

STRAND 2 ALGEBRA

Matrices

Introduction

- A matrix is a rectangular arrangement of numbers in rows and columns.
- For instance, matrix A below has two rows and three columns. The dimensions of this matrix are 2×3 (read "2 by 3").
- The numbers in a matrix are its entries. In matrix A , the entry in the second row and third column is 5.
- Some matrices (the plural of matrix) have special names because of their dimensions or entries.

Order of Matrix

-
-

Matrix consist of rows and columns. Rows are the horizontal arrangement while columns are the vertical arrangement. Order of matrix is being determined by the number of rows and columns. The order is given by stating the number of rows followed by columns.

Note;

- If the number of rows is m and the number of columns n , the matrix is of order $m \times n$.

E.g. If a matrix has m rows and n columns, it is said to be order $m \times n$.

e.g. $\begin{bmatrix} 2 & 0 & 3 & 6 \\ 3 & 4 & 7 & 0 \\ 1 & 9 & 2 & 5 \end{bmatrix}$ is a matrix of order 3×4 .

e.g. $\begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 5 \\ -1 & 3 & 0 \end{bmatrix}$ is a matrix of order 3.

e.g. $\begin{bmatrix} 2 & 3 & 4 \\ 1 & -8 & 5 \end{bmatrix}$ is a 2×3 matrix.

e.g. $\begin{bmatrix} 2 \\ 7 \\ -3 \end{bmatrix}$ is a 3×1 matrix.

Elements of Matrix

- The element of a matrix is each number or letter in the matrix. Each element is locating by stating its position in the row and the column.
- For example, given the 3×4 matrix

$$\begin{bmatrix} 2 & 0 & 3 & 6 \\ 3 & 4 & 7 & 0 \\ 1 & 9 & 2 & 5 \end{bmatrix}$$

- The element 1 is in the third row and first column.
- The element 6 is in the first row and forth column.

Note;

- A matrix in which the number of rows is equal to the number of columns is called a **square matrix**.

$$\begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & 5 \\ -1 & 3 & 0 \end{bmatrix}$$

- $[a_1, a_2 \dots a_n]$ Is called a row matrix or row vector.

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$$\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Is called a *column matrix* or *column vector*.

•

$$\begin{bmatrix} 2 \\ 7 \\ -3 \end{bmatrix}$$

Is a column vector of order 3×1 .

- $[-2 \ -3 \ -4]$ is a row vector of order 1×3 .
- Two or more matrices re equal if they are of the same order and their corresponding elements are equal. • Thus, if $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 1 & 5 \end{bmatrix}$ then, $a = 3$, $b = 4$ and $d = 5$.

Addition and Subtraction of Matrices

- Matrices can be added or subtracted if they are of the same order. The sum of two or more matrices is obtained by adding corresponding elements. Subtraction is also done in the same way.

Example

if $A = \begin{bmatrix} 2 & 5 \\ 0 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 3 \\ 6 & 2 \end{bmatrix}$ find :

17. $A + B$

18. $A - B$

Solution

a.

$$A+B = \begin{bmatrix} 2 & 5 \\ 0 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 2+1 & 5+3 \\ 0+6 & 7+2 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 6 & 9 \end{bmatrix}$$

b.

$$A - B = \begin{bmatrix} 2 & 5 \\ 0 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} 2-1 & 5-3 \\ 0-6 & 7-2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -6 & 5 \end{bmatrix}$$

Example

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 4 & 5 \\ 1 & 3 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 0 \\ 5 & 9 & 6 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 2 \\ 1 & 3 & 5 \\ 2 & 1 & 6 \end{bmatrix} = \begin{bmatrix} 3-2+8 & 2-4+0 & 1-1+2 \\ 0-1+1 & 4-2+3 & 5-0+5 \\ 1-5+2 & 3-9+1 & 2-6+6 \end{bmatrix}$$
$$= \begin{bmatrix} 9 & -2 & 2 \\ 0 & 5 & 10 \\ -2 & -5 & 2 \end{bmatrix}$$

Note;

After arranging the matrices you must use BODMAS

$$\begin{bmatrix} 2 & 7 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}$$

The matrix above cannot be added because they are not of the same order. $\begin{bmatrix} 2 & 7 \\ 4 & 9 \end{bmatrix}$ is of order 2×2 while $\begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}$ is of order 3×1

Matrix Multiplication

To multiply a matrix by a number, you multiply each element in the matrix by the number.

Example

$$3 \begin{bmatrix} -2 & 0 \\ 4 & -7 \end{bmatrix}$$

Solution

$$= \begin{bmatrix} -2(3) & 0(3) \\ 4(3) & -7(3) \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ 12 & -21 \end{bmatrix}$$

Example

$$-2 \begin{bmatrix} 1 & -2 \\ 0 & 3 \\ -4 & 5 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 6 & -8 \\ -2 & 6 \end{bmatrix}$$

Solution

$$\begin{bmatrix} -2 & 4 \\ 0 & -6 \\ 8 & -10 \end{bmatrix} + \begin{bmatrix} -4 & 5 \\ 6 & -8 \\ -2 & 6 \end{bmatrix}$$
$$= \begin{bmatrix} -6 & 9 \\ 6 & -14 \\ 6 & -4 \end{bmatrix}$$

Example

A woman wanted to buy one sack of potatoes, three bunches of bananas and two basket of onion. She went to kikuyu market and found the prices as sh 280 for the sack of potatoes ,sh 50 for a bunch of bananas and sh 100 for a basket of onions. At kondelee market the corresponding prices were sh 300, sh 48 and sh 80.

