LOUISIANA TECH UNIVERSITY

Design Report II:

Regenerative Heat Exchanger System

Kyle Beardsley ♦ Andrew Huck ♦ Colby Rybicki

MEEN 451 - 001

4 November 2019

Summary

The goal of this design project is to reduce the cost associated with running an ongoing process by adding a regenerative heat exchanger. Before implementing said heat exchanger, the process line consisted of a water supply flowing into a water heater and heating up in preparation for entering an unspecified process. After the water is used in this process, it is disposed of in a holding pond as a result of being contaminated during the process.

Our team has been tasked with designing a double-pipe heat exchanger. Due to the layout of the system and the desired efficiency, we are required to use a counterflow configuration. This heat exchanger will be used to heat the cold water supply using the contaminated hot water being output by the process. The process is designed to lower the heating required by the existing water heater by recapturing the heat from the contaminated water, thus lowering the overall amount of heating that would be used.

In order to analyze potential designs and ultimately find the best heat exchanger design for this situation, our team utilized MathCad. The end goal is to maximize the yearly saving of running the process by implementing this regenerative heat exchanger system. As such, the quality of the potential designs will be judged based on the annual savings they provide.

Problem Statement

LTU Chemicals, Inc. has asked the team to design a regenerative double-pipe heat exchanger for an existing chemical process. In the current design, water enters a water heater at 70°F, is heated to 125°F, and is then used in their chemical process. Upon finishing this process, the water will have been cooled to $115^{\circ}F$ and is then disposed of due to contamination. The intention of this project is to utilize the warm contaminated water to aid in the heating process via a heat exchanger. The system should be designed as a multi-modular unit, with several double-pipe heat exchangers operating in parallel with each other; each of these heat exchangers will have the same diameter, length, flow rate, and pressure drop. This pipe will operate in a counterflow configuration.

The system requires that the pressure drop across the fluid can not exceed 10 psi. Because the heat exchanger uses the same fluid path for both the heating and cooling, the total pressure drop across the system is the sum of both parts.

Due to the limited amount of space for the heat exchanger, the length of the pipes used in the heat exchanger can not exceed 50 ft, and the piping can only be procured in 5 ft increments. The pipe must be made of wrought iron and is only available in specific sizes. Each exchanger needs to have two tees to connect the flow to the annular region of the pipe and two bushings to prevent leaking. Prices for the pipes and tees are listed below in Table 1.

Nominal Size (in.)	1	2	3	4
Pipe Cost ($/ft)	5.42	4.26	26.33	32.05
Tee Cost ($)	4.26	14.10	48.34	101.07

Table 1. Cost of Piping & Tees

Prices for the bushings are listed in Table 2. Any bushings not listed may be estimated by summing the prices of the requested size.

Nominal Size (in.)	1x2	2x3	3x4
Bushing ($)	4.93	9.17	17.60

Table 2. Cost of Bushings

The pump has a 70% efficiency, and the initial cost of the pump is given as a function of its pump head,

$$C_{pump} = 500 * h_p^{0.5}$$

Equation 1. Cost of Pump

with C_{pump} as the cost of the pump in dollars and h_p is the pump head in feet. The cost of pumping the water is $0.11 per kilowatt-hour, and the operating cost of the water heater is $0.03 per kilowatt-hour. The system is expected to be running for 2000 hours per year.

The system has a planned lifespan of 15 years and uses an effective interest rate of 6%. The optimal design is based on the amount of cost savings relative to the initial plan. The total cost of the heat exchanger pipes, fittings, and an additional pump needs to be less than the amount of savings from the reduced use of the water heater.

Design Methodology

To find the best heat exchanger design, it is necessary to explore all potential options. With this in mind, our team decided to analyze every combination of pipe diameters at the maximum length of 50 feet and compare the maximum cost savings that can be achieved by varying the number of repeat units. Once the best combination of diameters was found, we began to change the length and observe how this affects the cost savings. Using this methodology, we arrived at a combination of length and pipe and annulus diameters, which gives the maximum yearly savings.

As mentioned above, the ultimate deciding factor is the yearly savings that the heat exchanger provides. This is to say that the initial cost of heating the water (Equation 4) must be reduced by enough to compensate for the added cost of the piping and pump, as well as the ongoing cost of running the additional pump.

First, a baseline cost of the previous system with which to compare the new system must be found. The requirements of the process are given as an inlet temperature and an inlet volumetric flow rate. This can be converted to an overall required mass flow rate for the process using Equation 2. Note Q_{req} is the required volumetric flow rate, and ρis the density of water at$125\ ^{\circ}F$.

$$\dot{m}_{overall} = Q_{req} \cdot \rho$$

Equation 2. Total Mass Flow Rate

Next, the power needed to heat the water can be found using Equation 3 where c_p is taken at the average fluid temperature of the water being heated.

$$\dot{q} = \dot{m}_{overall} \cdot c_p \cdot \left(T_{out} - T_{in}\right)$$

Equation 3. Power Required to Heat Fluid

Finally, the annual cost of the old system is found using Equation 4 by taking the product of the heater's operating cost, the total operating time (t = 2000 hr), and the heater's required power.

$$C = \left(\frac{\$0.03}{kW\ hr}\right)\dot{q}\ t$$

Equation 4. Annual Electrical Cost of Water Heater

After finding a baseline cost a generic mathcad sheet was set up which would allow any combination of diameters, lengths, and repeat units to be analyzed. This would allow the cost of the new systems to be compared against the established baseline cost.

In the following equations, any variable with the subscript 'p' refers to the inner pipe, for which the fluid properties are based on the hot water's average temperature. Conversely, any value with the subscript of 'a' refers to the annulus, for which the fluid properties are based on the cold water's average temperature, unless otherwise specified.

$$A_p = \frac{\pi}{4} \cdot {ID_p}^2 \qquad A_a = \frac{\pi}{4} \cdot \left({ID_a}^2 - {OD_p}^2\right)$$

Equation 5. Flow Area

Where ID and OD represent inner and outer diameter, respectively.

$$\dot{m}_{pipe} = \frac{\dot{m}_{overall}}{N}$$

Equation 6. Mass Flow Rate per Heat Exchanger

Where N is the number of heat exchangers used in parallel.

$$v_p = \frac{\dot{m}_{pipe}}{A_p \cdot \rho} \qquad v_a = \frac{\dot{m}_{pipe}}{A_a \cdot \rho}$$

Equation 7. Linear Fluid Velocity

The hydraulic diameter D_h is used in place of the annulus' diameter for hydraulic functions, such as pressure drop.

$$D_h = ID_a - OD_p$$

Equation 8. Hydraulic Diameter

The effective diameter D_e is used in place of the annulus' diameter for heat transfer functions.

