KONGUNADU COLLEGE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

SUBJECT NAME: DISCRETE MATHEMATICS

SUBJECT CODE: MA8351

UNIT – I

LOGIC AND PROOFS

1. **Prove that** $P, P \to Q, Q \to R \Rightarrow R$.

Solution:

Step	Premises	Rule	Reasons
1	$P \to Q$	P	Given Premises
2	P	P	Given Premises
3	Q	T	(1),(2), $P \to Q, P \Rightarrow Q$
4	$Q \to R$	P	Given Premises
5	R	T	(3),(4), $Q, Q \to R \Rightarrow R$

2. **Without using truth table show that** $P \to (Q \to P) \Leftrightarrow \neg P \to (P \to Q)$

Solution:

L.H.S: $P \to (Q \to P)$

Step	Premises	Reasons

1	$P \rightarrow (Q \rightarrow P)$	Conditional as disjunction
2	$\Leftrightarrow \neg P \vee (\neg Q \vee P)$	Commutative law
3	$\Leftrightarrow (\neg Q \vee P) \vee \neg P$	Associative law
4	$\Leftrightarrow (\neg Q \vee (P \vee \neg P))$	$P \vee \neg P \leftrightarrow T$
5	$\Leftrightarrow (\neg Q \vee T)$	$P \vee T = T$
6	T ……….(1)	

R.H.S: $(\neg P) \rightarrow (P \rightarrow Q)$

Step	Premises	Reasons
1	$\neg(\neg P) \vee (\neg P \vee Q)$	Conditional as disjunction
2	$\Leftrightarrow P \vee (\neg P \vee Q)$	Double negation law
3	$\Leftrightarrow P \vee (\neg P \vee Q)$	Associative law
4	$\Leftrightarrow (P \vee \neg P) \vee Q$	$P \vee \neg P \leftrightarrow T$
5	$\Leftrightarrow (T \vee Q)$	$P \vee T = T$

6	T ……….(2)	

From (1) &(2), we get

$$P \to (Q \to P) \Leftrightarrow (\neg P) \to (P \to Q)$$

3. Show that $P \to (Q \to R) \to ((P \to Q) \to (P \to R))$ **is a tautology.**

Solution: Let $A = P \to (Q \to R) \to ((P \to Q) \to (P \to R))$

$B = (P \to Q) \to (P \to R)$

$C = P \to (Q \to R)$

P	Q	R	$Q \to R$	$P \to Q$	$P \to R$	C	B	A
T	T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	T	F	T	T	T	T
T	F	F	T	F	F	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

Since the column of the truth table contains only T's

$P \to (Q \to R) \to ((P \to Q) \to (P \to R))$ is a tautology.

Which need not be a tautology.

4. Explain the two types of quantifiers through example.

Solution:

Universal Quantifiers	Existential Quantifiers
• The expression "all" is the universal quantifiers. We denote it by $(\forall x)$ .The symbol $(\forall x)$ represents each of the following phrases, having the same meaning as "all"	• The expression "somel" is the Existential quantifiers. We denote it by $(\exists x)$ .The symbol $(\exists x)$ represents each of the following phrases, having the same meaning 'some'.
• For all x	• For some x
• For every x	• For x such that
• For each x	• There exists an x such that
• Every thing x is such that	• There is an x such that
• Each thing x is such that	• There is atleast one x such that

5. Given an indirect proof of the theorem "if 3n+2 is odd, then n is odd".

Solution:

P: 3n+2 is odd

Q: n is odd

Hypothesis: Assume that $P \to Q$ is false.

(ie) Assume that P is true and Q is false. (ie) n is not odd $\Rightarrow$ n is even.

Analysis: If n is even then n= 2k for some integer k.

$3n+2 = 3(2k)+2= 6k+2 = 3(3k+1)$.

Conclusion: We observe that the R.H.S value of 3n+2 is divisible by 2. This means that 3n+2 is even. This contradicts the assumption P is true. In view of this contradiction, we infer that the given conditional $P \to Q$ is true.

6. **What are the contrapositive, the converse, and the inverse of the conditional statement.'If you work hard then you will be rewarded".**

Solution:

P : You work hard $\neg P$: You will not work hard.

Q : You will be rewarded $\neg Q$: You will not be rewarded.

Converse: $Q \to P$,You will be rewarded only if you work hard.

Contrapositive: $\neg Q \to \neg P$,If you will not be rewarded then you will not work hard.

Inverse: $\neg P \to \neg Q$,If you will not work hard then you will not be rewarded.