2. Express the woman's requirements as a row matrix
3. Express the prices in each market as a column matrix
4. Use the matrices in (a) and (b) to find the total cost in each market

Solution

a. Requirements in matrix form is $[1 \ 3 \ 2]$

b.

Price matrix for Kikuyu market is $\begin{bmatrix} 280 \\ 50 \\ 100 \end{bmatrix}$

Price matrix for kondelee market $\begin{bmatrix} 300 \\ 48 \\ 80 \end{bmatrix}$

c. Total cost in shillings at Kikuyu Market is

$$(1 \ 3 \ 2) \begin{bmatrix} 280 \\ 50 \\ 100 \end{bmatrix} = (1 \times 280 + 3 \times 50 + 2 \times 100) = (630)$$

Total cost in shillings at Kondelee Market is;

$$(1 \ 3 \ 2) \begin{bmatrix} 300 \\ 48 \\ 80 \end{bmatrix} = (1 \times 300 + 3 \times 48 + 2 \times 80) = (604)$$

The two results can be combined into one as shown below

$$(1 \ 3 \ 2) \begin{bmatrix} 280 & 300 \\ 50 & 48 \\ 100 & 80 \end{bmatrix} = (630 \ 604)$$

Note;

- The product of two matrices A and B is defined provided the number of columns in A is equal to the number of rows in B.
- If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then the product AB is an $m \times p$ matrix.

$$A \times B = AB$$

$$m \times n \times n \times p = m \times p$$

- Each time a row is multiplied by a column

Example

Find AB if $A = \begin{bmatrix} -2 & 3 \\ 1 & -4 \\ 6 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$

Solution

Because A is a 3×2 matrix and B is a 2×2 matrix, the product AB is defined and is a 3×2 matrix. To write the elements in the first row and first column of AB, multiply corresponding elements in the first row of A and the first column of B. Then add. Use a similar procedure to write the other entries of the product.

$$\begin{aligned} AB &= \begin{bmatrix} -2 & 3 \\ 1 & -4 \\ 6 & 0 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} (-2)(-1) + (3)(-2) & (-2)(3) + (3)(4) \\ (1)(-1) + (-4)(-2) & (1)(3) + (-4)(4) \\ (-6)(-1) + (0)(-2) & (6)(3) + (0)(4) \end{bmatrix} \\ &= \begin{bmatrix} -4 & 6 \\ 7 & -13 \\ -6 & 18 \end{bmatrix} \end{aligned}$$

Identity Matrix

- For matrices, the identity matrix or a unit matrix is the matrix that has 1's on the main diagonal and 0's elsewhere.
- The main diagonal is the one running from top left to bottom right .It is also called leading or principle diagonal. Examples are;

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

2 X 2 identity matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3 x 3 identity matrix

- If A is any n x n matrix and I is the n x n identity matrix, then $IA = A$ and $AI = A$.

Determinant Matrix

- The determinant of a matrix is the difference of the products of the elements on the diagonals.

Examples

The determinant of A, $\det A$ or $|A|$ is defined as follows:

$$\text{If } n=2, \det A = \begin{vmatrix} a_{11} & b_{12} \\ b_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - b_{12}b_{21}$$

Example

Find the determinant of $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

Solution

Subtract the product of the diagonals

$$1 \times 5 - 2 \times 3 = 5 - 6 = -1$$

Determinant is -1

Inverse of a Matrix

Two matrices of order n x n are inverse of each other if their product (in both orders) is the identity matrix of the same order n x n. The inverse of A is written as A^{-1}

Example

Show that

$$B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

Solution

$$\begin{aligned} AB &= \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 2 \times 3 + 1 \times -5 & 2 \times -1 + 1 \times 2 \\ 5 \times 3 + 3 \times -5 & 5 \times -1 + 3 \times 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

$$BA = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

Note;

To get the inverse matrix

- Find the determinant of the matrix. If it is zero, then there is no inverse
- If it is non zero, then;
 - Interchange the elements in the main diagonal
 - Reverse the signs of the element in the other diagonals
 - Divide the matrix obtained by the determinant of the given matrix

In summary

The inverse of the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ provided } ad-bc \neq 0$$

Example

Find the inverse of

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

Solution

$$A^{-1} = \frac{1}{6-4} \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{bmatrix}$$

Check

You can check the inverse by showing that $AA^{-1} = \text{identity matrix}$

$$\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ And } \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Solutions of Simultaneous Linear Equations Using Matrix Method

Using matrix method solve the following pairs of simultaneous equation

$$x + 2y = 4$$

$$3x - 5y = 1$$

$$\begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

Solution

$\begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$ is the coefficients matrix of the simultaneous equations

