[See this page in the course material.]

Learning Outcomes

- Simplify expressions using the Product Property of Exponents
- Simplify expressions using the Power Property of Exponents
- Simplify expressions using the Product to a Power Property of Exponents

We'll derive the properties of exponents by looking for patterns in several examples. All the exponent properties hold true for any real numbers, but right now we will only use whole number exponents. First, we will look at a few examples that leads to the Product Property.

For example, the notation 5^{4} can be expanded and written as $5\cdot5\cdot5\cdot5$, or 625. And don't forget, the exponent only applies to the number immediately to its left, unless there are parentheses.

What happens if you multiply two numbers in exponential form with the same base? Consider the expression ${2}^{3}{2}^{4}$. Expanding each exponent, this can be rewritten as $\left(2\cdot2\cdot2\right)\left(2\cdot2\cdot2\cdot2\right)$ or $2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2$. In exponential form, you would write the product as 2^{7}. Notice that 7 is the sum of the original two exponents, 3 and 4.

What about ${x}^{2}{x}^{6}$? This can be written as $\left(x\cdot{x}\right)\left(x\cdot{x}\cdot{x}\cdot{x}\cdot{x}\cdot{x}\right)=x\cdot{x}\cdot{x}\cdot{x}\cdot{x}\cdot{x}\cdot{x}\cdot{x}$ or x^{8}. And, once again, 8 is the sum of the original two exponents. This concept can be generalized in the following way:

	${x}^{2}\cdot{x}^{3}$
What does this mean? How many factors altogether?	
So, we have	${x}\cdot{x}\cdot{x}\cdot{x}\cdot{x}={x}^{5}$
Notice that 5 is the sum of the exponents, 2 and 3.	${x}^{2}\cdot{x}^{3}$ is ${x}^{2+3}$, or ${x}^{5}$
We write:	${x}^{2}\cdot {x}^{3}$

	[latex]{x}^{2+3}[/latex] [latex]{x}^{5}[/latex]

The base stayed the same and we added the exponents. This leads to the Product Property for Exponents.

The Product Property OF Exponents

For any real number [latex]x[/latex] and any integers *a* and *b*, [latex]\left(x^{a}\right)\left(x^{b}\right) = x^{a+b}[/latex].

To multiply exponential terms with the same base, add the exponents.

Caution! When you are reading mathematical rules, it is important to pay attention to the conditions on the rule. For example, when using the product rule, you may only apply it when the terms being multiplied have the same base and the exponents are integers. Conditions on mathematical rules are often given before the rule is stated, as in this example it says “For any number *x*, and any integers *a* and *b*.”

An example with numbers helps to verify this property.

[latex]\begin{array}{ccc}\hfill {2}^{2}\cdot {2}^{3}& \stackrel{?}{=}& {2}^{2+3}\hfill \\ \hfill 4\cdot 8& \stackrel{?}{=}& {2}^{5}\hfill \\ \hfill 32& =& 32\hfill \end{array}[/latex]

example

Simplify: [latex]{x}^{5}\cdot {x}^{7}[/latex]

Solution

	[latex]{x}^{5}\cdot {x}^{7}[/latex]
Use the product property, [latex]{a}^{m}\cdot {a}^{n}={a}^{m+n}[/latex].	[latex]x^{\color{red}{5+7}}[/latex]
Simplify.	[latex]{x}^{12}[/latex]

Example

Simplify.

[latex](a^{3})(a^{7})[/latex]

Show Solution

The base of both exponents is *a*, so the product rule applies.

[latex]\left(a^{3}\right)\left(a^{7}\right)[/latex]

Add the exponents with a common base.

[latex]a^{3+7}[/latex]

Answer

[latex]\left(a^{3}\right)\left(a^{7}\right) = a^{10}[/latex]

try it

See this interactive in the course material.

example

Simplify: [latex]{b}^{4}\cdot b[/latex]

Show Solution

Solution

	[latex]{b}^{4}\cdot b[/latex]
Rewrite, [latex]b={b}^{1}[/latex].	[latex]{b}^{4}\cdot {b}^{1}[/latex]
Use the product property, [latex]{a}^{m}\cdot {a}^{n}={a}^{m+n}[/latex].	[latex]b^{\color{red}{4+1}}[/latex]
Simplify.	[latex]{b}^{5}[/latex]

try it

See this interactive in the course material.

example

Simplify: [latex]{2}^{7}\cdot {2}^{9}[/latex]

Show Solution

Solution

	[latex]{2}^{7}\cdot {2}^{9}[/latex]
Use the product property, [latex]{a}^{m}\cdot {a}^{n}={a}^{m+n}[/latex].	[latex]2^{\color{red}{7+9}}[/latex]
Simplify.	[latex]{2}^{16}[/latex]

try it

See this interactive in the course material.

example

Simplify: [latex]{y}^{17}\cdot {y}^{23}[/latex]

Show Solution

Solution

	[latex]{y}^{17}\cdot {y}^{23}[/latex]
Notice, the bases are the same, so add the exponents.	[latex]y^{\color{red}{17+23}}[/latex]
Simplify.	[latex]{y}^{40}[/latex]

try it

[See this interactive in the course material.]