$$D_e = \frac{ID_a^{\,2} - OD_p^{\,2}}{OD_p}$$

Equation 9. Effective Diameter

Two Reynolds numbers were needed for the annulus. One Reynolds number was calculated using the effective diameter and would be used for heat exchanger calculations. The other Reynolds number was calculated using the hydraulic diameter and was used for hydraulic calculations. Here we use the effective diameter D_e because we are performing heat transfer calculations. Later, the hydraulic diameter, D_h , will be used when finding the pressure drop of the system.

$$Re_p = \frac{v_p \cdot ID_p}{\nu} \qquad Re_a = \frac{v_a \cdot D_e}{\nu}$$

Equation 10. Reynolds Number (Effective)

Where ν is the kinematic velocity of the fluid at the average stream temperature.

$$Nu_p = 0.023 \cdot Re_p^{4/5} \cdot Pr^{0.4} \qquad Nu_a = 0.023 \cdot Re_a^{4/5} \cdot Pr^{0.3}$$

Equation 11. Nusselt Number

The convection coefficient h indicates the heat energy transferred between the pipe surface and the moving fluid.

$$h_p = \frac{Nu_p \cdot k_f}{ID_p} \qquad h_a = \frac{Nu_a \cdot k_f}{D_e}$$

Equation 12. Convection Coefficient

Thermal capacitance C is the heat flow necessary to change the temperature rate of the fluids by one unit in one second.

$$C_c = \dot{m}_{pipe} \cdot c_{p,c} \qquad C_h = \dot{m}_{pipe} \cdot c_{p,h}$$

Equation 13. Thermal Capacitance

Going forward, the minimum thermal capacitance, C_{min}, refers to the lesser of the two thermal capacitances, and the maximum thermal capacitance, C_{max}, refers to the larger of the two.

$$C_r = \frac{C_{min}}{C_{max}}$$

Equation 14. Thermal Capacitance Ratio

The overall heat transfer coefficient U is the heat transferred per unit area per kelvin.

$$\frac{1}{UA} = \frac{1}{h_p \cdot \pi \cdot ID_p \cdot L} + \frac{ln(\frac{OD_p}{ID_p})}{2 \cdot \pi \cdot k_p \cdot L} + \frac{1}{h_a \cdot \pi \cdot OD_p \cdot L}$$

Equation 15. Overall Heat Transfer Coefficient.

Where k_p is the thermal conductivity of the pipe, L is the length of the heat exchanger, and $\pi \cdot D \cdot L$ is the area available for heat transfer. The system has an anticipated operating time of 10 hours per day, 5 days per week, and 40 weeks a year. Given that, it is assumed that the system can be cleaned in its downtime, and therefore the pipes will not incur enough fouling to detriment the flow rate and the heat transfer coefficient.

The number of transfer units, NTU, is used to determine the effectiveness, E, of the heat exchanger.

$$NTU = \frac{UA}{C_{min}}$$

Equation 16. Number of Transfer Units

$$E = \frac{1-e^{-NTU\cdot(1-C_r)}}{1-C_r\cdot e^{-NTU\cdot(1-C_r)}}$$

Equation 17. Effectiveness

The maximum possible heat transfer, q_{max}, represents the heat transfer that would occur in an infinitely long heat exchanger.

$$q_{max} = C_{min} \cdot (T_{hi} - T_{ci})$$

Equation 18. Maximum Possible Heat Transfer

The actual heat transfer in the heat exchanger is found using the calculated effectiveness and maximum possible heat transfer.

$$q = E \cdot q_{max}$$

Equation 19. Actual Heat Transfer

The rate of heat transfer has been found as a function of the average fluid temperature for the hot and cold streams.

The output temperatures of the cold and hot streams (T_{co} and T_{ho}, respectively) are determined by solving a system of two equations:

i.) $T_{co} = T_{ci} + \frac{q}{C_c}$

ii.) $C_c \cdot (T_{co} - T_{ci}) = C_h \cdot (T_{hi} - T_{ho})$

Now that the outlet temperatures are known, the new cost to power the water heater can be found using Equations 3 and 4.

To determine the cost of the pump, a few fluid properties must first be found. The friction factor, f, depends on the value of the Reynolds number, Re. Reynolds number indicates the behavior of a fluid. If the Reynolds number of a fluid is less than 2200, then the flow is considered to be laminar, and the friction factor is calculated using Equation 21. On the other hand, if the Reynolds number is greater than 2200, then the flow is considered to be turbulent, and the friction factor is calculated using Equation 22 or 23, known as the Chen Equation [1]. Note that the hydraulic diameter is used to calculate the Reynolds number Re and friction factor f of the annulus.

$$Re_{hyd} = \frac{v_a \cdot D_h}{\nu}$$

Equation 20. Reynolds Number (Hydraulic)

$$f = \frac{64}{Re}$$

Equation 21. Friction Factor ($Re < 2200$)

$$f_p = (-\ 2 \cdot log(\frac{\epsilon}{3.7065\, ID_p} - \frac{5.0452}{Re_p} \cdot log(\frac{1}{2.8257} \cdot (\frac{\epsilon}{ID_p})^{1.1098} + \frac{5.8506}{Re_p^{\ 0.8981}})))^{-2}$$

Equation 22. Chen Equation (Inner Pipe, $Re > 2200$)

$$f_a = (-\ 2 \cdot log(\frac{\epsilon}{3.7065\, D_h} - \frac{5.0452}{Re_{hyd}} \cdot log(\frac{1}{2.8257} \cdot (\frac{\epsilon}{D_h})^{1.1098} + \frac{5.8506}{Re_{hyd}^{\ 0.8981}})))^{-2}$$

Equation 23. Chen Equation (Annulus, $Re > 2200$)

The pressure drop $\Delta P$of the inner pipe and the annulus are found using Equations 24 and 25, respectively.

$$\Delta P_i = \rho \cdot \frac{g}{g_c} \cdot (f \cdot \frac{L}{ID_p} \cdot \frac{v^2}{2 \cdot g})$$

Equation 24. Pressure Drop (Inner Pipe)

$$\Delta P_a = \left(f \cdot \frac{L}{D_h} + 1\right) \cdot \frac{\rho \cdot v^2}{2 \cdot g_c}$$

Equation 25. Pressure Drop (Annulus)

Because the heat exchangers are in parallel, the pressure drop across each exchanger is equivalent.