7. **Is** $\neg p \wedge (p \vee q) \to q$ **a tautology.**

Solution: To Prove $\neg p \wedge (p \vee q) \to q \Leftrightarrow T$

$$\neg p \wedge (p \vee q) \to q \Leftrightarrow \neg(\neg p \wedge (p \vee q) \vee q \quad \text{(conversion formula)}$$

$$\Leftrightarrow p \vee \neg(p \vee q) \vee q$$

$$\Leftrightarrow \neg(p \vee q) \vee (p \vee q) \quad \text{(commutatively)}$$

$$\Leftrightarrow T$$

8. **Let** $E=\{-1,0,1,2\}$ **denote the universe of discourse. If** $p(x,y): x+y=1$ **,find the truth value of** $(\forall x)(\exists y)\, p(x,y)$ **.**

Solution:

Given p(x,y): x+y=1 and the inverse of discourse is $E=\{-1,0,1,2\}$

To find $(\forall x)(\exists y)\, p(x,y)=\forall x\ \exists y\,(x+y=1)$ is T.

If x=-1 then y=2 (ie) $\exists\, y=2$

If x=0, then y=1 (ie) $\exists\, y=1$

If x=1, then y=0 (ie) $\exists\, y=0$

If x=2, then y=-1 (ie) $\exists\, y=-1$

$\therefore (\forall x)(\exists y)\,(x+y=1)$ is true.

$\therefore$ the truth value of is T.s

9. **Show that** $(p\to r)\wedge(q\to r)$ **and** $(p\vee q)\to r$ **s are logically equivalent.** **(N/D 2014)**

Solution:

$$(p\to r)\wedge(q\to r)\equiv(\neg p\vee r)\wedge(\neg q\vee r)\equiv X \text{ , say}$$

$$(p\vee q)\to r\equiv\neg(p\vee q)\vee r\equiv(\neg p\wedge\neg q)\vee r\equiv Y \text{ , say}$$

We shall prove $X\equiv Y$ s by forming truth table

p	q	r	$\neg p$	$\neg q$	$\neg p \vee r$	$\neg q \vee r$	X	$\neg p \wedge \neg q$	Y
T	T	T	F	F	T	T	T	F	T
T	T	F	F	F	F	F	F	F	F
T	F	T	F	T	T	T	T	F	T
T	F	F	F	T	F	T	F	F	F
F	T	T	T	F	T	F	F	F	F
F	T	F	T	T	T	T	T	T	T
F	F	T	T	F	T	T	T	F	T
F	F	F	T	T	T	T	T	T	T

Since the columns of X and Y have the truth values, they are logically equivalent (ie) $X \equiv Y$.

10. Find a counter example. If possible, to these universally quantified statements, whose universe of discourse for all variables consists of all integers.

(a) $\forall x \ \forall y \ (x^2 = y^2 \rightarrow x = y)$

(b) $\forall x \ \forall y \ (xy \geq x)$ **s**

Solution:

(a) $x = 3, \ y = -3 \ (\therefore x^2 = y^2 \ , but \ \ x \neq y)$

(b) $x = 5, \ y = -2 \ (\therefore xy = -10 \ , but \ \ xy \neq\Box \ x)$

11. Construction a truth table for the compound proposition $(p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$

Solution:

$(p \to q) \equiv (\neg p \vee q)$ and $\neg p \to \neg q \equiv p \vee \neg q$

$\therefore (\neg p \vee q) \leftrightarrow (p \vee \neg q) \equiv P$, say

The truth table is shown here

p	q	$\neg p$	$\neg q$	$\neg p \vee q$	$p \vee \neg q$	P
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	T
F	F	T	T	T	T	T

12. Define functionally complete set of connectives and given an example .

Solution:

Any set of connective in which every formula can be expressed an another equivalent formula containing connectives from this set is called functionally complete set of connective.

(OR)

A collection of logical operators is called functionally complete if every compound proposition is logically equivalent to a compound proposition involving only these logical operators.

Example: The set of connectives { $\wedge, \neg$ } and { $\vee, \neg$ } are functionally complete $\{\neg\}, \{\vee\}, \{\wedge\}$ or { $\wedge, \vee$ } are not functionally complete.

Note : From the five connectives $\wedge, \vee, \neg, \to, \leftrightarrow$. We have obtained at least two sets of functionally complete connectives.

