A polar coordinate system is a plane with a point O, the pole, and a ray from O, the polar axis. each point P in the plane is assigned polar coordinates as follows: r is the directed distance from O to P, and θ is the directed angle whose initial side is on the polar axis and whose terminal side is on the line OP.

Example 1: Plotting Points in the Polar Coordinate System.
        Plot the points with the given polar coordinates:

   a.   P $(2, \frac{\pi}{3})$
   b.   Q $(-1, \frac{3\pi}{4})$
   c.   R (-3, -45 deg)
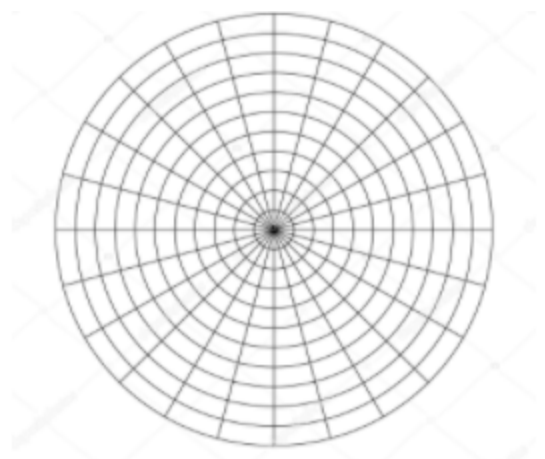
Finding all Polar Coordinates of a Point

        Let the point P have polar coordinates (r,θ). Any other polar coordinate of P must be of the form:

$$(r, \theta + 2\pi n) \text{ or } (-r, \theta + (2n + 1)\pi)$$

        where n is any integer. In particular, the pole has polar coordinates (0, θ), where θis any angle.

Example 2: Finding All Polar Coordinates for a Point

If the point P has polar coordinates $(4, \frac{\pi}{3})$, find all polar coordinates for P.

Coordinate Conversion Equations

Let the point P have polar coordinates (r,θ) and rectangular coordinates (x,y). Then,

$$x = r \cos \theta$$
$$y = r \sin \theta$$
$$r^2 = x^2 + y^2$$
$$\tan \theta = \frac{y}{x}$$

Example 3: Find the rectangular coordinates for the given polar coordinates:

a.   P $(2, \frac{7\pi}{6})$

b. Q $(4, \frac{4\pi}{3})$

c. R (4, -75 deg)

Example 4: Find two polar coordinate pairs for the point with the rectangular coordinates:

a.   S (-1, 1)

b. T (1, 2)

c. U (-4, 4)

Equation Conversions

We can use coordinate conversion equations to convert polar form to rectangular form and vice versa. Given the polar equation: r = 4 cos θ we can convert it to rectangular form…

$$r = 4 \cos \theta$$

Example 5: Convert the equation from polar form to rectangular form and identify the graph. Support your answer using your calculator polar graphing functions.

a.   r = 4 sec θ

b. r = 6 cosθ+ 4 sin θ

c. r = -2 csc θ

d. r = -5 sin θ

Convert the equation from rectangular form to polar form:

a. $x^2 + (y - 4)^2 = 16$

b. $(x + 1)^2 + (y - 3)^2 = 10$

c. x + 5y = 3

d. y = 3

Finding Distance Using Polar Coordinates

A radar tracking system sends out high-frequency radio waves and receives their reflection from an object. The distance and direction of the object from the radar is often given in polar coordinates.

Example 7: Using a radar Tracking System
    Radar detects two airplanes at the same altitude. Their polar coordinates are (8 mi, 110 deg) and (5 mi, 15 deg). How far apart are the planes?