We can extend the Product Property of Exponents to more than two factors.

example

Simplify: [latex]{x}^{3}\cdot {x}^{4}\cdot {x}^{2}[/latex]

Show Solution

Solution

	[latex]{x}^{3}\cdot {x}^{4}\cdot {x}^{2}[/latex]
Add the exponents, since the bases are the same.	[latex]x^{\color{red}{3+4+2}}[/latex]
Simplify.	[latex]{x}^{9}[/latex]

try it

See this interactive in the course material.

In the following video we show more examples of how to use the product rule for exponents to simplify expressions.

Video Link

Caution! Do not try to apply this rule to sums.

Think about the expression $\left(2+3\right)^{2}$

Does $\left(2+3\right)^{2}$ equal $2^{2}+3^{2}$?

No, it does not because of the order of operations!

$$\left(2+3\right)^{2}=5^{2}=25$$

and

$$2^{2}+3^{2}=4+9=13$$

Therefore, you can only use this rule when the numbers inside the parentheses are being multiplied (or divided, as we will see next).

[Video Link]

Simplify Expressions Using the Power Property of Exponents

We will now further expand our capabilities with exponents. We will learn what to do when a term with a power is raised to another power, and what to do when two numbers or variables are multiplied and both are raised to an exponent. We will also learn what to do when numbers or variables that are divided are raised to a power. We will begin by raising powers to powers. See if you can discover a general property.

Let's simplify $\left(5^{2}\right)^{4}$. In this case, the base is 5^2 and the exponent is 4, so you multiply 5^{2} four times:

[latex]\left(5^{2}\right)^{4}=5^{2}\cdot5^{2}\cdot5^{2}\cdot5^{2}=5^{8}[/latex] (using the Product Rule—add the exponents).

[latex]\left(5^{2}\right)^{4}[/latex] is a power of a power. It is the fourth power of [latex]5[/latex] to the second power. And we saw above that the answer is [latex]5^{8}[/latex]. Notice that the new exponent is the same as the product of the original exponents: [latex]2\cdot4=8[/latex].

So, [latex]\left(5^{2}\right)^{4}=5^{2\cdot4}=5^{8}[/latex] (which equals 390,625, if you do the multiplication).

Likewise, [latex]\left(x^{4}\right)^{3}=x^{4\cdot3}=x^{12}[/latex]

This leads to another rule for exponents—the **Power Rule for Exponents**. To simplify a power of a power, you multiply the exponents, keeping the base the same. For example, [latex]\left(2^{3}\right)^{5}=2^{15}[/latex].

	[latex]({x}^{2})^{3}[/latex]
	[latex]{x}^{2}\cdot{x}^{2}\cdot{x}^{2}[/latex]
What does this mean? How many factors altogether?	
So, we have	[latex]{x}\cdot{x}\cdot{x}\cdot{x}\cdot{x}\cdot{x}={x}^{6}[/latex]
Notice that [latex]6[/latex] is the product of the exponents, [latex]2[/latex] and [latex]3[/latex].	[latex]({x}^{2})^{3}[/latex] is [latex]{x}^{2\cdot3}[/latex] or [latex]{x}^{6}[/latex]
We write:	[latex]{\left({x}^{2}\right)}^{3}[/latex] [latex]{x}^{2\cdot 3}[/latex] [latex]{x}^{6}[/latex]

This leads to the Power Property for Exponents.

Power Property of Exponents

If [latex]x[/latex] is a real number and [latex]a,b[/latex] are whole numbers, then

[latex]{\left({x}^{a}\right)}^{b}={x}^{a\cdot b}[/latex]

To raise a power to a power, multiply the exponents.

Take a moment to contrast how this is different from the product rule for exponents found on the previous page.

An example with numbers helps to verify this property.

[latex]\begin{array}{ccc}\hfill {\left({5}^{2}\right)}^{3}& \stackrel{?}{=}& {5}^{2\cdot 3}\hfill \\ \hfill {\left(25\right)}^{3}& \stackrel{?}{=}& {5}^{6}\hfill \\ \hfill 15,625& =& 15,625\hfill \end{array}[/latex]

example

Simplify:

1. [latex]{\left({x}^{5}\right)}^{7}[/latex]

2. [latex]{\left({3}^{6}\right)}^{8}[/latex]

Show Solution

Solution

1.	
	[latex]{\left({x}^{5}\right)}^{7}[/latex]
Use the Power Property, [latex]{\left({a}^{m}\right)}^{n}={a}^{m\cdot n}[/latex].	[latex]x^{\color{red}{5\cdot{7}}}[/latex]
Simplify.	[latex]{x}^{35}[/latex]

2.	

	[latex]{\left({3}^{6}\right)}^{8}[/latex]
Use the Power Property, [latex]{\left({a}^{m}\right)}^{n}={a}^{m\cdot n}[/latex].	[latex]3^{\color{red}{6\cdot{8}}}[/latex]
Simplify.	[latex]{3}^{48}[/latex]

try it

[See this interactive in the course material.]