The total head loss of the system ΔH is the sum of the head losses in the inner pipe and annulus. The head loss of each heat exchanger is equivalent because they are in parallel.

$$\Delta H = \frac{\Delta P_i}{\rho(T_{h,avg}) \cdot g} + \frac{\Delta P_a}{\rho(T_{c,avg}) \cdot g}$$

Equation 26. Total Heat (System)

Now that the total head loss is known, the cost of the pump can be determined using Equation 27. To determine the annual cost of running the pump, the pump power required by the fluid $\dot{W}_p$ must first be found.

$$\dot{W}_p = \Delta H \cdot \dot{m}_{total} \cdot g$$

Equation 27. Power Imparted to the Fluid by the Pump

Now the input power required by the pump motor can be found:

$$\dot{W}_a = \frac{\dot{W}_p}{\eta}$$

Equation 28. Pump Motor Power

where η is the pump's efficiency. Finally, the annual cost of running the pump can be found:

$$C = \left(\frac{\$0.11}{kW\ hr}\right)\dot{W}_a \cdot t$$

Equation 29. Annual Electricity Cost of Pump

Once the initial cost of the piping, fittings, and pump (Tables 1 and 2, Equation 1) is known, it needs to be annualized to find the effective yearly cost

$$C_{eff,ann} = C_{init}\frac{(1+i)^n \cdot i}{(1+i)^n - 1}$$

Equation 30. Cost Annualization

Results & Discussion

As mentioned previously, the two primary analyses of the system consisted of finding the optimal pipe diameter configurations and the number of exchangers based on a fixed length, and on evaluating the effect of length on the amount of overall savings. For the first analysis, 12 different pipe diameter combinations and temperature positions were tested to find the maximum cost savings (Tables 3 and 4). Based on this, it was revealed that using a system with cold water in the inner pipe yielded slightly different results than directing warm water through the inner pipe. Additionally, having very similarly sized diameters for the inner and outer pipes used improved the amount of savings. At all of these recorded values, the total pressure drop was significantly lower than the 10 psi maximum.

For the second analysis, a 1x2 heat exchanger array with hot water flowing through the inner pipe was tested for cost savings using different overall lengths of pipe (Table 5). As the length was decreased, the maximum savings decreased by approximately $10,000 per five-foot reduction, because the additional length of the heat exchanger is able to add to the system's ability to transfer heat, which more than compensates for the cost of the additional piping.

Overall, the system behaves optimally as a 50-foot long heat exchanger system with a 1-inch nominal diameter for the inner pipe and a 2-inch nominal diameter for the outer pipe, with cold water flowing through the annulus and hot water flowing through the pipe. A system of 261 of these heat exchangers will provide $172,183 of savings every year. This layout will use $1223\ \frac{Btu}{hr}$ of pump work and $1.633 \cdot 10^7\ \frac{Btu}{hr}$ of heating for heating the water.

The problem statement never specifies a maximum number of heat exchangers that can be used in parallel, but 261 2-inch diameter pipes will take up a horizontal space of approximately 522 inches (43.5 feet). If this much space is not available, then it may be necessary to use a different design that will use fewer pipes, at the cost of reduced savings. Additionally, this design does not take into consideration the manifold and fixtures needed to split the flow evenly into 261 different heat exchangers, and the losses that such a split would incur.

If these factors make the solution unsatisfactory, then a better solution may be to use a 2x3 heat exchanger with cold fluid in the inner pipe. This solution still results in $146,884 of annual savings, but only uses 111 heat exchangers in parallel. This would reduce the amount of space required and the complexity of the manifold.

Drawings & Diagrams

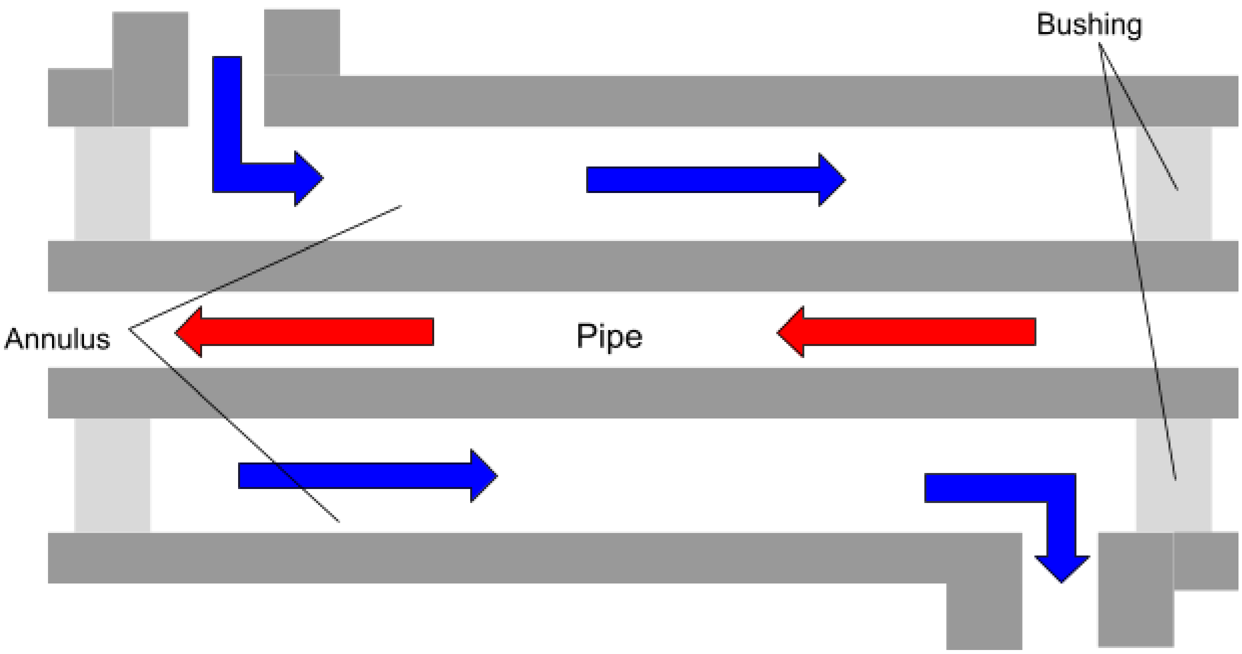

Figure 1. Diagram of a counterflow heat exchanger

Graphs & Tables

Inner-Pipe x Annulus Diameter (in.)	Savings	Number of Exchangers
1x2	170.763 k$	254
1x3	84.920 k$	100
1x4	47.743 k$	58
2x3	146.884 k$	111
2x4	93.391 k$	73
3x4	131.090 k$	72

Table 3. Total savings when the hot fluid stream is routed through the annulus at various double-pipe diameter combinations

Inner-Pipe x Annulus Diameter (in.)	Savings	Number of Exchangers
1x2	172.183 k$	261
1x3	85.148 k$	102
1x4	46.957 k$	58
2x3	146.338 k$	111
2x4	93.645 k$	74
3x4	129.738 k$	70

Table 4. Total savings when the cold fluid stream is routed through the annulus at various double-pipe diameter combinations

Length (ft)	Savings	Number of Exchangers
50	172.183 k$	261
45	164.086 k$	282
40	155.032 k$	310
35	144.818 $	340

