See this page in the course material.

Learning Outcomes

- Evaluate a sum given in summation notation.
- Find the partial sum of an arithmetic series.
- Solve an application problem using an arithmetic series.

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Using Summation Notation

To find the total amount of money in the college fund and the sum of the amounts deposited, we need to add the amounts deposited each month and the amounts earned monthly. The sum of the terms of a sequence is called a **series**. Consider, for example, the following series.

[latex]3+7+11+15+19+\cdots[/latex]

The **[latex]n\text{th }[/latex] partial sum** of a series is the sum of a finite number of consecutive terms beginning with the first term. The notation

[latex]\begin{align}&{S}_{n}\text{ represents the partial sum.} \\ &{S}_{1}=3 \\ &{S}_{2}=3+7=10 \\ &{S}_{3}=3+7+11=21 \\ &{S}_{4}=3+7+11+15=36\end{align}[/latex]

Summation notation is used to represent series. Summation notation is often known as sigma notation because it uses the Greek capital letter **sigma**, [latex]\Sigma[/latex], to represent the sum. Summation notation includes an explicit formula and specifies the first and last terms in the series. An explicit formula for each term of the series is given to the right of the sigma. A variable called the **index of summation** is written below the sigma. The index of summation is set equal to the **lower limit of summation**, which is the number used to generate the first term in the series. The number above the sigma, called the **upper limit of summation**, is the number used to generate the last term in a series.

If we interpret the given notation, we see that it asks us to find the sum of the terms in the series [latex]{a}_{k}=2k[/latex] for [latex]k=1[/latex] through [latex]k=5[/latex]. We can begin by substituting the terms for [latex]k[/latex] and listing out the terms of this series.

[latex]\begin{align} &{a}_{1}=2\left(1\right)=2 \\ &{a}_{2}=2\left(2\right)=4 \\ &{a}_{3}=2\left(3\right)=6 \\ &{a}_{4}=2\left(4\right)=8 \\ &{a}_{5}=2\left(5\right)=10 \end{align}[/latex]

We can find the sum of the series by adding the terms:

[latex]\sum\limits _{k=1}^{5}2k=2+4+6+8+10=30[/latex]

A General Note: Summation Notation

The sum of the first [latex]n[/latex] terms of a **series** can be expressed in **summation notation** as follows:

[latex]\sum\limits _{k=1}^{n}{a}_{k}[/latex]

This notation tells us to find the sum of [latex]{a}_{k}[/latex] from

[latex]k=1[/latex] to [latex]k=n[/latex].

[latex]k[/latex] is called the **index of summation**, 1 is the **lower limit of summation**, and [latex]n[/latex] is the **upper limit of summation**.

Q & A

Does the lower limit of summation have to be 1?

No. The lower limit of summation can be any number, but 1 is frequently used. We will look at examples with lower limits of summation other than 1.

How To: Given summation notation for a series, evaluate the value.

1. Identify the lower limit of summation.
2. Identify the upper limit of summation.
3. Substitute each value of [latex]k[/latex] from the lower limit to the upper limit into the formula.
4. Add to find the sum.

Example: Using Summation Notation

Evaluate [latex]\sum\limits _{k=3}^{7}{k}^{2}[/latex].

Show Solution

According to the notation, the lower limit of summation is 3 and the upper limit is 7. So we need to find the sum of [latex]{k}^{2}[/latex] from [latex]k=3[/latex] to [latex]k=7[/latex]. We find the terms of the series by substituting [latex]k=3\text{,}4\text{,}5\text{,}6[/latex], and [latex]7[/latex] into the function [latex]{k}^{2}[/latex]. We add the terms to find the sum.

[latex]\begin{align}\sum _{k=3}^{7}{k}^{2} & ={3}^{2}+{4}^{2}+{5}^{2}+{6}^{2}+{7}^{2} \\ & =9+16+25+36+49 \\ \\ & =135 \end{align}[/latex]

Try It

Evaluate [latex]\sum\limits _{k=2}^{5}\left(3k - 1\right)[/latex].

