BC Calculus Name:____________________

Day 4.4 – Logistic Growth Model

1) A rumor spreads through a community at the rate $\frac{dy}{dt} = 2y(1-y)$, where y is the percentage *(y = 1 means 100%)* of the population that has heard the rumor at time t weeks.

a) Suppose that at time $t = 0$, ten percent of the people in the community have heard the rumor. Use the solution of the differential equation above to write y as a function of t.

$y(t) =$ ______________________

b) What percentage of the population has heard the rumor at the time when it is spreading the fastest?

____________%

c) At what time t is the rumor spreading the fastest?

$t =$ ____________

d) Sketch the graph of y vs. t, scaling your axes appropriately and showing all of the important features of the graph.

2) Let g be a function with $g(4)=1$ such that all points (x, y) on the graph of g satisfy the logistic differential equation:

$$\frac{dy}{dx} = 2y(3-y)$$

a) Given that $g(4)=1$, find $\lim_{x\to\infty} g(x)$ and $\lim_{x\to\infty} g'(x)$.
(It is not necessary to solve for $g(x)$ or to show how you arrived at your answers.)

$\lim_{x\to\infty} g(x)$ = ____________

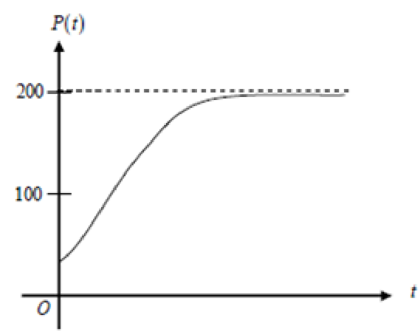
$\lim_{x\to\infty} g'(x)$ = ____________

b) For what value of y does the graph of g have a point of inflection? Find the slope of the graph of g at the point of inflection. (*As in (a), it is not necessary to actually solve for $g(x)$*)

$y =$ ____________

slope = ____________

3)



Which of the following differential equations for a population P could model the logistic growth shown in the figure above?

(A) $\frac{dP}{dt} = 0.2P - 0.001P^2$

(B) $\frac{dP}{dt} = 0.1P - 0.001P^2$

(C) $\frac{dP}{dt} = 0.2P^2 - 0.001P$

(D) $\frac{dP}{dt} = 0.1P^2 - 0.001P$

(E) $\frac{dP}{dt} = 0.1P^2 + 0.001P$

4) Let k be a positive constant. Which of the following is a logistic differential equation?

(A) $\frac{dy}{dt} = kt$

(B) $\frac{dy}{dt} = ky$

(C) $\frac{dy}{dt} = kt(1-t)$

(D) $\frac{dy}{dt} = ky(1-t)$

(E) $\frac{dy}{dt} = ky(1-y)$

5) A population y changes at a rate modeled by the differential equation $\frac{dy}{dt} = 0.2y(1000 - y)$, where t is measured in years. What are all values of y for which the population is increasing at a decreasing rate?

(A) 500 only

(B) $0 < y < 500$ only

(C) $500 < y < 1000$ only

(D) $0 < y < 1000$

(E) $y > 1000$

6) The rate of change, $\frac{dP}{dt}$, of the number of people on an ocean beach is modeled by a logistic differential equation. The maximum number of people allowed on the beach is 1200. At 10 A.M., the number of people on the beach is 200 and is increasing at the rate of 400 people per hour. Which of the following differential equations describes the situation?

(A) $\frac{dP}{dt} = \frac{1}{400}(1200 - P) + 200$

(B) $\frac{dP}{dt} = \frac{2}{5}(1200 - P)$

(C) $\frac{dP}{dt} = \frac{1}{500}P(1200 - P)$

(D) $\frac{dP}{dt} = \frac{1}{400}P(1200 - P)$

(E) $\frac{dP}{dt} = 400P(1200 - P)$