01 | Polynomial representations

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Today I am learning to rewrite polynomial and rational functions. So I can choose a form that reveals useful graph features. I'll know I have learned it when I can find zeros, restrictions, asymptotes and expand binomials. Essential question: How does changing a function's form reveal its behavior?

1. Warm-up. Factor $x^2 - 9$ and $x^2 + 5x + 6$. Expand $(x + 2)^2$.

2. Vocabulary. Standard form reveals degree and leading coefficient. Factored form reveals _____ and their multiplicities. A zero with even multiplicity usually makes the graph _____ the x-axis; odd multiplicity makes it cross.

3. I DO. Analyze $f(x) = 2x^3 - 16x^2 + 32x$. Factor, find degree, end behavior, zeros, y-intercept and where $f(x) \geq 0$.
Factor out 2x: $2x(x^2 - 8x + 16) = 2x(x - 4)^2$. Degree 3, positive leading coefficient: left down, right up. Zeros 0 (odd), 4 (even); y-intercept (0,0). Since the square is nonnegative, $f(x) \geq 0$ on $[0,\infty)$.

4. WE DO. $g(x) = x^3 - 2x^2 - 15x = x(x - ___)(x + ___)$. Find zeros and signs on the intervals they create.

5. YOU DO. Expand $2(x - 4)^2$. Find degree, zeros, y-intercept and where it is nonnegative.

6. Check. Which form reveals the y-intercept directly? Why does a double zero not change sign?

02 | Rational expressions: factor before analyzing

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KEY IDEA: A rational function is a quotient of polynomials. Record every original denominator zero BEFORE canceling. A canceled factor does not restore an excluded input.

7. Rule box. Domain excludes values making the original denominator _____. A factor that completely disappears from the denominator produces a _____. A factor remaining in the denominator produces a _____.

8. I DO. Analyze $\frac{x^2-4}{x^2+7x+10}$. Factor numerator and denominator; find domain, hole, vertical and horizontal asymptotes, zeros and intercepts.
$\frac{(x-2)(x+2)}{(x+2)(x+5)} = \frac{x-2}{x+5}$, $x \neq -2, -5$. Hole $(-2,-\frac{4}{3})$; VA $x = -5$; HA $y = 1$ (equal degrees); zero 2; y-intercept $(0,-\frac{2}{5})$. Domain $(-\infty,-5) \cup (-5,-2) \cup (-2,\infty)$.

9. WE DO. Analyze $\frac{x^2+7x+12}{x^2+x-6}$. Factor: numerator (x+___)(x+___); denominator (x+___)(x−___). Find all features as in #8.

10. YOU DO. Convert $\frac{2(x+3)(x-3)}{(x+2)(x+3)}$ to general form. Identify all features as in #8.

11. Check. Is x = −3 a zero in #10? Can a hole lie on the x-axis?

03 | Repeated factors and end behavior

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Compare the exponent of a shared factor in numerator (m) and denominator (n). If n > m, a vertical asymptote remains. If 0 < n ≤ m, there is a hole. If m > n > 0, the hole lies on the x-axis.

12. I DO. Compare $\frac{x^3+2x}{x^2(x-3)}$ and $\frac{(x+3)^2(x-2)}{(x+3)(x+1)}$. Identify holes and vertical asymptotes.
First: $\frac{x^2+2}{x(x-3)}$, x≠0,3. VA x=0,3; no holes. HA y=1. Second: $\frac{(x+3)(x-2)}{x+1}$, x≠−3,−1. Hole (−3,0); VA x=−1; no HA; slant y=x (remainder −6).

13. WE DO. $\frac{2x(x+4)(x-1)}{x^2-2x+1}$. Factor denominator. Is x=1 a hole or VA? Explain.

14. YOU DO. $\frac{(x-2)^3(x+1)}{(x-2)^2(x+4)}$. Find domain, hole, zero and VA.

15. End-behavior rules. Compare degrees p (numerator) and q (denominator): p<q → y=___; p=q → ratio of leading coefficients; p=q+1 → _____; p>q+1 → neither horizontal nor slant.

16. Check. Classify the end behavior: A $\frac{3x-7}{x^2+4x+8}$; B $\frac{5x^2+3x-2}{2x^2-3}$; C $\frac{4x^2-8}{2x+1}$; D $\frac{x^4+1}{x^2+1}$.

04 | Polynomial long division and slant asymptotes

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Division algorithm: $P(x) = D(x)Q(x) + R(x)$, or $\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)}$. The remainder has smaller degree than the divisor. Divide → multiply → subtract → bring down → repeat.

17. Warm-up. 675 ÷ 21. Find quotient and remainder.

18. I DO. Divide $(6x^2 + 7x + 5)$ by $(2x + 1)$.
$6x^2 \div 2x = 3x$. Multiply: $6x^2+3x$; subtract: $4x+5$. Divide $4x \div 2x = 2$. Multiply: $4x+2$; subtract: 3. Quotient $3x+2$; remainder 3. Result $3x+2+\frac{3}{2x+1}$. Slant $y=3x+2$.