Show Solution

[See this interactive in the course material.]

Arithmetic Series

Just as we studied special types of sequences, we will look at special types of series. Recall that an **arithmetic sequence** is a sequence in which the difference between any two consecutive terms is the **common difference**, [latex]d[/latex]. The sum of the terms of an arithmetic sequence is called an **arithmetic series**. We can write the sum of the first [latex]n[/latex] terms of an arithmetic series as:

[latex]{S}_{n}={a}_{1}+\left({a}_{1}+d\right)+\left({a}_{1}+2d\right)+…+\left({a}_{n}-d\right)+{a}_{n}[/latex].

We can also reverse the order of the terms and write the sum as

[latex]{S}_{n}={a}_{n}+\left({a}_{n}-d\right)+\left({a}_{n}-2d\right)+…+\left({a}_{1}+d\right)+{a}_{1}[/latex].

If we add these two expressions for the sum of the first [latex]n[/latex] terms of an arithmetic series, we can derive a formula for the sum of the first [latex]n[/latex] terms of any arithmetic series.

[latex]\begin{align}{S}_{n}&={a}_{1}+\left({a}_{1}+d\right)+\left({a}_{1}+2d\right)+…+\left({a}_{n}-d\right)+{a}_{n} \\
+{S}_{n}&={a}_{n}+\left({a}_{n}-d\right)+\left({a}_{n}-2d\right)+…+\left({a}_{1}+d\right)+{a}_{1} \\
\hline
2{S}_{n}&=\left({a}_{1}+{a}_{n}\right)+\left({a}_{1}+{a}_{n}\right)+…+\left({a}_{1}+{a}_{n}\right)
\end{align}[/latex]

Because there are [latex]n[/latex] terms in the series, we can simplify this sum to

[latex]2{S}_{n}=n\left({a}_{1}+{a}_{n}\right)[/latex].

We divide by 2 to find the formula for the sum of the first [latex]n[/latex] terms of an arithmetic series.

[latex]{S}_{n}=\dfrac{n\left({a}_{1}+{a}_{n}\right)}{2}[/latex]

This is generally referred to as the **Partial Sum** of the series.

A General Note: Formula for the Partial Sum of an Arithmetic Series

An **arithmetic series** is the sum of the terms of an arithmetic sequence. The formula for the partial sum of an arithmetic sequence is

[latex]{S}_{n}=\dfrac{n\left({a}_{1}+{a}_{n}\right)}{2}[/latex]

How To: Given terms of an arithmetic series, find the partial sum

1. Identify [latex]{a}_{1}[/latex] and [latex]{a}_{n}[/latex].
2. Determine [latex]n[/latex].
3. Substitute values for [latex]{a}_{1},{a}_{n}[/latex], and [latex]n[/latex] into the formula [latex]{S}_{n}=\dfrac{n\left({a}_{1}+{a}_{n}\right)}{2}[/latex].
4. Simplify to find [latex]{S}_{n}[/latex].

Example: Finding the partial sum of an Arithmetic Series

Find the partial sum of each arithmetic series.

1. [latex]5 + 8 + 11 + 14 + 17 + 20 + 23 + 26 + 29 + 32[/latex]
2. [latex]20 + 15 + 10 + \dots + -50[/latex]
3. [latex]\sum\limits _{k=1}^{12}3k - 8[/latex]

Show Solution

1. We are given [latex]{a}_{1}=5[/latex] and [latex]{a}_{n}=32[/latex].Count the number of terms in the sequence to find [latex]n=10[/latex].Substitute values for [latex]{a}_{1},{a}_{n},[/latex] and [latex]n[/latex] into the formula and simplify.

   [latex]\begin{align} \\ &{S}_{n}=\dfrac{n\left({a}_{1}+{a}_{n}\right)}{2} \\ &{S}_{10}=\dfrac{10\left(5+32\right)}{2}=185 \\ \text{ }\end{align}[/latex]
2. We are given [latex]{a}_{1}=20[/latex] and [latex]{a}_{n}=-50[/latex].Use the formula for the general term of an arithmetic sequence to find [latex]n[/latex].