Table 5. Total savings for a 1x2 heat exchanger of various lengths (cold fluid stream routed through annulus)

Part	Cost/unit	Number of Units	Cost
Pipe (1 in nominal diameter)	$5.42/foot	13050	$70,731.00
Pipe (2 in nominal diameter)	$11.06/foot	13050	$144,333.00
Tee (2 in nominal diameter)	$14.10/tee	522	$7,360.20
Bushing (1x2 in nominal diameters)	$4.93/bushing	522	$2,573.46
Pump	$500*Hp^0.5	0.72 ft	$424
		Total:	$225,422

Table 6. Capital Costs

Operation	Cost/interval	Duration	Heating	Annual Cost
Heater	$0.03 \frac{\$}{kW\cdot hr}$	2000 hr/year	$1.633 \cdot 10^{7} \frac{Btu}{hr}$	$292,459
Pump	$0.11 \frac{\$}{kW\cdot hr}$	2000 hr/year	$1.223 \cdot 10^{3} \frac{Btu}{hr}$	$79
			Total:	$292,538

Table 7. Operating Costs

References

[1] Janna, William S. Design of Fluid Thermal Systems. 4th ed., Cengage Learning, 2015.

Mathcad Code

GOLBAL FUNCTIONS & VARIABLES

MATHCAD SPECIFIC

$\$:= 1$

$K\$:= 10^3\ \$$

$M\$:= 10^6\ \$$

$B\$:= 10^9\ \$$

$\text{ORIGIN} := 1$

$gph := \frac{gal}{hr}$

FLUID TABLE PROPERTIES (water)

Density: $\rho(T) := \left(-3 \cdot 10^{-12} \cdot \left(\frac{T}{1\ \Delta°F}\right)^4 + 3 \cdot 10^{-9} \cdot \left(\frac{T}{1\ \Delta°F}\right)^3 - 2 \cdot 10^{-6} \cdot \left(\frac{T}{1\ \Delta°F}\right)^2 + 0.0001 \cdot \left(\frac{T}{1\ \Delta°F}\right) + 1.0005\right) 62.4\ \frac{lbm}{ft^3}$

Specific Heat: $C_p(T) := \left(6 \cdot 10^{-14} \cdot \left(\frac{T}{1\ \Delta°F}\right)^5 - 7 \cdot 10^{-11} \cdot \left(\frac{T}{1\ \Delta°F}\right)^4 + 3 \cdot 10^{-8} \cdot \left(\frac{T}{1\ \Delta°F}\right)^3 - 5 \cdot 10^{-6} \cdot \left(\frac{T}{1\ \Delta°F}\right)^2 + 0.0003 \cdot \left(\frac{T}{1\ \Delta°F}\right) + 1.0008\right) \frac{Btu}{lbm \cdot R}$

Kinematic Viscosity: $\nu(T) := \left(577.38 \cdot \left(\frac{T}{1\ \Delta°F}\right)^{-.965}\right) \left(10^{-6}\right) \frac{ft^2}{s}$

Thermal Conductivity: $k_f(T) := \left(-4 \cdot 10^{-12} \cdot \left(\frac{T}{1\ \Delta°F}\right)^4 + 6 \cdot 10^{-9} \cdot \left(\frac{T}{1\ \Delta°F}\right)^3 - 4 \cdot 10^{-6} \cdot \left(\frac{T}{1\ \Delta°F}\right)^2 + 0.001 \cdot \left(\frac{T}{1\ \Delta°F}\right) + 0.2897\right) \frac{Btu}{hr \cdot ft \cdot R}$

Prandtl Number: $Pr(T) := \left(2.062985 \cdot 10^{-14} \cdot \left(\frac{T}{1\ \Delta°F}\right)^6 - 4.239212 \cdot 10^{-11} \cdot \left(\frac{T}{1\ \Delta°F}\right)^5 + 3.492750 \cdot 10^{-8} \cdot \left(\frac{T}{1\ \Delta°F}\right)^4 - 1.476119 \cdot 10^{-5} \cdot \left(\frac{T}{1\ \Delta°F}\right)^3 + 3.401231 \cdot 10^{-3} \cdot \left(\frac{T}{1\ \Delta°F}\right)^2 - 0.4173886 \cdot \left(\frac{T}{1\ \Delta°F}\right) + 23.85026\right)$

PIPE PROPERTIES (wrought iron, sch. 40)

Roughness Factor: $\varepsilon := 0.00015\ ft$

Thermal Conductivity: $k_p := 34.1\ \frac{Btu}{hr \cdot ft \cdot R}$

GIVEN

SYSTEM REQUIREMENTS

Required flow rate: $Q_{req} := 60000\ gph$

Hot fluid inlet temp.: $T_{h\_i} := 115\ \Delta°F$

Cold fluid inlet temp.: $T_{c\_i} := 70\ \Delta°F$

Max pressure drop: $\Delta P_{max} := 10\ psi$

Max HXer length: $L_{max} := 50\ ft$

Pump efficiency: $\eta := 70\%$

Annual operation time: $t := 2000\ hr$

COST INFO.

Water heater operation: $C_{heater}(Q_{dot}) := \left(0.03\ \frac{\$}{kW \cdot hr}\right) \cdot Q_{dot} \cdot t$

Pump operation: $C_{po}(W_{dot\_a}) := \left(0.11\ \frac{\$}{kW \cdot hr}\right) \cdot W_{dot\_a} \cdot t$

Initial pump cost: $C_{pump}(h_p) := 500 \left(\frac{h_p}{ft}\right)^{0.5}\ \$$ where h_p is the required head in feet

Lifetime (years): $n := 15$

Effective interest rate: $i := 6\%$

Piping, tee, & bushing cost:

$$C_{pipe}(D_{nom}) := \left\| \begin{array}{l} \text{if } D_{nom} = 1\ in \\ \quad \| 5.42 \\ \text{else if } D_{nom} = 2\ in \\ \quad \| 11.06 \\ \text{else if } D_{nom} = 3\ in \\ \quad \| 26.33 \\ \text{else if } D_{nom} = 4\ in \\ \quad \| 32.05 \\ \text{else} \\ \quad \| 0 \end{array} \right| \cdot \frac{\$}{ft}$$

$$C_{tee}(D_{nom}) := \left\| \begin{array}{l} \text{if } D_{nom} = 1\ in \\ \quad \| 4.26 \\ \text{else if } D_{nom} = 2\ in \\ \quad \| 14.10 \\ \text{else if } D_{nom} = 3\ in \\ \quad \| 48.34 \\ \text{else if } D_{nom} = 4\ in \\ \quad \| 101.07 \\ \text{else} \\ \quad \| 0 \end{array} \right| \cdot \$$$