19. WE DO. Divide $(6x^2 + x + 5)$ by $(2x + 1)$. First quotient term = _____. After subtracting, remainder expression = _____. Finish.

20. YOU DO. Analyze $\frac{x^3+3x^2-10x}{x^2+8x+15}$. Factor; find all asymptotes and any hole.

21. Check. Why is the quotient the slant line? What should you insert when a power is missing?

05 | Graph analysis: from equation to graph

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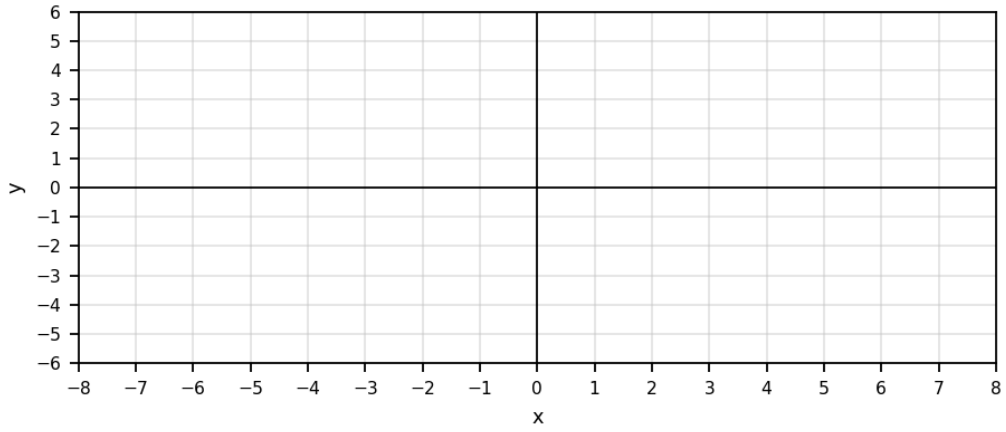
Graphing checklist: factor; record domain; find intercepts and holes; locate asymptotes; test symmetry using f(−x); make sign chart including all excluded inputs; plot and connect branches without crossing vertical asymptotes.

22. I DO. Analyze $\frac{x^2-x-20}{x^3+x^2-12x}$. Find reduced form, domain, intercepts, hole, asymptotes, symmetry and signs.

$\frac{(x-5)(x+4)}{x(x+4)(x-3)} \to \frac{x-5}{x(x-3)}$, $x \neq -4,0,3$. Hole $(-4,-\frac{9}{28})$; zero 5; no y-intercept; VAs x=0,3; HA y=0. Neither symmetry. Signs on intervals split at −4,0,3,5: −,−,+,−,+. Near 0: left −∞, right +∞; near 3: left +∞, right −∞.

23. WE DO. Use the grid to sketch #22. Mark dashed asymptotes, open circle and intercept.



24. Check. Why must a calculator graph be checked algebraically? Can a graph cross a horizontal asymptote?

06 | From graphs and limits to equations

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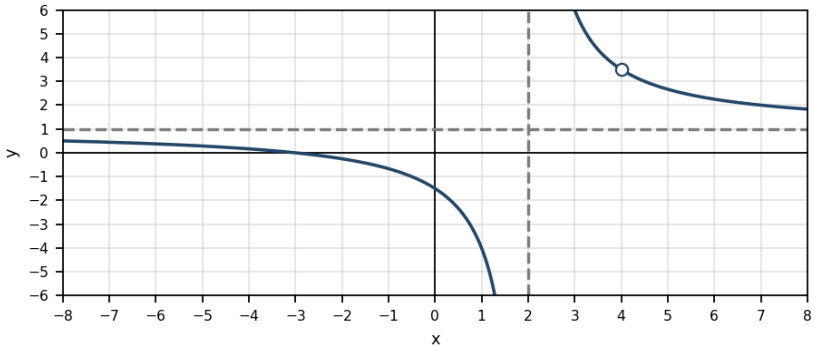
A zero suggests a numerator factor, a vertical asymptote suggests a denominator factor, and a hole requires a common factor. A scale factor can be determined using a known point or the horizontal asymptote.

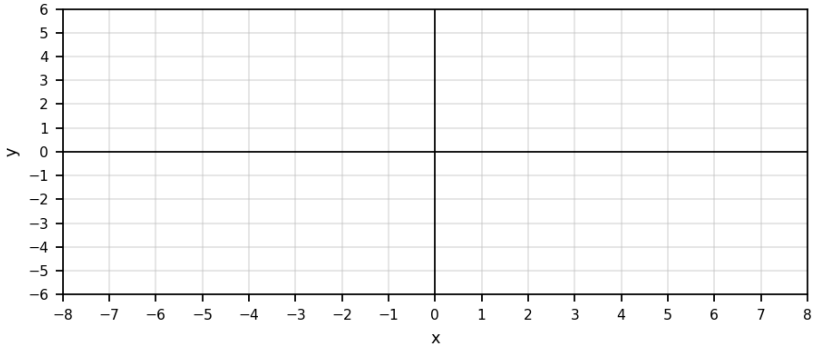
25. I DO. Use the displayed graph to write a possible factored equation. Label zero, hole and asymptotes.