UNIT II

COMBINATORICS

1. Find the recurrence relation of the sequence $s(n) = a^n : n \geq 1$

Solution:

$$S(n) = a^n;\ n \geq 1$$

$$S(n-1) = a^{n-1}$$

$$S(n-1) = \frac{a^n}{a}$$

$$aS(n-1) = S(n)$$

$\therefore$ The recurrence relation is $S(n) - a\,S(n-1) = 0\ ;\ n \geq 1$

2. How many bit strings of length ten contain (i) exactly four 1's (ii) at least four 1's?

Solution:

(a) A bit string of length 10 can be considered to have 10 positions. These 10 positions should be filled with four 1's and 0's.

$\therefore$ No of required bit strings $= \frac{10!}{4!6!} = 210$

(b) The ten positions are to be filled up with 4, 1's and 6 , 0's (or) 5, 1's and 5, 0's etc (or) ten 1's and no 0's.

$\therefore$ No of required bit strings $= \frac{10!}{4!6!} + \frac{10!}{5!5!} + \frac{10!}{6!4!} + \frac{10!}{7!3!} + \frac{10!}{8!2!} + \frac{10!}{9!1!} + \frac{10!}{10!0!} = 848$

3. Use Mathematical induction to solve that $1+2+3+.....+n=\frac{n(n+1)}{2}$

Solution:

Let $P(n)$ denote the Proposition(or equation) $1+2+3+.....+n=\frac{n(n+1)}{2}$

We have to prove that $P(n)$ is true for all $n\geq 1$.

Basic step:

Here $n_0=1$

$\therefore$ P(1) is the proposition.

$\therefore$ P(1) is $1=\frac{1.(1+1)}{2}\Rightarrow 1=1$. Which is true So, P(1) is true.

Inductive step: Assume P(k) is true (k>1)

$\Rightarrow 1+2+3+....+k=\frac{k(k+1)}{2}$ is true ………………..(1)

To Prove: P(k+1) is true

To prove $1+2+3+....+k+(k+1)=\frac{(k+1)(k+2)}{2}$ is true

$$\Rightarrow (1+2+3+....+k)+(k+1)=\frac{k(k+1)}{2}+(k+1)$$

L.H.S: $=(k+1)\left(\frac{k}{2}+1\right)=\frac{(k+1)(k+2)}{2}=R.H.S$

$\therefore$ P(k+1) is true. Thus P(n) is true $\Rightarrow$ P(k+1) is true. Hence by the first principle of induction

P(n) is true $\forall n\geq 1 \Rightarrow 1+2+3+.....+n=\frac{n(n+1)}{2}$

4. **How many ways a 2xn rectangular board be tiled using 1x2 and 2x2 pieces?**

Solution:

Let a_n be the number of ways of the 2xn rectangle be tiled by 1x2 and 2x2 tiles

If $n = 1,\ a_1 = 1$, since only one 1x2 tile

If $n = 2,\ a_2 = 2\,(1\times 2)\,(or)\,1(2\times 2) = 2+1 = 3$ ways

If $n = 3,\ a_3 = 3\,(1\times 2)\,(or)\,1(2\times 2)\,and\,1(1\times 2) = 3+2 = 5$ ways

If $n = 4,\ a_4 = 4\,(1\times 2)\,(or)\,2(2\times 2)\,(or)\,1(2\times 2)\,and\,2\,(1\times 2) = 5+2+4 = 11$ ways

If $n = 5,\ a_5 = 5\,(1\times 2tiles)\,(or)\,2(2\times 2)\,and\ 1(1\times 2)\,(or)\,1\,(2\times 2)\ and\,3(1\times 2) = 9+3+9 = 21$ ways

And so on.

$\therefore\ a_n$ is the n[th]

Term of the sequence 1,3,5,11,21……….

5. **State the principle of strong induction?**

Solution:

It is sometimes convenient to replace the induction hypothesis P(k) by the stronger assumption P(1), P(2), P(3),…..P(k) are true.

The resulting principle known as the principle of strong mathematical induction.

Step 1: Inductive base: To prove P(1) is true.

Step: 2 Strong Inductive hypothesis: Assume that P(n) is true for all integers $1 \le n \le k$

Step:3 Inductive step: To Prove that P(k+1) is true on basis of the strong inductive hypothesis.

