

Calculus 2 — Thursday & Friday Guided Worksheets

Answer Key

THURSDAY — Organizing the Definite Integral

2. Understanding the Notation

For $\int_{-2}^4 \cos(t) dt$: Lower limit = -2 ; upper limit = 4 ; integrand = $\cos(t)$; variable of integration = t .

3. Positive and Negative Contributions

Try It: *Negative. If $f(x)$ stays below the x -axis on $[a,b]$, every signed contribution is negative, so $\int_a^b f(x) dx < 0$.*

4. Think From the Graph

1. $\sin(x)$ is positive on $(0, \pi)$ and negative on $(\pi, 2\pi)$.
2. $\int_0^{2\pi} \sin(x) dx = 0$ because the positive and negative signed areas have equal magnitude and cancel.
3. No. The geometric area is not 0; only the net signed area is 0.

5. Properties of Definite Integrals

Reversing Bounds — Example 1: $\int_4^1 f(x) dx = -5$.

Reversing Bounds — Example 2: $\int_2^7 g(x) dx = 6$.

Same Bounds — Example 1: $\int_3^3 f(x) dx = 0$.

Same Bounds — Example 2: $\int_{\pi}^{\pi} \sin(x) dx = 0$.

Constant Multiple — Example 1: $\int_0^3 5f(x) dx = 5(4) = 20$.

Constant Multiple — Example 2: $\int_0^3 [-2f(x)] dx = -2(4) = -8$.

Sum/Difference — Example 1: $\int_0^3 (f+g) dx = 4 + (-2) = 2$.

Sum/Difference — Example 2: $\int_0^3 (2f-3g) dx = 2(4) - 3(-2) = 14$.

Adjacent Intervals — Example 1: $\int_1^7 f = 5 + (-2) = 3$.

Adjacent Intervals — Example 2: $\int_7^1 f = -\int_1^7 f = -3$.

6. Periodic Graphs and Integrals

1. The two integrals are equal by periodicity, and both equal 0.

2. $\int_0^{4\pi} \sin(x) dx = 0 + 0 = 0$.

3. $\int_4^8 f(x) dx = 3$; $\int_0^8 f(x) dx = 6$.

4. $\int_3^6 g(x) dx = -2$; $\int_0^9 g(x) dx = -6$.

FRIDAY — FTC → Antiderivatives → u-Substitution

1. Fundamental Theorem of Calculus

Example 1: $\int_0^2 3x^2 \, dx \, [x^3]_0^2 = 8$.

Example 2: $\int_1^3 (2x+1) \, dx \, [x^2+x]_1^3 = 12 - 2 = 10$.

2. Find the Antiderivative

Example 1: $\int_0^\pi \cos(x) \, dx \, [\sin x]_0^\pi = 0$.

Example 2: $\int_1^2 4x^3 \, dx \, [x^4]_1^2 = 16 - 1 = 15$.

3. u-Substitution

Try It: $\int 3x^2(x^3+2)^4 \, dx$ Let $u=x^3+2$, $du=3x^2 \, dx$. Then $\int u^4 du = u^5/5 + C = (x^3+2)^5/5 + C$.

4. Definite u-Substitution and Bounds

Example 1: $\int_0^1 2x(x^2+2)^2 \, dx$ $u=x^2+2$, $du=2x \, dx$. New bounds: $x=0 \rightarrow u=2$; $x=1 \rightarrow u=3$. $\int_2^3 u^2 du = [u^3/3]_2^3 = 19/3$.

Example 2: $\int_0^1 3x^2(x^3+1)^3 \, dx$ $u=x^3+1$, $du=3x^2 \, dx$. New bounds: $x=0 \rightarrow u=1$; $x=1 \rightarrow u=2$. $\int_1^2 u^3 du = [u^4/4]_1^2 = 15/4$.

Answer Summary

-2, 4, $\cos(t)$, t , negative; 0; -5, 6, 0, 0, 20, -8; 2, 14, 3, -3; 0, 0, 3, 6, -2, -6; 8, 10, 0, 15; $(x^3+2)^5/5 + C$; 19/3; 15/4