Example

Simplify [latex]6\left(c^{4}\right)^{2}[/latex].

Show Solution

Since you are raising a power to a power, apply the Power Rule and multiply exponents to simplify. The coefficient remains unchanged because it is outside of the parentheses.

[latex]6\left(c^{4}\right)^{2}[/latex]

Answer

[latex]6\left(c^{4\cdot 2}\right)=6c^{8}[/latex]

Watch the following video to see more examples of how to use the power rule for exponents to simplify expressions.

[Video Link]

Simplify Expressions Using the Product to a Power Property

We will now look at an expression containing a product that is raised to a power. Look for a pattern.

Simplify this expression.

[latex]\left(2a\right)^{4}=\left(2a\right)\left(2a\right)\left(2a\right)\left(2a\right)=\left(2\cdot2\cdot2\cdot2\right)\left(a\cdot{a}\cdot{a}\cdot{a}\cdot{a}\right)=\left(2^{4}\right)\left(a^{4}\right)=16a^{4}[/latex]

Notice that the exponent is applied to each factor of [latex]2a[/latex]. So, we can eliminate the middle steps.

[latex]\begin{array}{l}\left(2a\right)^{4} = \left(2^{4}\right)\left(a^{4}\right)\text{, applying the }4\text{ to each factor, }2\text{ and }a\\\\\,\,\,\,\,\,\,\,\,\,\,\,\,=16a^{4}\end{array}[/latex]

The product of two or more numbers raised to a power is equal to the product of each number raised to the same power.

	[latex]{\left(2x\right)}^{3}[/latex]
What does this mean?	[latex]2x\cdot 2x\cdot 2x[/latex]
We group the like factors together.	[latex]2\cdot 2\cdot 2\cdot x\cdot x\cdot x[/latex]
How many factors of [latex]2[/latex] and of [latex]x?[/latex]	[latex]{2}^{3}\cdot {x}^{3}[/latex]
Notice that each factor was raised to the power.	[latex]{\left(2x\right)}^{3}\text{ is }{2}^{3}\cdot {x}^{3}[/latex]
We write:	[latex]{\left(2x\right)}^{3}[/latex] [latex]{2}^{3}\cdot {x}^{3}[/latex]

The exponent applies to each of the factors. This leads to the Product to a Power Property for Exponents.

Product to a Power Property of Exponents

If [latex]a[/latex] and [latex]b[/latex] are real numbers and [latex]m[/latex] is a whole number, then

[latex]{\left(ab\right)}^{m}={a}^{m}{b}^{m}[/latex]

To raise a product to a power, raise each factor to that power.

How is this rule different from the power raised to a power rule? How is it different from the product rule for exponents shown above?

An example with numbers helps to verify this property:

[latex]\begin{array}{ccc}\hfill {\left(2\cdot 3\right)}^{2}& \stackrel{?}{=}& {2}^{2}\cdot {3}^{2}\hfill \\ \hfill {6}^{2}& \stackrel{?}{=}& 4\cdot 9\hfill \\ \hfill 36& =& 36\hfill \end{array}[/latex]

example

Simplify: [latex]{\left(-11x\right)}^{2}[/latex]

Show Solution

Solution

	[latex]{\left(-11x\right)}^{2}[/latex]
Use the Power of a Product Property, [latex]{\left(ab\right)}^{m}={a}^{m}{b}^{m}[/latex].	[latex](-11)^{\color{red}{2}}x^{\color{red}{2}}[/latex]
Simplify.	[latex]121{x}^{2}[/latex]

try it

See this interactive in the course material.

example

Simplify: [latex]{\left(3xy\right)}^{3}[/latex]

Show Solution

Solution

	[latex]{\left(3xy\right)}^{3}[/latex]
Raise each factor to the third power.	[latex]3^{\color{red}{3}}x^{\color{red}{3}}y^{\color{red}{3}}[/latex]
Simplify.	[latex]27{x}^{3}{y}^{3}[/latex]

try it

[See this interactive in the course material.]

Example

Simplify. [latex]\left(2yz\right)^{6}[/latex]

Show Solution

Apply the exponent to each number in the product.

[latex]2^{6}y^{6}z^{6}[/latex]

Answer

[latex]\left(2yz\right)^{6}=64y^{6}z^{6}[/latex]

If the variable has an exponent with it, use the Power Rule: multiply the exponents.

Example

Simplify. [latex]\left(−7a^{4}b\right)^{2}[/latex]

Show Solution

Apply the exponent 2 to each factor within the parentheses.

[latex]\left(−7\right)^{2}\left(a^{4}\right)^{2}\left(b\right)^{2}[/latex]

Square the coefficient and use the Power Rule to square [latex]\left(a^{4}\right)^{2}[/latex].

[latex]49a^{4\cdot2}b^{2}[/latex]

Simplify.

[latex]49a^{8}b^{2}[/latex]

Answer

[latex]\left(-7a^{4}b\right)^{2}=49a^{8}b^{2}[/latex]

In the next video we show more examples of how to simplify a product raised to a power.

Video Link

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