6. **Find the recurrence relation satisfying the equation** $y_n = A(3)^n + B(-4)^n$

Solution:

Given : $y_n = A(3)^n + B(-4)^n$ ………………(1)

$$y_{n+1} = A(3)^{n+1} + B(-4)^{n+1} \quad \text{………..(2)}$$

$$y_{n+2} = A(3)^{n+2} + B(-4)^{n+2}$$

$$= 9\,A(3)^n + 16B(-4)^n \quad \text{…………(3)}$$

Eliminating A and B from (1), (2) and (3), we get

$$\begin{vmatrix} y_n & 1 & 1 \\ y_{n+1} & 3 & -4 \\ y_{n+2} & 9 & 16 \end{vmatrix} = 0$$

$$y_n(48+36) - 1(16y_{n+1} + 4y_n + 2) + 1(9y_{n+1} - 3y_{n+2}) = 0$$

$$84y_n - 16y_{n+1} - 4y_{n+2} + 9y_{n+1} - 3y_{n+2} = 0$$

$$84y_n - 7y_{n+1} - 7y_{n+2} = 0$$

$$12y_n - 7y_{n+1} - y_{n+2} = 0$$

$$y_{n+2} + y_{n+1} - 12y_n = 0$$

7. **If seven colours are used to paint 50 bicycles, then show that atleast 8 bicycles will be the same colour**

Solution:

Here Number of Pigeon=m = Number of bicycle = 50

Number of Holes = n = Number of colours = 7

By generalized Pigeon Hole principle, we get

$$\frac{50-1}{7}+1=8$$

Atleast 8 bicycles will have the same colour.

8. **Solve the recurrence relation y(k)-8y(k-1)+16y(k-2)=0 for $k \geq 2$,where y(2)=16 and y(3)=80**

Solution:

The recurrence relation can be written as

$$y_k - 8y_{k-1} + 16y_{k-2} = 0$$

The characteristic equation is

$$r^2 - 8r + 16 = 0$$

$$(r-4)^2 = 0$$

$$\Rightarrow r = 4, 4$$

$\therefore$ The solution is $y(k) = (\alpha_1 + \alpha_2 k)4^k$ ……………..(1)

Given $y_2 = 16$

Put k=2, in (1), we get

$$y(2) = (\alpha_1 + \alpha_2 2)4^2 = 16$$

$$16(\alpha_1 + 2\alpha_2) = 16$$

$(\alpha_1 + 2\alpha_2) = 1$ ………………..(2)

Put k=3, in (1), we get

$$y(3)=(\alpha_1+\alpha_2.3)4^3=80$$

$$64(\alpha_1+3\alpha_2)=80$$

$$(\alpha_1+3\alpha_2)=\frac{80}{64}=\frac{5}{4} \quad \ldots\ldots\ldots\ldots\ldots..(3)$$

Solving (2) and (3), we get

$$\alpha_1=\frac{1}{2},\ \alpha_2=\frac{1}{4} \quad \ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Substituting (4) in (1), we get

$$y(k)=\left(\frac{1}{2}+\frac{1}{4}k\right)4^k$$

$$=(2+k)4^{k-1}$$

Which is the requited solution

9. Find the recurrence relation satisfying the equation $y(k)=A(3)^n+B(-4)^n$

Solution:

Given: $y(k)=A(3)^n+B(-4)^n$ ……………….(1)

$$y_{n+1}=A3^{n+1}+B(-4)^{n+1}$$

$$y_{n+1}=3A3^n-4B(-4)^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

$$y_{n+2}=A3^{n+2}+B(-4)^{n+2}$$

$$=9A3^n+16B(-4)^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Eliminating A and B from (1),(2) and (3), we get

$$\begin{vmatrix} y_n & 1 & 1 \\ y_{n+1} & 3 & -4 \\ y_{n+2} & 9 & 16 \end{vmatrix} = 0$$

$$y_n(48+36) - 1(16y_{n+1} + 4y_n + 2) + 1(9y_{n+1} - 3y_{n+2}) = 0$$

$$84y_n - 16y_{n+1} - 4y_{n+2} + 9y_{n+1} - 3y_{n+2} = 0$$

$$84y_n - 7y_{n+1} - 7y_{n+2} = 0$$

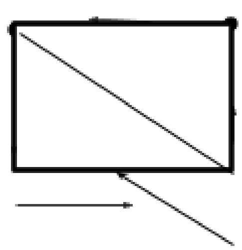
$$12y_n - y_{n+1} - y_{n+2} = 0$$

$$y_{n+1} + y_{n+2} - 12y_n = 0$$

Unit – III

GRAPHS

1. **Is the directed graph given below strongly connected? why or why not?**



Solution:

It is strongly connected graph.

