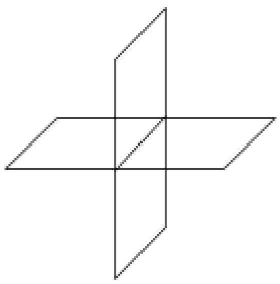
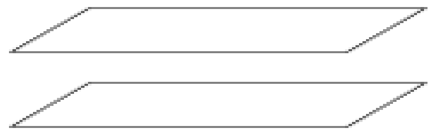
9.3 THE INTERSECTION OF TWO PLANES

Possible Intersections for Two Planes

Case 1: Two planes intersecting along a line



Case 2: Two Parallel Planes



Case 3: Two Coincident Planes

Solutions for a System of Equations Representing Two Planes
The system of equations corresponding to the intersection of two planes will have either _____________ or an ________________ number of solutions.

It is not possible for two planes to intersect at a ____________________ __________________.

Ex. 1 Determine the solution to the system of equations defined by the two planes $\pi_1: x - y + z = 4$ and $\pi_2: 2x - 2y + 2z = 10$. Discuss how these planes are related to each other.

Ex. 2 Determine the solutions to the following system of equations defined by the two planes $\pi_1: x + 2y - 3z = -1$ and $\pi_2: 4x + 8y - 12z = -4$.

Intersection of Two Planes and their Normals

If the planes π_1 and π_2 have $\vec{n}_1$ and $\vec{n}_2$ as their respective normals, we know the following:

- If $\vec{n}_1 = k\vec{n}_2$ for some scalar, k, the planes are coincident or they are parallel and non-coincident. If they are coincident, there are an _________________ number of points of intersection. If they are parallel and non-coincident, there are _______ points of intersection.
- If $\vec{n}_1 \neq k\vec{n}_2$, the two planes intersect in a ____________. This results in a ___________ number of points of intersection.

Ex. 3 Determine solutions to the following system of equations defined by the two planes: $\pi_1: x - y + z = 3$ and $\pi_2: 2x + 2y - 2z = 3$.

Ex. 4 Determine the solution to the following system of equations defined by the two planes: $\pi_1: 2x - y + 3z = -2$ and $\pi_2: x - 3z = 1$

Ex. 5 Determine an equation of a line that passes through the point P(5,-2,3) and is parallel to the line of intersection of the planes $\pi_1 : x + 2y - z = 6$ and $\pi_2 : y + 2z = 1$.