Date______
Period_____

5B1 Gauss's Method

Gauss Method: The following section is meant to help you develop what is know as the Gauss Method for finding the sum of a series of numbers.

<u>Example</u> Problem: Add up the integers from 1 to 10.
Solution: Let the sum of the integers from 1 to 10 be called S. (S for sum.)
Then $S = 1+2+3+4+5+6+7+8+9+10$ if we went in order or it could also be $S = 10+9+8+7+6+5+4+3+2+1$ if we went in reverse order.

$$\begin{array}{lcllllllllll} S & = & 1 & +2 & +3 & +4 & +5 & +6 & +7 & +8 & +9 & +10 \\ +S & = & 10 & +9 & +8 & +7 & +6 & +5 & +4 & +3 & +2 & +1 \\ \hline 2S & = & 11 & +11 & +11 & +11 & +11 & +11 & +11 & +11 & +11 & +11 \end{array}$$

This means that

$$2S = 10(11)$$
$$S = \frac{10}{2}(11) = 55$$

Use the example above as a model for each of the following problems. You will need to use your own paper. (I realize that I gave you problems you could probably just type in your calculator and quickly get the answer. The purpose is to explore the method so that it can be used to find the sum of the first 1000 positive integers for example.)

1. Add up all of the integers from 0 to 4.

$S = 0+1+2+3+4$
$+\ S = 4+3+2+1+0$
$2S = 4+4+4+4+4$
$2S = 5(4)$
$S = \frac{5(4)}{2} = 10$

2. Add up all of the integers from -3 to 4.

$S = -3+-2+-1+0+1+2+3+4$
$+\ S = 4+3+2+1+0+-1+-2+-3$
$2S = 8(1)$
$S = \frac{8(1)}{2} = 4$

3. Add up the first 5 positive odd integers.

$S = 1+3+5+7+9$
$S = 9+7+5+3+1$
$2S = 5(10) \longrightarrow S = \frac{5(10)}{2} = 25$

Try to skip to the last line in Gauss's method to find each of the following sums. I have included the answer in parentheses so you can check your method.

4. Add up all of the integers from 1 to 1000. (500,500)

$S = \frac{(1+1000)(1000)}{2} = 500,500$

5. Add up all of the integers from 0 to 15,023 (11,2852,776)

$S = \frac{(0+15023)(15024)}{2}$

$S = 11,2,852,776$

6. Add up all of the odd integers from 1 to 99. (2500)

$S = \frac{(1+99)(50)}{2} = 2500$

7. Try to use Gauss's Method to find the sum of $4^0 + 4^1 + 4^2 + 4^3$. Write it all out. Do not skip to the shortcut. What is different about this problem than the previous problems, which does not allow us to use Gauss's method?

$$S = 1 + 4 + 16 + 64$$
$$+ \quad S = 64 + 16 + 4 + 1$$
$$2S = 65 + 20 + 20 + 65$$

The series is not arithmetic so the sum of each pair is not the same.

8. Generalize: Use Gauss's Method to show how you would find the sum of the integers from 1 to n.

$$S = \frac{(1+n)n}{2}$$

9. Generalize: Use Gauss's Method to show how you would find the sum of the integers from 0 to n.

$$S = \frac{(0+n)(n+1)}{2} = \frac{n(n+1)}{2}$$

10. Apply Gauss's Method on to find the sum of the arithmetic series: 5+8+11+...+158. (4238)

$$S = 5 + 8 + 11 + \ldots 158$$
$$S = 158 + 155 + 152 + \ldots 5$$
$$2S = 163(52)$$
$$S = \frac{163(52)}{2} = 4238$$

$f(n) = 3n + 5$

$158 = 3n + 5$

$51 = n$

$\Rightarrow$ 52 terms

11. Generalize Gauss's method (using words) to find the sum of any arithmetic series.

$$S = \frac{(\text{First Term} + \text{Last Term})(\#\text{ of terms})}{2}$$

Exercises: Apply Gauss's Method to find each sum if possible. If Gauss's Method cannot be used, state the reason.

12. $5+8+11+14+\ldots+32$

$f(n)=5+3n$

$32=5+3n$

$9=n$

$S=\frac{10(5+32)}{2}=185$

13. $-2+(-5)+(-8)+(-11)+\ldots+(-20)$

$f(n)=-2-3n$

$-20=-2-3n$

$6=n$

$S=\frac{7(-2-20)}{2}=-77$

14. $1+4+9+16+\ldots144$

Cannot use Gauss's Method - The series is not arithmetic - no constant difference

15. $5+7+9+11+\ldots+27$

$f(n)=5+2n$

$27=5+2n$

$11=n$

$S=\frac{12(5+27)}{2}=192$

16. $-4+1+6+11+\ldots91$

$f(n)=-4+5n$

$91=-4+5n$

$19=n$

$S=\frac{20(-4+91)}{2}=870$

17. $13+20+27+34+\ldots+272$

$f(n)=13+7n$

$272=13+7n$

$37=n$

$S=\frac{38(13+272)}{2}=5415$

18. $5+10+20+\ldots+5120$

x2 x2

This is a Geometric Series so Gauss's Method cannot be used

19. $89+86+83+80+\ldots+20$

$f(n)=89-3n$

$20=89-3n$

$-69=-3n$

$23=n$

$S=\frac{24(89+20)}{2}=1308$