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Total No. of Printed Pages: 2

Total No. of Questions: [09]

B.Sc. (Non-Medical) (Semester – 3rd)

REAL ANALYSIS-I

Subject Code: BSNMS1306

Paper ID: [22131416]

Time: 03 Hours

Maximum Marks: 60

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A

(2 marks each)

Q1. Attempt the following:

- a) If $x, y \in \mathbb{R}$ and $x < y$, then there exists a rational number $p \in \mathbb{Q}$ such that $x < p < y$.
- b) Find the value of b for which $1 + e^b + e^{2b} + e^{3b} + \dots = 9$.
- c) State Cauchy convergence criterion for sequences.
- d) If $a, b \in \mathbb{R}$, then $|a + b| \leq |a| + |b|$.
- e) Determine the radius of convergence of power series $\sum \frac{(n+1)^{n+1} x^{n+1}}{(n+1)!}$
- f) State Bolzano Weier strass Approximation theorem.
- g) Check the convergence/divergence of series $\sum_{n=1}^{\infty} n$.
- h) Give an example of a uniformly continuous function of $[0, 1]$ that is differentiable on $(0, 1)$ but whose derivative is not bounded on $(0, 1)$.
- i) If $A = \left\{ \frac{3}{2^n} : n \in \mathbb{N} \right\}$. Find inf and sup of A . Is A bounded?
- j) Check the convergence of the series $\sum_{n=1}^{\infty} \frac{6^n}{4^n + 3}$.

Section – B

(5 marks each)

- Q2. (i) Show that $\mathbb{Q}$ does not have least upper bound property.
(ii) The set $\mathbb{Q}$ of all rational numbers is denumerable.
- Q3. Define a Monotone sequence. Using Monotone convergence theorem, check the convergence of sequence $\left\{ \frac{2n+1}{n+5} \right\}$.
- Q4. State the direct comparison test and limit comparison test. Also, check the convergence of the series $\sum_{n=1}^{\infty} \frac{3n+1}{(n+1)(n+2)(n+3)}$.
- Q5. Discuss the pointwise and uniform convergence of a sequence of functions $\{f_n\}, f_n : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f_n(x) = \frac{nx}{1+n^2 x^2}$, for all $x \in \mathbb{R}$.

Q6. (i) Does $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ converge?

(ii) Suppose $\{x_n\}, \{y_n\}$ and $\{z_n\}$ are real sequences. If $x_n \rightarrow x$ and $y_n \rightarrow x$ and $x_n \leq z_n \leq y_n$ for all $n \geq N_0$, then $z_n \rightarrow x$.

Section – C

(10 marks each)

Q7. Define conditional convergent and absolute convergent. Check whether the following series converge absolutely or conditionally.

(i) $\sum_{n=1}^{\infty} (-1)^n \frac{\tan^{-1} n}{n^2 + 1};$

(ii) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin \sin n}{n^2}.$

Q8. (i) Define a bounded sequence. Show that a convergent sequence is bounded.

(ii) Show that R is uncountable.

Q9. Assuming that the power series expansion for $(1 + x^2)^{-1}$ as $(1 + x^2)^{-1} = 1 - x^2 + x^4 - x^6 + \dots$. Obtain the power series expansion for $\log \log 1 + x$. Deduce that

(i) $1 - \left(\frac{1}{2}\right) + \left(\frac{1}{3}\right) - \left(\frac{1}{4}\right) + \dots = \log 2$

(ii) $\frac{1}{2}[\log \log (1 + x)]^2 = \left(\frac{1}{2}\right)x^2 - \left(\frac{1}{3}\right)\left(1 + \frac{1}{2}\right)x^3 + \left(\frac{1}{4}\right)\left(1 + \frac{1}{2} + \frac{1}{3}\right)x^4 - \dots$, for $-1 \leq x \leq 1$.