   [latex]\begin{align}\\ {a}_{n}&={a}_{1}+\left(n - 1\right)d \\ -50&=20+\left(n - 1\right)\left(-5\right) \\ -70&=\left(n - 1\right)\left(-5\right) \\ 14&=n - 1 \\ 15&=n \\ \text{ }\end{align}[/latex]

   Substitute values for [latex]{a}_{1},{a}_{n}\text{,}n[/latex] into the formula and simplify.

   [latex]\begin{align}\\ &{S}_{n}=\dfrac{n\left({a}_{1}+{a}_{n}\right)}{2} \\ &{S}_{15}=\dfrac{15\left(20 - 50\right)}{2}=-225 \\ \text{ }\end{align}[/latex]
3. To find [latex]{a}_{1}[/latex], substitute [latex]k=1[/latex] into the given explicit formula.

   [latex]\begin{align}\\ {a}_{k}&=3k - 8 \\ {a}_{1}&=3\left(1\right)-8=-5 \\ \text{ }\end{align}[/latex]

   We are given that [latex]n=12[/latex]. To find [latex]{a}_{12}[/latex], substitute [latex]k=12[/latex] into the given explicit formula.

   [latex]\begin{align} \\{a}_{k}&=3k - 8 \\ {a}_{12}&=3\left(12\right)-8=28 \\ \text{ }\end{align}[/latex]

   Substitute values for [latex]{a}_{1},{a}_{n}[/latex], and [latex]n[/latex] into the formula and simplify.

   [latex]\begin{align}\\{S}_{n}&=\dfrac{n\left({a}_{1}+{a}_{n}\right)}{2} \\ {S}_{12}&=\dfrac{12\left(-5+28\right)}{2}=138 \end{align}[/latex]

Try It

Use the formula to find the partial sum of each arithmetic series.

[latex]1.4+1.6+1.8+2.0+2.2+2.4+2.6+2.8+3.0+3.2+3.4[/latex]

Show Solution

[latex]26.4[/latex]

[latex]12+21+29\dots + 69[/latex]

Show Solution

[latex]328[/latex]

[latex]\sum\limits _{k=1}^{10}5 - 6k[/latex]

Show Solution

[latex]-280[/latex]

See this interactive in the course material.

See this interactive in the course material.

Example: Solving Application Problems with Arithmetic Series

On the Sunday after a minor surgery, a woman is able to walk a half-mile. Each Sunday, she walks an additional quarter-mile. After 8 weeks, what will be the total number of miles she has walked?

Show Solution

This problem can be modeled by an arithmetic series with [latex]{a}_{1}=\frac{1}{2}[/latex] and [latex]d=\frac{1}{4}[/latex]. We are looking for the total number of miles walked after 8 weeks, so we know that [latex]n=8[/latex], and we are looking for [latex]{S}_{8}[/latex]. To find [latex]{a}_{8}[/latex], we can use the explicit formula for an arithmetic sequence.

[latex]\begin{align} {a}_{n}&={a}_{1}+d\left(n - 1\right) \\ {a}_{8}&=\dfrac{1}{2}+\dfrac{1}{4}\left(8 - 1\right)=\dfrac{9}{4} \end{align}[/latex]

We can now use the formula for arithmetic series.

[latex]\begin{align} {S}_{n}&=\dfrac{n\left({a}_{1}+{a}_{n}\right)}{2} \\ {S}_{8}&=\dfrac{8\left(\frac{1}{2}+\frac{9}{4}\right)}{2}=11 \end{align}[/latex]

She will have walked a total of 11 miles.

Try It

A man earns $100 in the first week of June. Each week, he earns $12.50 more than the previous week. After 12 weeks, how much has he earned?

Show Solution

$2,025

See this interactive in the course material.

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