Zero −3, VA x=2, HA y=1, hole at x=4. Start with $\frac{x+3}{x-2}$; multiply numerator and denominator by (x−4). Result $\frac{(x+3)(x-4)}{(x-2)(x-4)}$. Hole $(4,\frac{7}{2})$.



26. WE DO. A graph has zero 1, VAs x=0 and x=3, hole x=2, HA y=0 and passes through (−1,1). Write a possible equation.

27. YOU DO. Sketch a graph with end limits 2, left limit at −3 equal to −∞, right limit +∞, and a hole (5,3).



28. Check. If both sides approach −∞ at x=−1, what multiplicity is suggested? Give a function with left end +∞ and right end −∞.

07 | Pascal's triangle and binomial expansion

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A binomial has two terms. Pascal's triangle supplies coefficients. Each inside number equals the sum of the two above it. Row numbering begins at zero.

29. Complete Pascal's triangle (rows 0–8). Row 0: 1; row 1: 1 1; row 2: 1 2 1; row 3: 1 3 3 1.

1

1 1

1 2 1

1 3 3 1

1 ___ ___ ___ 1

1 ___ ___ ___ ___ 1

1 ___ ___ ___ ___ ___ 1

1 ___ ___ ___ ___ ___ ___ 1

1 ___ ___ ___ ___ ___ ___ ___ 1

30. Formula box. $(a+b)^n = \Sigma\ C(n,r)a^{n-r}b^r$, r=0 to n. There are ___ terms. The exponent of a decreases; the exponent of b _____. Optional: $C(n,r)=\frac{n!}{r!(n-r)!}$.

31. I DO. Expand $(x+3)^3$ and $(x+3)^2$.

Row 3: $x^3+3x^2(3)+3x(3^2)+3^3 = x^3+9x^2+27x+27$. Row 2: $x^2+2x(3)+3^2=x^2+6x+9$.

32. WE DO. Expand $(2x-1)^4$. Coefficients: 1, ___, ___, ___, 1.

33. YOU DO. Expand $(x-3)^5$.

34. Check. Is C(6,2) the second term's coefficient? Why do subtraction signs alternate?

08 | Find a specific term or coefficient

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Use $T_{r+1} = C(n,r)a^{n-r}b^r$. For the kth term, r = k−1. To locate a power of x, solve for the exponent before computing the coefficient.

35. I DO. Find the third term of $(2x+3)^6$.
The third term means r=2. C(6,2)=15. $15(2x)^4(3)^2 = 15 \cdot 16 \cdot 9\ x^4 = 2160x^4$.

36. WE DO. Find the coefficient of x^4 in $(x+5)^6$. Set 6−r=4, so r=___.

37. YOU DO. Find the coefficient of x^3 in $(x-3)^8$.

38. Check. Find the last term in $(2x+3)^4$. Evaluate C(6,2) using its row and position.

39. AP-style MCQ. What is the coefficient of the fourth term of $(x-2)^6$? A −160 B −120 C 120 D 160. Explain.

40. AP-style MCQ. Second term of $(2a-b)^4$? A $-32a^3b$ B $-4a^2b^2$ C $-16ab^3$ D $-ab^4$.

09 | Practice: basic and application

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Show work in each box. Use exact values and interval notation. Level 1: recall and apply. Level 2: select a representation and justify.

41. Level 1. Factor $-3x^3+27x$. Find zeros, end behavior and where $g(x) \geq 0$.

42. Level 1. Expand $(x+2)^4$ and $(2x+1)^3$.

43. Level 2. Analyze $\frac{t^3-4t^2}{t^2+2t-15}$: factored form, domain, zeros, holes, all asymptotes, y-intercept.

44. Level 2. Expand $(3x-2)^4$. Find the fourth term of $(x-2)^7$.

45. Level 2. Find the fifth term of $(3x-1)^8$. Find the coefficient of x^5 in $(x+3)^7$.

10 | AP-level challenge and exit ticket

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Level 3: explain your reasoning. Original AP-style practice, not official College Board exam questions.

46. FRQ. Let $\frac{x^2-x-12}{x^3+x^2-20x}$. Find the factored form, domain, zeros, hole, VAs, HA and y-intercept. Explain why a canceled zero is excluded.

47. MCQ. Which has right-end limit $+\infty$? A $\frac{(x+3)^3}{x(x^3+1)}$ B $-2(x-1)^3(x+5)$ C $\frac{x(x+3)^3}{x(x^3+1)}$ D $x(x+2)^3(x-1)$. Justify.

48. Summary. Factoring reveals ____. Original denominator zeros are ____. Long division reveals the ____ asymptote. Pascal's triangle gives ____.

49. Exit ticket — concept. When does a common factor create a hole instead of a VA?

50. Exit ticket — application. Find the third term in $(x+5)^5$.

S1. Exit ticket — reasoning. Explain why $\frac{x^2-1}{x-1}$ and x+1 do not have identical graphs.