OLD SYSTEM

FLUID PROPERTIES

Inlet & outlet fluid temp.: $T_{h\_in} := 70\ \Delta°F \quad T_{h\_out} := 125\ \Delta°F$

Average fluid temp.: $T_{avg\_old} := \dfrac{T_{h\_in} + T_{h\_out}}{2} = 97.5\ \Delta°F$

Density: $\rho_{old} := \rho(T_{h\_out}) = 61.581\ \dfrac{lb}{ft^3}$

Specific heat: $C_{p\_old} := C_p(T_{avg\_old}) = 1.005\ \dfrac{Btu}{lbm \cdot R}$

SYSTEM ANALYSIS

Mass flow rate: $m_{dot\_tot} := Q_{req} \cdot \rho_{old} = 137.203\ \dfrac{lbm}{s}$

Hot water heater power required: $q_{dot} := m_{dot\_tot} \cdot C_{p\_old} \cdot (T_{h\_out} - T_{h\_in}) = (2.729 \cdot 10^7)\ \dfrac{Btu}{hr}$

COST ANALYSIS

Total annual cost: $C_{ann\_old} := C_{heater}(q_{dot}) = 4.799 \cdot 10^5$

$C_{ann\_old} = 480\ K\$$

$D_{nom\_p} := 1\ in \quad D_{nom\_a} := 2\ in \quad L := 50\ ft \quad N := 261$ Pipe (inner): hot fluid

NEW SYSTEM - Temperature

SYSTEM PROPERTIES

Pipe (inner): hot fluid
Annulus (outer): cold fluid

Number of Pipes: $N = 261$

Length of Heat Exchanger: $L = 50\ ft$ $L = 50\ ft$

Nominal Diameter (Pipe): $D_{nom\_p} = 1\ in$

Inner Diameter (Pipe): $ID_p := ID(D_{nom\_p})$ $ID_p = 1.049\ in$

Outer Diameter (Pipe): $OD_p := OD(D_{nom\_p})$ $OD_p = 1.315\ in$

Flow Area (Pipe): $A_i := \dfrac{\pi}{4} ID_p^{\,2}$ $A_i = 0.864\ in^2$

Nominal Diameter (Annulus): $D_{nom\_a} = 2\ in$

Inner Diameter (Annulus): $ID_a := ID(D_{nom\_a})$ $ID_a = 2.068\ in$

Outer Diameter (Annulus): $OD_a := OD(D_{nom\_a})$ $OD_a = 2.375\ in$

Flow Area (Annulus): $A_o := \dfrac{\pi}{4} \cdot (ID_a^{\,2} - OD_p^{\,2})$ $A_o = 1.999\ in^2$

Hydraulic Diameter: $D_h := ID_a - OD_p$ $D_h = 0.753\ in$

Effective Diameter: $D_e := \dfrac{ID_a^{\,2} - OD_p^{\,2}}{OD_p}$ $D_e = 1.936\ in$

FLUID BEHAVIOR

Average Cold Fluid Temp: $T_{c\_avg}(T_{c\_o}) := 0.5\ (T_{c\_i} + T_{c\_o})$

Average Hot Fluid Temp: $T_{h\_avg}(T_{h\_o}) := 0.5\ (T_{h\_i} + T_{h\_o})$

Mass Flow Rate (Total): $m_{dot\_tot} = 137.203\ \frac{lbm}{s}$

Mass Flow Rate (Per Hxer): $m_{dot} := \frac{m_{dot\_tot}}{N} = 0.526\ \frac{lbm}{s}$

Flow Velocity (Pipe): $v_p(T) := \frac{m_{dot}}{A_i \cdot \rho(T)}$

Flow Velocity (Annulus): $v_a(T) := \frac{m_{dot}}{A_o \cdot \rho(T)}$

Reynolds Number (Pipe): $Re_p(T) := \frac{v_p(T) \cdot ID_p}{\nu(T)}$

Reynolds Number (Annulus): $Re_a(T) := \frac{v_a(T) \cdot D_e}{\nu(T)}$

Nusselt Number (Pipe): $Nu_p(T) := \begin{cases} 1.86 \cdot \left(\frac{ID_p \cdot Re_p(T) \cdot Pr(T)}{L}\right)^{\frac{1}{2}} & \text{if } Re_p(T) < 2200 \\ 0.023 \cdot Re_p(T)^{\frac{4}{5}} \cdot Pr(T)^{0.3} & \text{else} \end{cases}$

Nusselt Number (Annulus): $Nu_a(T) := \begin{cases} 1.86 \cdot \left(\frac{D_e \cdot Re_a(T) \cdot Pr(T)}{L}\right)^{\frac{1}{2}} & \text{if } Re_a(T) < 2200 \\ 0.023 \cdot Re_a(T)^{\frac{4}{5}} \cdot Pr(T)^{0.4} & \text{else} \end{cases}$

Convection Coeff. (Pipe): $h_i(T) := \frac{Nu_p(T) \cdot k_f(T)}{ID_p}$

Convection Coeff. (Annulus): $h_o(T) := \frac{Nu_a(T) \cdot k_f(T)}{D_e}$

Thermal Capacitance (Pipe): $C_h(T) := v_p(T) \cdot A_i \cdot \rho(T) \cdot C_p(T)$

Thermal Capacitance (Ann.): $C_c(T) := v_a(T) \cdot A_o \cdot \rho(T) \cdot C_p(T)$

Thermal Capacitance (Pipe): $C_h(T) := v_p(T) \cdot A_i \cdot \rho(T) \cdot C_p(T)$

Thermal Capacitance (Ann.): $C_c(T) := v_a(T) \cdot A_o \cdot \rho(T) \cdot C_p(T)$

Thermal Capacitance (Min.):
$$C_{min}(T_c, T_h) := \begin{array}{l} \text{if } C_c(T_c) > C_h(T_h) \\ \quad C_h(T_h) \\ \text{else if } C_c(T_c) < C_h(T_h) \\ \quad C_c(T_c) \\ \text{else} \\ \quad 0 \end{array}$$

Thermal Capacitance (Max.):
$$C_{max}(T_c, T_h) := \begin{array}{l} \text{if } C_c(T_c) > C_h(T_h) \\ \quad C_c(T_c) \\ \text{else if } C_c(T_c) < C_h(T_h) \\ \quad C_h(T_h) \\ \text{else} \\ \quad 0 \end{array}$$

Thermal Capacitance Ratio:
$$C_r(T_c, T_h) := \frac{C_{min}(T_c, T_h)}{C_{max}(T_c, T_h)}$$

$$UA(T_c, T_h) := \left(\frac{1}{h_i(T_h) \cdot \pi \cdot ID_p \cdot L} + \frac{\ln\left(\frac{OD_p}{ID_p}\right)}{2\,\pi \cdot k_p \cdot L} + \frac{1}{h_o(T_c) \cdot \pi \cdot OD_p \cdot L} \right)^{-1}$$

