started in second grade by extending the range to landmarks of the *Next 100/1000/10,000,* etc. and from any number to another. This connects directly to the *Adding Up to Subtract* strategy, e.g. 435 - 178 can be solved as 178 + c = 435. 178 + 22 + 235 = 435 where 270 is the answer for c [22 + 235].

Fourth and fifth graders extend the use of this activity by integrating decimals and fractions, e.g. *You are at .67; how much to get to 1.0?*

Sixth-graders continue the same organizing around landmarks as the number domain expands to include negative integers.

Using *Arrow Notation* to Capture Students Thinking

Leads to arrow (→) - The arrow is gaining traction in the mathematics education community as an acceptable symbol to capture the action by the individual as the computation unfolds. The symbol is referred to as a "*leads to"* arrow or "*gets me to…"* arrow. The benefits for introducing arrow notation to students is two-fold: it avoids abuse of the equal sign and it potentially captures movement on the number line

Avoiding abuse of the equal sign - Arrow notation is an informal representation used to capture students' thinking as the increment forward or backward in the numbers. This notation prevents students from abusing the equal sign in capturing the steps in their thinking. While my thinking may be to split 5 into 3 and 2 so that I can add 3 to 7 and then add 2 to get to 12, *it is incorrect to write* that as…

7 + 3 = 10 + 2 = 12

7 + 3 does equal 10, but 7 + 3 ≠ 10 + 2. Students need to view the equal sign as a *comparison symbol* of two equal mathematical expressions, not as an operation symbol. Thus the above example can be captured as…

$7 + \underline{3} \rightarrow 10 + \underline{2} \rightarrow 12$; 12 is my answer

Mimicking movement on a number line - *Arrow Notation* can be used to capture the direction of movement on the number line depending on how it is used, particularly in subtraction. Consider the two ways it is used below to capture a solution to 42 + *c* = 100.

$$42 + 50 \longrightarrow 92 + 8 \longrightarrow 100$$

$$42 \xrightarrow{\qquad\qquad +50 \qquad\qquad} 92 \xrightarrow{+8} 100$$

c = 58

In the first instance, the arrows are capturing a line of thinking. In the second, a relative value is being reflected as to the length of distance each span represents. While it is not an exact representation and the proportions of a true number line, it reflects an individual's understanding of both the direction of movement and relative span of distance. The same thing holds for the use of arrow notation to capture one's thinking in solving a subtraction problem.

$$72 - 48$$

$$72 - 40 \longrightarrow 32 - 2 \longrightarrow 30 - 6 \longrightarrow 24$$

$$24 \xleftarrow{-6} 30 \xleftarrow{-2} 32 \xleftarrow{\qquad\qquad -40 \qquad\qquad} 72$$

In the first solution, the arrow notation is being used to capture the continuous flow of removing increments of 48; first the 40, then 2 of the eight and then the final 6 of the eight. In the second, the use of the arrow notation captures how the solution moves along a number line. Both representations are accurate. However, each reflects a different mathematical idea. If your role in an instructional context with students is to be the recorder of a student's thinking, you need to decide and record which representation is most mathematically productive to the student(s) to consider at that time.

Range of strategies - When posing the *Get to a Number* activity with students, know that there is more than one strategy a student can use to find a solution. It is essential that students understand that flexibility in thinking is acceptable and admired. What may involve you is the support of students in how to record those various strategies. Once recorded, a conversation that compares and contrasts the different strategies focuses students on the various possibilities available to them in the future. Compare the following strategies to solve the following. *You are at 678, how much to get to 1000.*

$$678 \xrightarrow{+20} 692 \xrightarrow{+8} 700 \xrightarrow{\qquad\qquad +300 \qquad\qquad} 1000\text{; The answer is } \mathit{328}$$

$$678 \xrightarrow{\qquad\qquad +300 \qquad\qquad} 978 \xrightarrow{+2} 980 \xrightarrow{+20} 1000$$

$$678 \xrightarrow{\qquad\qquad\qquad +400 \qquad\qquad\qquad} \cancel{1000} \longrightarrow 1078$$

$$\xleftarrow{-78} \qquad\qquad \mathit{400 - 78 = 328}$$

Each of these strategies is efficient and productive. The first strategy uses smaller increments to get to the next 100, the second increments a large quantity first and then smaller amounts to 1000, the third, and example of *the compensation strategy*, goes beyond the target amount of 1000 and then adjusts at the end to find the final solution.

Examples for subtraction for 23.14 – 19.89

The Difference Between Strategy (subtraction)

$19.89 \xleftarrow{-.11} 20 \xleftarrow{\quad -3 \quad} 23 \xleftarrow{-.14} 23.14$; The answer is 3 + .11 + .14 = 3.25

Separate Result Unknown

.89 = .14 + .75

3.25 $\xleftarrow{-.75} 4 \xleftarrow{\quad -19 \quad} 23 \xleftarrow{-.14} 23.14$ The answer is 3.25

Compensation Strategy

$3.14 \xleftarrow{\quad -20 \quad} 23.14$

$\xrightarrow{+.11}$ *3.25* The answer is 3.25

The difference between (addition)

$19.89 \xrightarrow{+.11} 20 \xrightarrow{+3.14} 23.14$ The answer is 3.14 + .11 = 3.25

The intent is to honor the students' thinking and to support them in representing their strategy in as mathematically accurate as possible.

Standard Representation of Mathematical Equations - Eventually, students can move to more conventional mathematical forms of representation. This does not mean that vertical/column forms of representation are more powerful than horizontal. Remember, upper-level mathematics is all represented in a horizontal format. It is in accounting where vertical formats predominate. Students *should be comfortable moving across both formats* of representation. The mathematical relationships show up differently in the two formats and it is powerful for students to understand how the same mathematical idea is captured in both representational forms.

Language of Value - As you and your students work with numbers, it is essential that all of you talk in values rather than in digits. Speak in terms of 20s and 30s rather than in 2 or 3 or even 2 tens and 3 tens. This is particularly important when working with decimals. In the examples above .14 is always to be read as *fourteen hundredths* not "point one four." The more students speak and think in values the stronger and more secure their place value understanding becomes. Speaking in digits inadvertently freezes students in the immature notion that multi-digit numbers are just the literal single values mechanically manipulated.

Summary -

- Support students in developing the capacity to decompose numbers into efficient combinations to organize around landmarks of 10. This will move them off the younger developmental stage of counting on/back by ones
- Learning to decompose and reconfigure numbers is a core mathematical concept and skill that allows many algebraic principles to be understood and used

- Learning to organize around landmarks of 10 develops various efficient addition and subtraction strategies based on a strong number sense
- Arrow notation is an informal yet robust means to represent one's computational thinking and helps students avoid abusing the use of the equal sign
- Arrow notation can be used in two ways, to simply record the linear progression of one's mathematical actions or to represent movement on the number line. Both are valid.
- More than one strategy can be used to solve addition and subtraction problems using arrow notation.
- Consistently learning to speak in value while operating on numbers allows students' place value knowledge to become more secure sooner and develop more robustly
- These math activities are just as powerful for middle and secondary students as it is for elementary students. It is all a question of the numbers, number domains, and number ranges in which the students are able to work