$\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ is the constants matrix

We need to calculate the inverse of $A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$

$$A^{-1} = \frac{1}{(1)(-5)-(2)(3)} \begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix} = -\frac{1}{11} \begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix}$$

$$\text{Hence } A^{-1}B = -\frac{1}{11} \begin{pmatrix} -5 & -2 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$= -\frac{1}{11} \begin{pmatrix} -22 \\ 11 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Hence the value of $x = 2$ and the value of $y = 1$ is the solution of the simultaneous equation

$$A = \begin{pmatrix} 1 & 3 \\ 5 & 3 \end{pmatrix}, B = \begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix}, C = \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}$$

$$\begin{pmatrix} x & 0 \\ 5 & y \end{pmatrix}$$

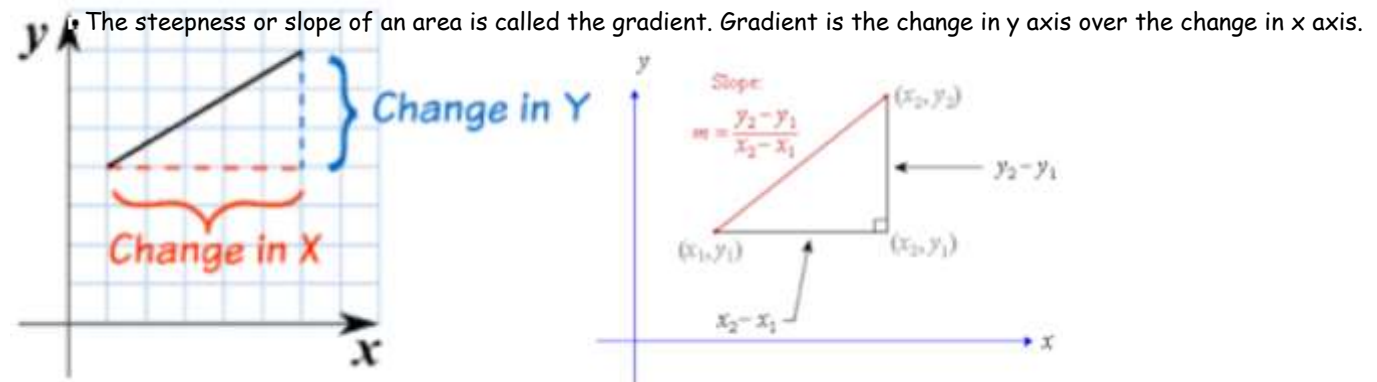
Questions

1. A and B are two matrices. If $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ find B given that $A^2 = A + B$
2. Given that $A = \begin{pmatrix} 1 & 2 \\ 4 & 3 \end{pmatrix}$ and $AB = BC$, determine the value of P
3. A matrix A is given by $A = a$. Determine A^2
- b. If $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ determine the possible pairs of values of x and y
4.
 - a. Find the inverse of the matrix $\begin{pmatrix} 9 & 8 \\ 1 & -1 \end{pmatrix}$
 - b. In a certain week a businessman bought 36 bicycles and 32 radios for total of Kshs 227 280. In the following week, he bought 28 bicycles and 24 radios for a total of Kshs 1 74 960. Using matrix method, find the price of each bicycle and each radio that he bought
 - c. In the third week, the price of each bicycle was reduced by 1 0% while the price of each radio was raised by 1 0%. The businessman bought as many bicycles and as many radios as he had bought in the first two weeks. Find by matrix method, the total cost of the bicycles and radios that the businessman bought in the third week.
5. Determine the inverse T-1 of the matrix $\begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$ Hence find the coordinates to the point at which the two lines $x + 2y = 7$ and $x - y = 1$
6. Given that $A = \begin{pmatrix} 0 & -1 \\ 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 2 & -4 \end{pmatrix}$ Find the value of x of
 - i. $A - 2x = 2B$
 - ii. $3x - 2A = 3B$
 - iii. $2A - 3B = 2x$
7. Find the non- zero value of k for which $\begin{pmatrix} k+1 & 2 \\ 4k & 2k \end{pmatrix}$ is an inverse.
9. A clothes dealer sold 3 shirts and 2 trousers for Kshs. 840 and 4 shirts and 5 trousers for Kshs 1680. Form a matrix equation to represent the above information. Hence find the cost of 1 shirt and the cost of 1 trouser.

Equation of a Straight Line

Introduction

Gradient

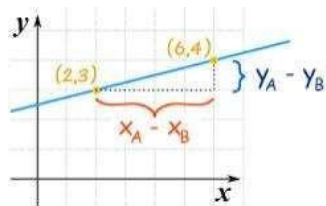


$$\frac{\text{change in y co-ordinates}}{\text{change in x co-ordinates}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Note:

- If an increase in the x coordinates also causes an increase in the y coordinates the gradient is positive.
- If an increase in the x coordinates causes a decrease in the value of the y coordinate, the gradient is negative.
- If, for an increase in the x coordinate, there is no change in the value of the y coordinate, the gradient is zero.

