Exam 2 Review

Sec 4.1, 4.2, 3.1, 3.2, 3.3, 5.1, 5.2, ~~5.3~~

Update 3/22/24: Section 5.3 will not be included in the exam (this means problems 9 and 10 below can be safely ignored)

The problems on this review sheet will help you practice some of the algorithmic and computational skills required for this exam. However, these technical skills are not a substitute for conceptual understanding! On the exam you may be asked to write and explain your thinking (not just solve problems).

What do you need to know? Some sample questions and important concepts:

- What is the basic principle behind Newton's Law of Cooling? Translate that principle into a differential equation, explaining the meaning of each variable and constant.
- What is the main idea of Euler's Method?
- Compare the Improved Euler's Method with Euler's Method. What is the difference? In what way is the method an improvement?
- Compare the Runge-Kutta Method to the other methods (Euler's Method and Improved Euler's Method). What is the difference?
- Why would we choose to use one of the numerical methods, instead of solving the differential equation to obtain an exact answer?
- You should know how to carry out the steps for each of the Numerical Methods (Euler's Method, Improved Euler's Method, and Runge-Kutta Method). That is, you should know the procedure for going from one point (x_i, y_i) to the next point (x_{i+1}, y_{i+1})in each method.
- You should be able to identify and solve homogeneous linear second-order differential equations with constant coefficients
- You should be able to identify linear second-order differential equations and test a proposed solution to see if it is correct. You should understand the relationship between basic solutions and the general solution to these equations.
- Problems like 3-10 below begin with a differential equation, for example y''-8y'+16y=0, but when solving the problem we actually work with another equation, the characteristic equation, in this case r^2-8r+16=0. Where does this characteristic equation come from? What is the connection between the characteristic equation and the original differential equation?

Some practice problems:

1. A bottle of cold water at 34℉ is carried into a warm 85℉ room. After 30 seconds, the temperature of the water is 36.2℉.
   a. Assume the time t is measured in seconds, find a formula for the temperature of the water $T(t)$ at time t.
   b. What will the temperature be after 10 minutes?
   c. How long does it take the water to reach 75℉?
2. When a hot object is placed in a water bath whose temperature is 25℃, it cools from 100℃ to

50℃ in 180s. In another bath, the same cooling occurs in 160s. Find the temperature of the second bath.

Problems 3-10. Find the general solution. If an initial condition is given, find the specific solution satisfying the condition.

3. $4y'' - 12y' + 45y = 0$
4. $y'' - 7y' - 8y = 0,\ y(0) = 8,\ y'(0) = 19$
5. $y'' - 14y' + 65y = 0, y(0) = 11,\ y'(0) = 69$
6. $y'' - 8y' + 16y = 0$
7. $25y'' - 60y' + 36y = 0, y(0) = 5/3,\ y'(0) = 3/2$
8. $6y'' + 19y' - 11y = 0$
9. ~~THIS PROBLEM WILL NOT BE ON THE EXAM:~~ $y'' - 2y' - 3y = 3e^{2t}$
10. ~~THIS PROBLEM WILL NOT BE ON THE EXAM~~ $y'' + 6y' + 9y =- 578 \sin 5t$

11. Suppose you are using the Runge-Kutta method to approximate the solution to an initial value problem. After several steps, you generate the point $(1.25, -1.7)$. What is the next point in the sequence, assuming the step size is $h = 0.25$ and $y' = 2^{xy} - 3x$?

Exam 2 Review ANSWER KEY

If you discover an error please let me know, either in class, on the OpenLab, or by email to jreitz@citytech.cuny.edu. Corrections will be posted on the "Exam Reviews" page.

1. a. The temperature decay constant is $k \approx 0.00146984$
   The temperature at time t (in seconds) is given by: $T = 85 - 51e^{-0.00146984t}$
   b. After 10 minutes (time $t = 600$ seconds), the temperature is $T(600) = 63.8863$℉
   c. The temperature will reach 75℉ after 1108.5 seconds, or 18.457 minutes
2. The temperature of the second bath is 19.7933℃.
3. General solution: $y(t) = e^{3t/2}(c_1 \cos 3t + c_2 \sin 3t)$
4. General solution: $y(t) = c_1 e^{8t} + c_2 e^{-t}$
   Specific solution satisfying the given initial conditions: $y(t) = 3e^{8t} + 5e^{-t}$
5. General solution: $y(t) = e^{7t}(c_1 \cos 4t + c_2 \sin 4t)$
   Specific solution satisfying the given initial conditions: $y(t) = e^{7t}(11 \cos 4t - 2 \sin 4t)$
6. General solution: $y(t) = c_1 e^{4t} + c_2 te^{4t}$
7. General solution: $y(t) = c_1 e^{6t/5} + c_2 te^{6t/5}$
   Specific solution satisfying the given initial conditions: $y(t) = \frac{5}{3}e^{6t/5} - \frac{1}{2}te^{6t/5}$
8. General solution: $y(t) = c_1 e^{-11t/3} + c_2 e^{t/2}$
9. $y(t) = c_1 e^{-t} + c_2 e^{3t} - e^{2t}$
10. $y(t) = c_1 e^{-3t} + c_2 te^{-3t} + 8 \sin 5t + 15 \cos 5t$
11. The next point in the sequence is $(1.5, -2.698100619)$.
    Values found along the way:
    x_i= 1.25
    y_i= -1.7
    k1= -3.520748989
    k2= -3.994928998
    k3= -4.002073184
    k4= -4.439661504
    y_(i+1)= -2.698100619