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B.Sc. (Hons.) Mathematics (Semester – 4th)
THEORY OF PROBABILITY
Subject Code: BMATS1424
Paper ID: [22131218]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) Define a random variable.

b) Explain the classical approach of probability.

c) State Bayes' theorem.

d) An experiment consists of rolling a fair six-sided die. Let A be the event of rolling an even number, and B be the event of rolling a number greater than 3. Find the probability of the event A ∪ B.

e) Let X be a random variable with probability density function given by $f(x) = 3x^2$ for 0 < x <1. Find the mean and variance of X.

f) Define conditional probability. Provide the formula for conditional probability.

g) How can you determine if two events are independent?

h) Let X and Y be independent continuous random variables with probability density functions $f_X(x) = 2x$ for $0 < x < 1$ and $f_Y(y) = 3y^2$ for $0 < y < 1$. Find the joint probability density function of X and Y.

i) Provide an example where the Poisson distribution is applicable.

j) Let X and Y be continuous random variables with joint probability density function given by $f(x, y) = 6xy$ for $0 < x < 1$ and $0 < y < 1$. Find the marginal probability density function of X.

Section – B **(5 marks each)**

Q2. Let the joint density of random variable X and Y is given by

$$f(x, y) = \{24xy, \quad 0 < x < 1,\ 0 < y < 1,\ 0 < x + y < 1\ 0, \quad otherwise\ .$$

Find the moment generating function of X and Y, and hence, find whether X and Y are independent?

Q3. Let $X \sim Bin(n,\ p)$. Find the mean and variance of X.

Q4. A box I contains 4 **white** and 6 **black** balls while another box II contains 4 **white** and 3 **black** balls. One ball is drawn at random from one of the boxes, and it is found to be **black**. Find the probability that it was drawn from **box I.**

Q5. Explain the difference between discrete and continuous distribution with an example.

Q6. Find the moment generating function (MGF) of normal distribution.

Section – C **(10 marks each)**

Q7. Let X and Y are two random variables. Prove that $E(X + Y) = E(X) + E(Y)$.

Q8. Show that Poisson distribution is a limiting case of the binomial distribution.

Q9. Let the joint density of random variable X and Y is given by

$$f(x,y) = \{k, \quad 4 \le x \le 10, \ 0 \le y \le 5 \ 0, \ otherwise \ .$$

Find the following:

i) Value of k,

ii) $P(X \le 8, 3 \le Y \le 4)$,

iii) $P(9 \le X \le 13, \ Y \le 1)$.