For vertical line, the gradient is not defined



Solution

$$\begin{aligned}\text{Gradient} &= \frac{\text{change in y axis}}{\text{change in x axis}} \\ &= \frac{4 - 3}{6 - 2} \\ &= \frac{1}{4}\end{aligned}$$

QUESTIONS

(1). For each of the following pairs of points. find the change in the y coordinate and the corresponding change in the x coordinate. Hence, find the gradients of the lines passing through them;

a. A(2, 3), B(5, 6)

b. C(5,10), D(12, 20)

c. E(-5, 60), F(2,1)

d. G(4, 5), H(6, 5)

e. I(8, 0), J(12, -6)

f. K(5, -2), L(6,2)

g. M(6, 3), N(-6, +2)

h. P(2, -5), Q(2, 3)

(2). Find the gradients of the lines passing through the following pairs of points:

a. A(3, 2), B(-1,1)

b. C(7, 2), D(4, 3)

c. E(-1, -3), F(-2,-2)

d. G(0, 5), H(2, 5)

e. I($\frac{1}{4}$, $\frac{1}{3}$), J($\frac{1}{3}$, $\frac{1}{4}$)

f. K(0.5, 0.3), (L-0.2, -0.7)

(3). Find the gradient of each of the following lines:

a. $y = \frac{1}{2}x + 3$

b. $3y - 4x = 5$

c. $y = -2x + 2$

d. $y + 2x - 3 = 0$

e. $\frac{1}{3}x + \frac{1}{4}y = \frac{1}{12}$

f. $y = -10$

Equation of a Line

Equation of a Straight Line.

Given Two Points

Example.

Find the equation of the line through the points A (1, 3) and B (2, 8)

Solution

The gradient of the required line is $8 - 3 = 5$

$2 - 1$

Take any point p (x, y) on the line. Using... points P and A, the gradient is $y - 3 / x - 1$

Therefore $y - 3 = 5$

$x - 1$

Hence $y = 5x - 2$

Given the Gradient and One Point on the Line

Example

Determine the equation of a line with gradient 3, passing through the point (1, 5).

Solution

Let the line pass through a general point (x, y). The gradient of the line is $y - 5 = 3x - 1$

Hence the equation of the line is $y = 3x + 2$

QUESTIONS

(1). Find the equations of lines with the given gradients and passing through the given points:

a. 4; (2, 5)

b. $\frac{3}{4}$; (-1, 3)

c. -2; (7, 2)

d. $-\frac{1}{3}$; (6, 2)

e. 0; (-3, -5)

f. $-\frac{3}{2}$; (0, 7)

g. m; (1, 2)

h. m; (a, b)

(2). Find the equation of the following line passing through the given points.

a. (0, 0) and (1, 3)

b. (0, -4) and (1, 2)

c. (0, 4) and (-1, -2)

d. (1,0) and (, 1)

e. (3, 7) and (5, 7)

f. (-1, 7) and (3, 3)

g. (11, 1) and (14, 4)

h. (5, -2.5) and (3.5, -2)

i. (a, b) and (c, d)

j. (x_1, y_1) and (x_2, y_2)

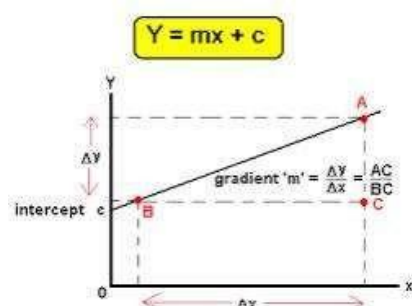
Linear Equation $y = mx + c$

We can express linear equation in the form $y = mx + c$.

Illustrations.

For example $4x + 3y = -8$ is equivalent to $y = -\frac{4}{3}x - \frac{8}{3}$

In the linear equation below gradient is equal to m while c is the y intercept.



Using the above statement we can easily get the gradient.

Example

Find the gradient of the line whose equation is $3y - 6x + 7 = 0$

Solution

Write the equation in the form of $y = mx + c$

$$3y = 6x - 7$$

$$y = 2x - \frac{7}{3}$$

$m = 2$ and also gradient is 2.

QUESTIONS

(1). For each of the following straight lines, determine the gradient and the y-intercept. Do not draw the line:

a. $3y = 7x$

b. $2y = 6x + 1$

c. $7 - 2x = 4y$

d. $3y = 7$

e. $2y - 3x + 4 = 0$

f. $3(2x - 1) = 5y$

g. $y + 3x + 7 = 0$

h. $5x - 3y + 6 = 0$

i. $\frac{3}{2}y - 15 = \frac{2}{3}x$

j. $2(x + y) = 4$

k. $\frac{1}{3}x + \frac{2}{5}y + \frac{1}{6} = 0$

l. $-10(x + 3) = 0.5y$

m. $ax + by + c = 0$

Graph of a Straight Line

Draw the graph of a line passing through (0, 4) and has a gradient of 2.

