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B.SC. Hons. MATH (Semester – 4th)

NUMERICAL METHODS

Subject Code: BMATS1404

Paper ID: [19131219]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. Define diagonally dominant system. Is the following system is strictly diagonally dominant system?

$$6x_1 + 3x_2 + x_3 = 10\, x_1 + 6x_2 + 4x_3 = 11\, 3x_1 - 2x_2 + 8x_3 = 9$$

b. Write the formula for the Secant method and its order of convergence.

c. If Δ is a forward difference operator and ∇ is a backward difference operator, then prove that $\Delta \equiv \nabla(1 - \nabla)^{-1}$.

d. Perform three iterations of the Picard method to solve the following the initial value problem

$$\frac{dy}{dx} = x + y,\ y(0) = 1.$$

e. Solve the initial value problem $\frac{dy}{dx} = x - y^2,\ y(1) = 2$ to obtain the value of $y(1.2)$. Use Euler method with step size h = 0.1.

f. Construct a backward difference table for the following data.

x	1	2	3	4	5
y	1	4	9	16	25

g. State intermediate value theorem and use it to prove that the equation $x^3 - 4x - 9 = 0$ has a root in the interval (2, 3).

h. Use Lagrange interpolating polynomial to compute the value of $f(1.3)$ for the following data points.
(0, -1) , (1, 1) , (2,9) , (3, 29).

i. Write the algorithm to evaluate definite integral using Weddle's rule.

j. Use Trapezoidal rule to compute the integral for the following data set.

x:	1	1.1	1.2	1.3	1.4
f(x):	0.367879	0.298197	0.236928	0.18452	0.140858

Section – B **(5 marks each)**

Q2. Find the smallest root of the equation e-x = 2 cos x using Newton-Raphson method.

Q3. Use Newton divided difference formula to derive interpolating polynomial for the following data points

(0,-1), (1, 1), (2, 9), (3, 29), (5, 129),

And hence compute the value of the point y(4).

Q4. Compute the value of integral $\int_1^2 e^{-x^2} dx$ using Simpson 1/3 and Simpson 3/8 techniques.

Q5. Use Taylor series method to solve the initial value problem $\frac{dy}{dx} = x + y,\ y(0) = 1$. Also, compute the value y(0.1).

Q6. Compute the second iteration of the Jacobi method for the following system of linear equations

$7x_1 - 3x_2 + 2x_3 + x_4 = 12\ 2x_1 - 6x_2 + x_3 + 2x_4 = 6 \quad x_1 + x_2 + 5x_3 + 2x_4 = 1$

With initial approximation, $x_1^{(0)} = 0, x_2^{(0)} = 0, x_3^{(0)} = 0, x_4^{(0)} = 0.$

Section – C **(10 marks each)**

Q7. Solve the initial value problem $\frac{dy}{dx} = x + \sqrt{y}, y(0) = 1$, for the value y(0.5), using Runge-Kutta fourth order method with step size h = 0.1.

Q8. Solve the following system of linear equations corrects up to three decimal places using the Gauss-Seidel iterative procedure. Take zero vector as the initial solution vector.

$$3x_1 + x_2 - x_3 = 3\ 2x_1 + 4x_2 + x_3 = 7\ x_1 - x_2 + 4x_3 = 4$$

Q9. Use Newton forward difference formula to find the value of f(1.2) for the following data set

x:	1	1.25	1.5	1.75	2
f(x):	0	0.223144	0.405465	0.559616	0.693147