Number of Transfer Units:
$$NTU(T_c, T_h) := \frac{UA(T_c, T_h)}{C_{min}(T_c, T_h)}$$

Effectiveness:
$$E(T_c, T_h) := \frac{1 - e^{-NTU(T_c, T_h) \cdot (1 - C_r(T_c, T_h))}}{1 - C_r(T_c, T_h) \cdot e^{-NTU(T_c, T_h) \cdot (1 - C_r(T_c, T_h))}}$$

Heat Transfer (Max.): $q_{max}(T_c, T_h) := C_{min}(T_c, T_h) \cdot (T_{h\_i} - T_{c\_i})$

Heat Transfer (Actual): $q(T_c, T_h) := E(T_c, T_h) \cdot q_{max}(T_c, T_h)$

Outlet Temp. (Cold/Pipe):

Guess Values

$T_{c\_o} := 90\ \Delta°F \qquad T_{h\_o} := 100\ \Delta°F$

Constraints

$$T_{c\_o} = T_{c\_i} + \frac{q\left(T_{c\_avg}\left(T_{c\_o}\right), T_{h\_avg}\left(T_{h\_o}\right)\right)}{C_c\left(T_{c\_avg}\left(T_{c\_o}\right)\right)}$$

$$C_c\left(T_{c\_avg}\left(T_{c\_o}\right)\right) \cdot \left(T_{c\_o} - T_{c\_i}\right) = C_h\left(T_{h\_avg}\left(T_{h\_o}\right)\right) \cdot \left(T_{h\_i} - T_{h\_o}\right)$$

Solver

$$sol := \text{find}\left(T_{c\_o}, \mathrm{T_{h\_o}}\right) = \begin{bmatrix} 92.383 \\ 92.59 \end{bmatrix} \Delta°F$$

$T_{c\_o} := sol_1 = 92.383\ \Delta°F \qquad T_{h\_o} := sol_2 = 92.59\ \Delta°F$

$$T_{c\_film} := \frac{\left(T_{c\_o} + T_{c\_i}\right)}{2} = 81.192\ \Delta°F \qquad T_{h\_film} := \frac{\left(T_{h\_i} + T_{h\_o}\right)}{2} = 103.795\ \Delta°F$$

NEW SYSTEM - Pump Head

Area

Hydraulic Reynolds Number: $Re_{a\_hyd} := \frac{v_a\left(T_{c\_film}\right) \cdot D_h}{\nu\left(T_{c\_film}\right)} = 4.602 \cdot 10^3 \qquad \nu\left(T_{c\_film}\right) = \left(8.294 \cdot 10^{-6}\right) \frac{ft^2}{s}$

Friction Factor:

$$f(Re, D) := \begin{cases} \dfrac{64}{Re} & \text{if } Re < 2100 \\ \left(-2 \log\left(\dfrac{\varepsilon}{3.7065\ D} - \dfrac{5.0452}{Re} \cdot \log\left(\dfrac{1}{2.8257} \cdot \left(\dfrac{\varepsilon}{D}\right)^{1.1098} + \dfrac{5.8506}{Re^{0.8981}}\right)\right)\right)^{-2} & \text{else} \end{cases}$$

Pressure Change (Pipe): $\Delta P_i := \rho\left(T_{h\_film}\right) \cdot \left(f\left(Re_p\left(T_{h\_film}\right), ID_p\right) \cdot \frac{L}{ID_p} \cdot \frac{\left(v_p\left(T_{h\_film}\right)\right)^2}{2}\right) = 0.227\ psi$

Pressure Change (Annulus): $\Delta P_o := \left(f\left(Re_{a\_hyd}, D_h\right) \cdot \frac{L}{D_h} + 1\right) \cdot \frac{\rho\left(T_{c\_film}\right) \cdot v_a\left(T_{c\_film}\right)^2}{2} = 0.083\ psi$

Total Pressure Drop: $\Delta P_{total} := \Delta P_i + \Delta P_o = 0.31\ psi \qquad \Delta P_{max} = 10\ psi$

$$H_{tot} := \frac{\Delta P_i}{\rho\left(T_{h\_film}\right) \cdot g} + \frac{\Delta P_o}{\rho\left(T_{c\_film}\right) \cdot g} = 0.72\ ft$$

NEW SYSTEM - Pricing

Area

Hot water heater power required: $q_{dot} := m_{dot\_tot} \cdot C_p \left(\frac{(125\ \Delta°F + T_{c\_o})}{2} \right) \cdot (125\ \Delta°F - T_{c\_o}) = (1.618 \cdot 10^7)\ \frac{Btu}{hr}$

Annual Heating cost: $C_{ann\_heater} := C_{heater}(q_{dot}) = 284.426\ K\$$

Pump work imparted to fluid: $W_{dot\_p} := H_{tot} \cdot m_{dot\_tot} \cdot g = 457.134\ \frac{Btu}{hr}$

$C_p \left(\frac{(125\ \Delta°F + T_{c\_o})}{2} \right) = 1.004\ \frac{Btu}{lbm \cdot R}$

Pump input power: $W_{dot\_a} := \frac{W_{dot\_p}}{\eta} = 653.049\ \frac{Btu}{hr}$

Annual pump operation cost: $C_{ann\_po} := C_{po}(W_{dot\_a}) = 0.042\ K\$$

Inital Pump cost: $C_{pump} := C_{pump}(H_{tot}) = 0.424\ K\$$

Cost of piping: $C_{pipe} := (C_{pipe}(D_{nom\_a}) + C_{pipe}(D_{nom\_p})) \cdot N \cdot L = 215.064\ K\$$

Cost of tees: $C_{tee} := C_{tee}(D_{nom\_a}) \cdot 2 \cdot N = 7.36\ K\$$

Cost of bushings: $C_{bush} := C_{bush}(D_{nom\_p}, D_{nom\_a}) \cdot 2 \cdot N = 2.573\ K\$$

Total initial cost: $C_{initial} := C_{pump} + C_{pipe} + C_{tee} + C_{bush} = 225.422\ K\$$

Initial costs converted to effective annual cost: $C_{eff\_ann} := \left(\frac{(1+i)^n\ i}{(1+i)^n - 1} \right) C_{initial} = 23.21\ K\$$

Total effective annual costs: $C_{ann} := C_{ann\_heater} + C_{ann\_po} + C_{eff\_ann}$