Solution

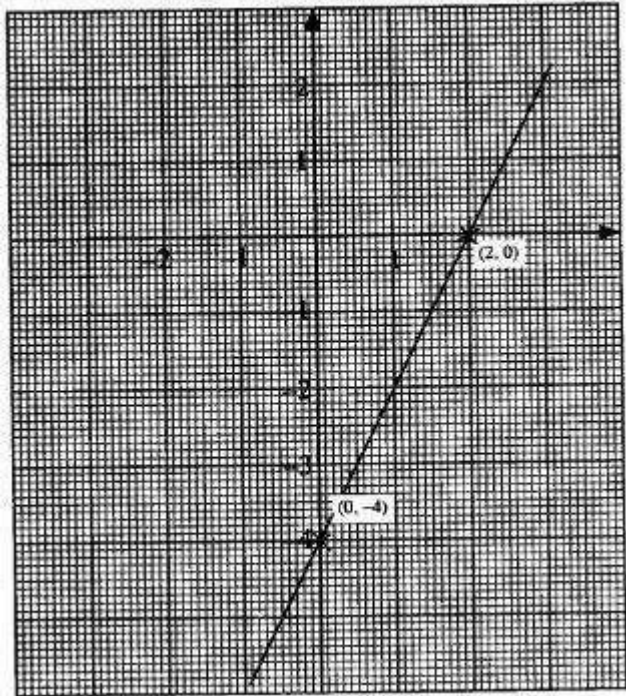
The equation of the line is;

$$\frac{y + 4}{x} = 2$$

$$y + 4 = 2x$$

$$y = 2x - 4$$

The x-intercept is 2, and the y-intercept is -4. the line cuts the x-axis at (2, 0). and the y-axis at (0, -4).



Questions

(1). Draw the lines passing through the given points and having the given gradients:

- a. (0, 3); 3
- b. (0, 2); 5
- c. (4, 3); 2

(2). Draw the graph of the line passing through:

- a. (5, 0) and the gradient is 2
- b. (3, 0); $g=5$
- c. (2, 0); $g= \frac{1}{3}$

(3). Draw graphs of the lines represented by the following equations using the x and y-intercepts.

- a. $y = \frac{1}{2}x + 3$
- b. $3y - 4x = 5$
- c. $y + 2x - 3 = 0$
- d. $y = -2x + 2$

Perpendicular Lines

Perpendicular Lines

- If the products of the gradient of the two lines is equal to - 1, then the two lines are perpendicular to each other.

Example

Find if the two lines are perpendicular

$$y = \frac{1}{3}x + 1; y = -3x - 2$$

Solution

The gradients are

$$M = \frac{1}{3} \text{ and } M = -3$$

The product is

$$\frac{1}{3} \times -3 = -1$$

The answer is -1 hence they are perpendicular.

Example

$$y = 2x + 7$$

$$y = -2x + 5$$

The products of their gradients is $2 \times -2 = -4$ hence the two lines are not perpendicular.

QUESTIONS

- (1). A line L_1 passes through point (1, 2) and has a gradient of 5. Another line L_2 , is perpendicular to L_1 and meets it at a point where $x = 4$. Find the equation for L_2 in the form of $y = mx + c$
- (2). P (5, -4) and Q (-1, 2) are points on a straight line. Find the equation of the perpendicular bisector of PQ:
giving the answer in the form $y = mx + c$.
- (3). The equation of a line $-\frac{3}{5}x + 3y = 6$. Find the:
 - a. Gradient of the line
 - b. Equation of a line passing through point (1, 2) and perpendicular to the given line b
- (5). Find the equation of the perpendicular to the line $x + 2y = 4$ and passes through point (2,1)

Parallel Lines

Parallel Lines

Parallel lines have the same gradients e.g.

$$y = 2x +$$

$$7 \quad y = 2x$$

$$- 9$$

Both lines have the same gradient which is 2 hence they are parallel

QUESTIONS

(1). In each of the following, find the equation of the line through the given point and parallel to the given line:

a. $(0, 0)$; $y = \frac{2}{3}x + 1$

b. $(-3.5, 0)$; $x + y = 10$

c. $(5, 2)$; $5y - 2x - 115 = 0$

d. $(-3, 5)$; $7y = 3x$

e. $(-\frac{1}{3}, \frac{3}{4})$; $2(y - 2x) = 1.1$

f. $(0, -3)$; $2x + y = 3$

g. $(3\frac{1}{2}, -1\frac{1}{2})$; $15(1 - x) = 22y$

h. $(-3, 4)$; $x = 101$

i. $(2, 3)$; $y = 0$

Inequalities

Representation

Introduction

Inequality symbols

■ $>$ Greater Than

■ $\geq$ Greater Than or Equal

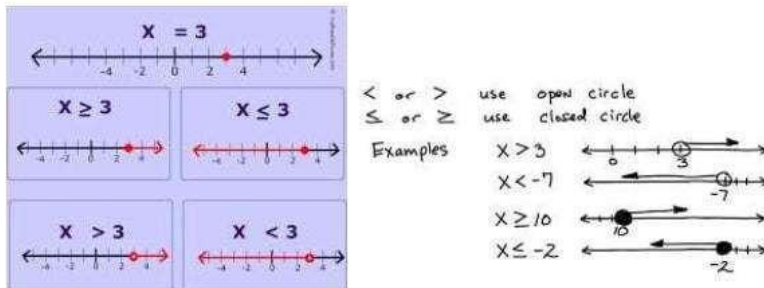
To ■ $<$ Less Than

■ $\leq$ Less Than or Equal To

Statements connected by these symbols are called inequalities

Simple Statements

- Simple statements represents only one condition as follows



- $x = 3$ represents specific point which is number 3
- $x > 3$ does not include 3 it represents all numbers to the right of 3 meaning all the numbers greater than 3 as illustrated above.
- $x < 3$ represents all numbers to left of 3 meaning all the numbers less than 3.
- The empty circle means that 3 is not included in the list of numbers to greater or less than 3.
- The expression $x \geq 3$ or $x \leq 3$ means that means that 3 is included in the list and the circle is shaded to show that 3 is included.

QUESTIONS

Illustrate each of the following inequalities on the number line:

(1).

a. $x < 7$

b. $x > -3$

c. $x \leq 0$

d. $x \leq -5$

(2).

a. $x < -10$

b. $x < -4$

c. $x \geq -6$

d. $x < 2.5$

(3).

a. $x \leq -\frac{1}{2}$

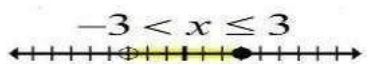
b. $x \leq -2.3$

Compound Statements

Compound Statements

- A compound statement is a two simple inequalities joined by "and" or "or." Here are two

Examples.



- $3 \geq x$ and $x > -3$ Combined into one to form $-3 < x \leq 3$
- All real numbers that are greater than - 3 but less or equal to 3 ■ $x >$

-6 and $x < 3$ forms $-6 < x < 3$



- All real numbers that greater than - 6 but less than 3

QUESTIONS

(1). Write each of the following pairs of simple statements into compound statements and illustrate them on a number line.

a. $x > -4, x \leq 2$

b. $x < 0.5, x > 0$

(2). Write each of the following pairs of simple statements as a compound statement:

a. $x > 2, x < 5$

b. $x \geq 3, x < 6$

c. $x \geq 1, x \geq 7$

d. $x > -4, x \leq 0$

e. $x \geq -3, x \leq -1$

Solving Simple Inequalities

Solution to Simple Inequalities

Example

Solve the inequality

$$x - 1 > 2$$

Solution

Adding 1 to both sides gives;

$$x - 1 + 1 > 2 + 1$$

Therefore, $x > 3$

QUESTIONS

Solve each of the following inequalities and represent your solutions on a number line.

(1).

a. $2x + 4 > 10$

b. $3x - 5 < 2$

(2).

a. $5x + 3 > 4$

19. $3x - 4 \leq -13$

(3).

a. $3x - 7 \geq 5$

b. $1 - 4x \geq 9$

Multiple Inequalities with a Negative Number

Multiplication and Division by a Negative Number

- Multiplying or dividing both sides of an inequality by positive number leaves the inequality sign unchanged ▪

Multiplying or dividing both sides of an inequality by negative number reverses the sense of the inequality sign.

Example

Solve the inequality $1 - 3x < 4$

Solution

$$-3x - 1 < 4 - 1$$

$$-3x < 3$$

$$-3x > 3$$

$$-3 \div -3$$

Note that the sign is reversed $x > -1$

QUESTIONS

Solve the inequalities below

a. $6 - \frac{1}{2}x > 12$

b. $3 - 2x < 17$

c. $\frac{1}{3} - 2x \leq -8\frac{1}{3}$

d. $3(1 - x) + 4(x + 3) \geq 30$

e. $2x + 3 < -1$

f. $-3x - 4 \geq 2$

Simple Simultaneous Inequalities

Simultaneous Inequalities

Example

Solve the following

$$3x - 1 > -4$$

$$2x + 1 \leq 7$$

Solution

Solving the first inequality

$$3x - 1 > -4$$

$$3x > -3$$

$$x > -1$$

Solving the second inequality

$$2x + 1 \leq 7$$

$$2x \leq 6 \text{ Therefore } x \leq 3 \text{ The combined inequality is } -1 < x \leq 3$$

QUESTIONS

Solve the following inequalities

(1).

a.

$$x + 3 > 5$$

$$x - 4 < 4$$

b.

$$x + 10 \geq 6$$

$$x - 2 \leq 3$$

(2).

a.

$$-\frac{1}{2}x - 2 \leq 1$$

$$-3x - 9 > -6$$

b.

$$-5x + 7 < 12$$

$$\frac{1}{3}x + 2 \leq 5$$

(3).

a.

$$-7x - 1 < 6$$

$$\frac{x}{3} + 1 < \frac{4}{3}$$

b.

$$3x - \frac{1}{2} > 4$$

$$x - \frac{1}{5} < \frac{2}{5}x + 1$$

(4).

a.

$$\frac{x}{5} + \frac{1}{3} < 1$$

$$x - \frac{4}{5} > \frac{1}{8}x$$

b.

$$5 \leq 3x + 2$$

$$3x - 14 < -2$$

Compound Simultaneous Inequalities

Compound Simultaneous Inequalities

Simultaneous inequalities can as well be written as a compound statement.