$C_{ann} = 307.678\ K\$$

Old system's annual costs: $C_{ann\_old} = 479.861\ K\$$

Annual Savings: $C_{save} := C_{ann\_old} - C_{ann}$

$C_{save} = 172.183\ K\$$

Sample Calculations

$$\dot{m}_{overall} = Q_{req} \cdot \rho(125\,^{\circ}F) = 60000\,\frac{gal}{hr} \cdot \frac{0.1336\,ft^3}{gal} \cdot \frac{1\,hr}{3600\,s} \cdot 61.581\,\frac{lbm}{ft^3} = 137.203\,\frac{lbm}{s}$$

$$A_p = \frac{\pi}{4} \cdot ID_p^{\,2} = \frac{\pi}{4} \cdot (1.049\,in)^2 = 0.864\,in^2$$

$$A_a = \frac{\pi}{4} \cdot \left(ID_a^{\,2} - OD_p^{\,2}\right) = \frac{\pi}{4} \cdot \left((2.068\,in)^2 - (1.315\,in)^2\right) = 1.999\,in^2$$

These sample calculations would normally require iteration to determine the cold and hot outlet temperatures. For the purpose of brevity, the correct outlet temperatures are used.

$$T_{h,avg} = \frac{1}{2} \cdot (T_{h,i} + T_{h,o}) = \frac{1}{2} \cdot (115\,^{\circ}F + 92.59\,^{\circ}F) = 103.759\,^{\circ}F$$

$$\dot{m}_{pipe} = \frac{\dot{m}_{overall}}{N} = \frac{137.203\frac{lbm}{s}}{261} = 0.526\frac{lbm}{s}$$

$$v_p = \frac{\dot{m}_{pipe}}{A_i \cdot \rho(103.795\,^{\circ}F)} = \frac{0.526\,\frac{lbm}{s}}{0.864\,in^2 \cdot (61.922\frac{lbm}{ft^3}) \cdot \frac{1\,ft^2}{144\,in^2}} = 1.414\,\frac{ft}{s}$$

$$D_h = ID_a - OD_p = 2.068\,in - 1.315\,in = 0.753\,in$$

$$D_e = \frac{ID_a^{\ 2} - OD_p^{\ 2}}{OD_p} = \frac{(2.068\ in)^2 - (1.315\ in)^2}{1.315\ in} = 1.936\ in$$

$$Re_p = \frac{v_p \cdot ID_p}{\nu(103.795\ ^\circ F)} = \frac{1.414\ \frac{ft}{s} \cdot 1.049\ in \cdot \frac{1\ ft}{12\ in}}{6.544 \cdot 10^{-6}\ \frac{ft^2}{s}} = 1.889\ \cdot\ 10^4$$

$$Nu_p = 0.023\ \cdot\ Re_p^{\ \frac{4}{5}}\ \cdot\ Pr(103.795\ ^\circ F)^{0.3} = 0.023\ \cdot\ (1.889\ \cdot\ 10^4)^{\frac{4}{5}}\ \cdot (4.233)$$

$$Nu_p = 93.493$$

$$h_p = \frac{Nu_p \cdot k_f(103.795\ ^\circ F)}{ID_p} = \frac{93.493 \cdot 0.357\ \frac{Btu}{hr \cdot ft \cdot R}}{1.049\ in \cdot \frac{1\ ft}{12\ in}} = 381.424 \frac{Btu}{hr \cdot ft^2 \cdot R}$$

$$C_h = \dot{m}_{pipe}\ \cdot\ C_p(103.795\ ^\circ F) = 0.738\ \frac{lbm}{s}\ \cdot\ 1.004\ \frac{Btu}{lbm \cdot R}\ \cdot\ \frac{3600\ s}{1\ hr} = 1900\ \frac{Btu}{hr \cdot R}$$

$$C_r = \frac{C_{min}}{C_{max}} = \frac{1900}{1903} = 0.999$$

$$UA = \left(\frac{1}{h_p \cdot \pi \cdot ID_p \cdot L} + \frac{ln(\frac{OD_p}{ID_p})}{2 \cdot \pi \cdot k_p \cdot L} + \frac{1}{h_a \cdot \pi \cdot OD_p \cdot L}\right)^{-1}$$

$$UA = \left(\frac{1}{381.424\ \frac{Btu}{hr \cdot ft^2 \cdot R} \cdot \pi \cdot 1.049\ in \cdot \frac{1\ ft}{12\ in} \cdot 50\ ft} + \frac{ln(\frac{1.315\ in}{1.049\ in})}{2 \cdot \pi \cdot 34.1\ \frac{Btu}{hr \cdot ft \cdot R} \cdot 50 ft} + \frac{1}{182.273\ \frac{Btu}{hr \cdot ft^2 \cdot R} \cdot \pi \cdot 1.315\ in \cdot}\right.$$

$$UA = 1893 \frac{Btu}{hr \cdot R}$$

$$NTU = \frac{UA}{C_{min}} = \frac{1893 \frac{Btu}{hr \cdot R}}{1900\ \frac{Btu}{hr \cdot R}} = 0.996$$

$$E = \frac{1 - e^{-NTU \cdot (1 - C_r)}}{1 - C_r \cdot e^{-NTU \cdot (1 - C_r)}} = \frac{1 - e^{-0.996 \cdot (1 - 0.999)}}{1 - 0.999 \cdot e^{-0.996 \cdot (1 - 0.999)}} = 0.499$$

$$q_{max} = C_{min}\ \cdot\ (T_{hi} - T_{ci}) = 1900\ \frac{Btu}{hr \cdot R}\ \cdot\ (115\ ^\circ F\ -\ 70\ ^\circ F) = 8.522\ \cdot\ 10^4\ \frac{Btu}{hr}$$

$$q = E \cdot q_{max} = 0.499 \cdot 8.522 \cdot 10^4 \frac{Btu}{hr} = 4.269 \cdot 10^4 \frac{Btu}{hr}$$

$$T_{co} = T_{ci} + \frac{q}{C_c} = 70\ ^{\circ}F + \frac{4.269 \cdot 10^4 \frac{Btu}{hr}}{1903 \frac{Btu}{hr \cdot R}} = 92.383\ ^{\circ}F$$

$$C_c \cdot (T_{co} - T_{ci}) = C_h \cdot (T_{hi} - T_{ho})$$

$$\rightarrow 1903 \frac{Btu}{hr \cdot R} \cdot (92.59\ ^{\circ}F - 70\ ^{\circ}F) = 1900 \frac{Btu}{hr \cdot R}(115\ ^{\circ}F - 92.983\ ^{\circ}F)$$

$$Re_{hyd} = \frac{v_a \cdot D_h}{\nu(103.795\ ^{\circ}F)} = \frac{0.609 \frac{ft}{s} \cdot 0.753\ in \cdot \frac{1\ ft}{12\ in}}{8.294 \cdot 10^{-6} \frac{ft^2}{s}} = 4602$$

$$f_p = (-2 \cdot log(\frac{\epsilon}{3.7065\ ID_p} - \frac{5.0452}{Re_p} \cdot log(\frac{1}{2.8257} \cdot (\frac{\epsilon}{ID_p})^{1.1098} + \frac{5.8506}{Re_p^{\ 0.8981}})))^{-2}$$