For instance, $x - 4 < 4x < 4$ should be interpreted as:

i. $x - 4 < 4x$

ii. $4x < 4$

Solving the first inequality,

$$x < 4x + 4$$

$$-3x < 4$$

$$x > \frac{4}{3}$$

Solving the second inequality

$$4x < 4$$

$$x < \frac{1}{4}$$

$$x < 1$$

The solution which satisfies both inequalities is, therefore; -

$$\frac{1}{3} < x < 1$$

QUESTIONS

Solve the following inequalities;

(1).

a. $\frac{1}{2} - \frac{1}{4}x \leq x \leq 2$

b. $12 - x \geq 5 \geq 2x - 2$

(2).

a. $-4x < 6 \leq 180x$

b. $3x - 2 \geq -4 < -1 - 2x$

(3).

a. $6x - 13 \leq 17 < 8x - 7$

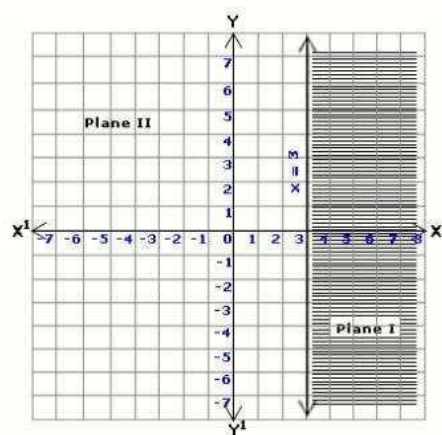
b. $2x + 3 > 5x - 3 > -8$

Graphs of Simple Inequalities

Graphical Representation of Inequality

Consider the following;

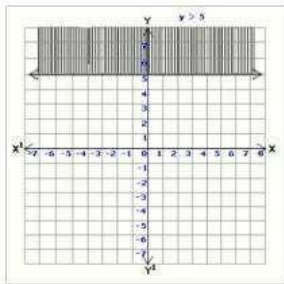
$$x \leq 3$$



- The line $x = 3$ satisfy the inequality ≤ 3 , the points on the left of the line satisfy the inequality. ■ We don't need the points to the right hence we shade it

Note:

- We shade the unwanted region
- The line is continuous because it forms part of the region e.g it starts at 3. For $\leq$ or $\geq$ inequalities the line must be continuous For $<$ or $>$ the line is not continuous, its dotted. This is because the value on the line **does not** satisfy the inequality.



QUESTIONS

Show the regions that satisfy each of the following inequalities on a squared paper:

(1).

a. $x \leq 4$

b. $x > -2$

(2).

a. $x < -1$

b. $y \leq 3$

(3).

a. $y \geq -4$

b. $y \leq 0$

(4).

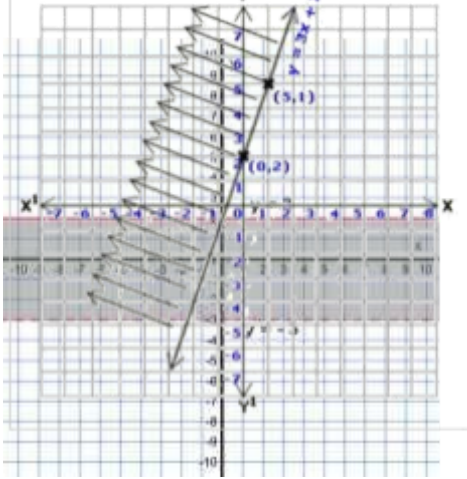
a. $y + 2 < -5$

b. $x + 2 \geq -1$

Graphs of Compound Inequalities

Graphical Representation of Inequality

- A compound inequality can also be represented graphically



QUESTIONS

Represent the following inequalities on the graph

(1).

a. $-1 \leq 3x - 1 < 5$

b. $\frac{x-3}{4} > \frac{x+5}{2}$

(2).

a. $\frac{1}{3} - \frac{1}{5}x < \frac{1}{2}x + 1$

b. $\frac{2}{3}x - 7 + \frac{1}{5}x \geq -1$

(3).

a. $\frac{1}{2}x + \frac{1}{2} > \frac{1}{4}$

b. $x^2 - 4x \geq x(x - 1) - 18$

Linear Inequality of Two Unknowns

Linear Inequality of Two Unknown

- Consider the inequality $y \leq 3x + 2$ the boundary line is $y = 3x + 2$

- If we pick any point above the line eg $(-3, 3)$ then substitute in the equation $y - 3x \leq 2$ we get $12 \leq 2$ which is not true so the values lies in the unwanted region hence we shade that region .

QUESTIONS

Graph each of the following inequalities:

(1).

a. $2x + y > 3$

b. $x - y < 4$

(2).

a. $3x + 2y > 12$

b. $3y + x \leq -5$

(3).

a. $y + 4x < 3$

b. $y - \frac{1}{2}x \geq 1$

(4).

a. $2x > y + 4$

b. $\frac{1}{6}x + \frac{1}{3}y \leq \frac{1}{4}$

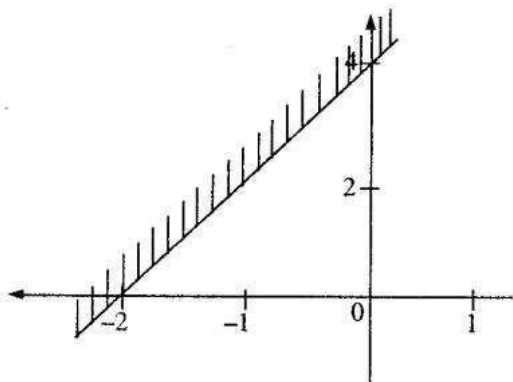
(5).

a. $2y - 3x - 5 \leq 0$

b. $\frac{1}{6}x + \frac{1}{3}y \leq -1$

Reading Inequalities from the Graph

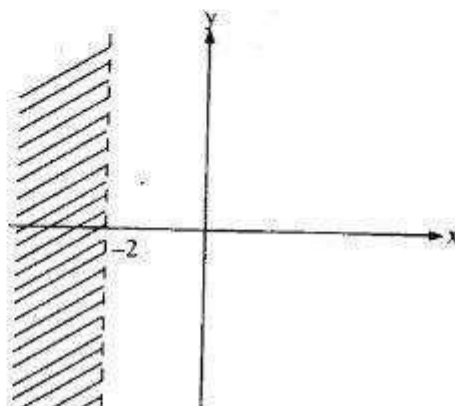
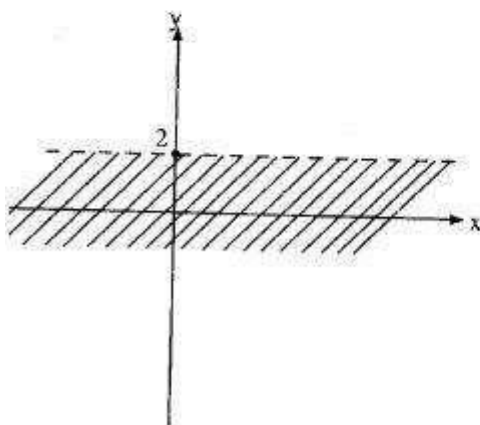
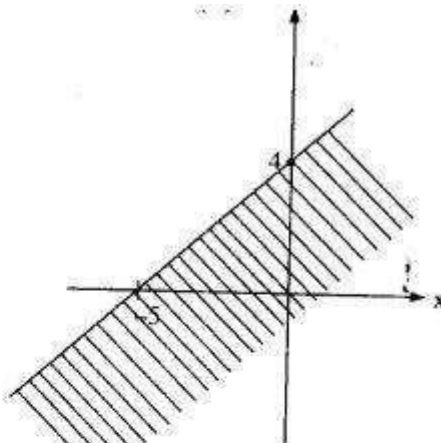
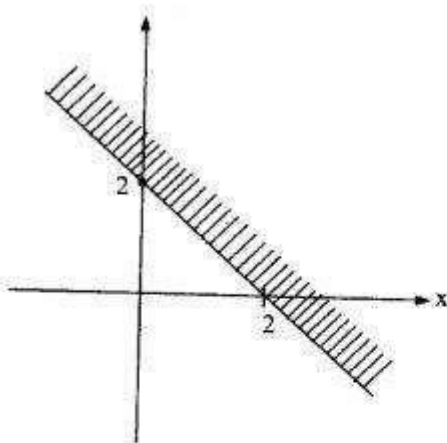
- Given a region satisfied by an inequality, the inequality can be found.



- The equation of the line is $y = 2x + 4$.
- Consider a point in the wanted region such as the origin $(0, 0)$. The x co-ordinate and y co-ordinate are both zero. Substituting these values in the equation $y = 2x + 4$, we get zero on the left hand side and four on the right.
- Since $0 < 4$ and the line $y = 2x + 4$ is continuous, the required inequality is $y < 2x + 4$.

QUESTIONS

(1). Find the inequalities shown



Watch the video for this lesson on the link below.

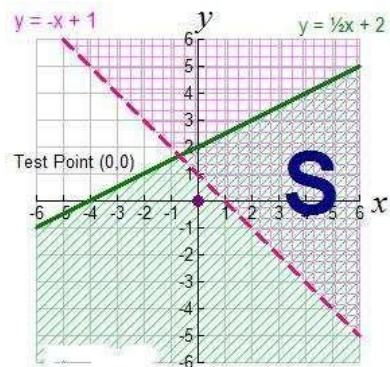
Intersecting Regions

Intersecting Regions

- These are identities regions which satisfy more than one inequality simultaneously.

Example

Draw a region which satisfy the following inequalities $y + x \geq 1$ and $y - \frac{1}{2}x \geq 2$



QUESTIONS

In each of the questions, draw the regions which satisfy all the inequalities.

(1).

a.

$$x + y \geq 0$$

$$x < 2$$

$$y > 0$$

b.

$$2x + y \geq 6$$

$$x < 3$$

$$y < 6$$

(2).

a.

$$4x - 3y \leq 12$$

$$x > 0$$

$$y > 0$$

b.

$$4x - 3y < 12$$

$$y \geq 0$$

$$y \leq 6$$