$$f_p = (-2 \cdot log(\frac{0.00015\ ft}{3.7065 \cdot \frac{1.049}{12} ft} - \frac{5.0452}{1.889 \cdot 10^4} \cdot log(\frac{1}{2.8257} \cdot (\frac{0.00015\ ft}{\frac{1.049}{12} ft})^{1.1098} + \frac{}{(1.88}$$

$$f_p = 0.03$$

$$\Delta P_i = \rho(103.795\ ^{\circ}F) \cdot \frac{g}{g_c} \cdot (f_p \cdot \frac{L}{ID_p} \cdot \frac{v^2}{2 \cdot g})$$

$$\Delta P_i = 61.922 \frac{lbm}{ft^3} \cdot \frac{32.2 \frac{ft}{s^2}}{32.2 \frac{lbm \cdot ft}{lbf \cdot s^2}} \cdot (0.03 \cdot \frac{50\ ft}{\frac{1.049}{12} ft} \cdot \frac{(1.414 \frac{ft}{s})^2}{2 \cdot 32.2 \frac{ft}{s^2}}) \cdot \frac{1\ ft^2}{144\ in^2} = 0.227\ ps$$

$$\Delta P_a = (f \cdot \frac{L}{D_h} + 1) \cdot \frac{\rho(81.192\ ^{\circ}F) \cdot v_a^{\ 2}}{2 \cdot g_c} = (0.038 \cdot \frac{50\ ft}{\frac{0.753}{12} ft} + 1) \cdot \frac{62.207 \frac{lbm}{ft^3} \cdot (0.609 \frac{ft}{s}}{2 \cdot 32.2 \frac{ft}{s^2}}$$

$$\Delta P_a = 0.083\ psi$$

$$\Delta P_{total} = \Delta P_i + \Delta P_a = 0.227\ psi + 0.083\ psi = 0.31\ psi$$

$$H_{tot} = \frac{\Delta P_i}{\rho(103.795\,^{\circ}F)\cdot g} + \frac{\Delta P_a}{\rho(81.192\,^{\circ}F)\cdot g}$$

$$H_{tot} = \left(\frac{0.227\,psi}{61.922\,\frac{lbm}{ft^3}\cdot 32.2\,\frac{ft}{s^2}} + \frac{0.083\,psi}{62.207\,\frac{lbm}{ft^3}\cdot 32.2\,\frac{ft}{s^2}}\right) \cdot \frac{144\,in^2}{1\,ft^2} \cdot 32.2\,\frac{lbm\cdot ft}{lbf\cdot s^2} = 0.72\,ft$$

$$\dot{q}_{heater} = \dot{m}_{overall} \cdot C_p(10.2385\,^{\circ}F) \cdot (125\,^{\circ}F - T_{c,o})$$

$$\dot{q}_{heater} = 137.203\,\frac{lbm}{s} \cdot 1.004\,\frac{Btu}{lbm\cdot R} \cdot (125\,^{\circ}F - 92.383\,^{\circ}F) \cdot \frac{3600\,s}{hr}$$

$$\dot{q}_{heater} = 1.618 \cdot 10^7\,\frac{Btu}{hr}$$

$$C_{ann,heater} = 0.03\,\frac{\$}{kW\cdot hr} \cdot t \cdot \dot{q}_{heater}$$

$$C_{ann,heater} = 0.03\,\frac{\$}{kW\cdot hr} \cdot 2000\,hr \cdot 1.618 \cdot 10^7\,\frac{Btu}{hr} \cdot \frac{1\,kW}{3412.14\,\frac{Btu}{hr}} = 284.426$$

$$\dot{W}_p = H_{tot} \cdot \dot{m}_{overall} \cdot g$$

$$\dot{W}_p = 1.336\,ft \cdot 137.203\,\frac{lbm}{s} \cdot \frac{32.2\,\frac{ft}{s^2}}{32.2\,\frac{lbm\cdot ft}{lbf\cdot s^2}} \cdot \frac{1\,\frac{Btu}{hr}}{0.216\,\frac{ft\cdot lbf}{s}} = 457.134\,\frac{Btu}{hr}$$

$$\dot{W}_a = \frac{\dot{W}_p}{\eta} = \frac{457.134\,\frac{Btu}{hr}}{0.7} = 653.049\,\frac{Btu}{hr}$$

$$C_{ann,pump} = 0.11\,\frac{\$}{kW\cdot hr} \cdot t \cdot \dot{W}_a$$

$$C_{ann,pump} = 0.11\,\frac{\$}{kW\cdot hr} \cdot 2000\,hr \cdot 653.049\,\frac{Btu}{hr} \cdot \frac{1\,kW}{3412.14\,\frac{Btu}{hr}} = \$78$$

$$C_{init,pump} = 500\,\frac{\$}{ft^{0.5}} \cdot (H_{tot})^{0.5} = 500\,\frac{\$}{ft^{0.5}} \cdot (0.72\,ft)^{0.5} = 424\,\$$$

$$C_{pipe} = (C_{pipe,1in} + C_{pipe,2in}) \cdot N \cdot L$$

$$C_{pipe} = (5.42\,\frac{\$}{ft} + 11.06\,\frac{\$}{ft}) \cdot 261 \cdot 50\,ft = 215.064\,k\$$$

$$C_{tee} = C_{tee,2in} \cdot 2\frac{tee}{pipe} \cdot N = 14.10\,\frac{\$}{tee} \cdot 2\,\frac{tee}{pipe} \cdot 261\,pipes = 7.36\,k\$$$

$$C_{bush} = C_{bush,1x2} \cdot 2\,\frac{bush}{pipe} \cdot N = 4.93\,\frac{\$}{bush} \cdot 2\,\frac{bush}{pipe} \cdot 261\,pipes = 2.573\,k\$$$

$$C_{init} = C_{pump} + C_{pipe} + C_{tee} + C_{bush}$$

$$C_{init} = (0.424 + 215.064 + 7.36 + 2.573)\,k\$ = 225.422\,\mathrm{k\$}$$

$$C_{eff,ann} = C_{init}\frac{(1+i)^{n}\cdot i}{(1+i)^{n}-1} = 225.422\,k\$ \cdot \frac{(1.06)^{15}\cdot 0.06}{(1.06)^{15}-1} = 23.21\,k\$$$

$$C_{ann,total} = C_{ann,heater} + C_{ann,pump} + C_{eff,\,ann}$$

$$C_{ann,\,total} = (284.426 + 0.078 + 23.21)\,k\$ = 307.35\,k\$$$

$$C_{save} = C_{ann,\,old} - C_{ann,\,total} = (479.961 - 307.35)\,k\$ = 172.183\,k\$$$