For, the possible pairs of vertices of the graph are (V_1, V_2) $(V_{1,}V_3)$ $(V_{1,}V_4)$ $(V_{2,}V_3)$ $(V_{2,}V_4)$and $(V_{3,}V_4)$

i) Consider the pair (V_1, V_2)

Then there is a path from $V_{1\rightarrow} V_2$ and path from $V_2 \rightarrow V_1$ via $V_{2\rightarrow} V_{3\rightarrow} V_1$

ii) Consider the pair (V_1, V_2)

Then there is a path from $(V_1 to V_3)$,via $V_{1\rightarrow} V_{2\rightarrow} V_3$and path from $V_{3\rightarrow} V_{1.}$ Similarly we can prove 1 for the remaining pair of vertices each vertices is reachable from other.

Therefore given graph is strongly connected.

2. Represent the graph using an adjacency matrix

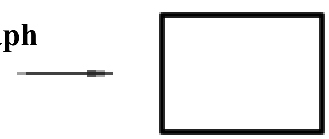
0 1 0 1 0 1 0 1 0 0 0 1

1 0 1 0

Solution:

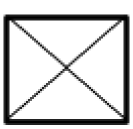
The adjacency matrices are 0 1 0 1 0 1 0 1 0 0 0 1 **Graph**

1 0 1 0



3. **Give an example of a non eulerian graph which is Hamiltonian.**

Solution:



$deg(v_1)= deg(v_2)= deg(v_3) =deg(v_4)=3$

Here the vertices are not even degree .therefore given is non eulerian graph .

$deg(v_1)+ deg(v_2)$ n -1

$deg(v_2)+ deg(v_3)$ n -1

$deg(v_3)+ deg(v_4)$ n -1

$deg(v_4)+ deg(v_1)$ n -1

The given graph is hamiltonian.the Hamiltonian circuit is v_1, v_2, v_3, v_4, v_1.

4. State the handshaking theorem:

Solution:

The sum of all vertex degree is equal to twice the number of edges or the sum of the degrees of the vertices of G is even. Let W be the set of vertices of odd degree and U be the set of vertices of even degree . then $\sum_{v \in U} \deg deg\ v = \sum_{v \in W} \deg deg\ v + \sum_{v \in U} \deg deg\ v = 2\,|E|$

But $\sum_{v \in U} \deg deg\ v$ is even

Hence $\sum_{v \in W} \deg deg\ v$ is even.

5. Define isomorphism between two graphs.

Solution:

Isomorphism of graphs : let $G_1 = (V(G_1), E(G_1))$ and $G_2 = (V(G_2), E(G_2))$ be two graphs.

Afunction f: $G_1 \Rightarrow G_2$ is called on isomorphism if,

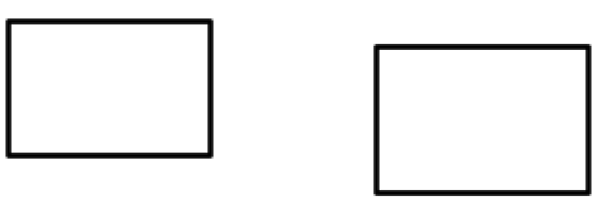
i) F is one to one

ii) F is onto

iii) $(x,y) \in E(G_1)$ iff $f(x), f(y) \in E(G_2)$ two vertices x and y are adjacent in G_1 if f(x) and f(y) are adjacent in G_2. if the graph G_1 is isomorphic to G_2 then we write $G_1 \approx G_2$.

6. Give an example of an euler graph

Solution:

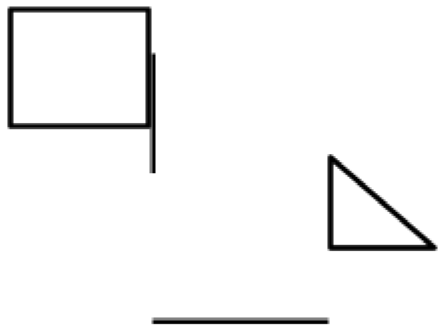


n =10 , e = 13 , f= 5

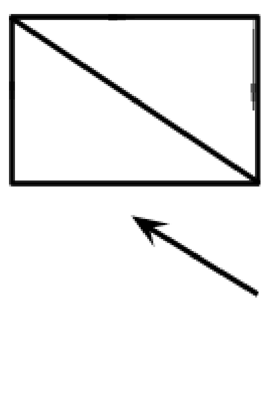
n – e+ f = 2

7. Give an example of a non Eulerian graph which is Hamiltonian.

Solution:



8. **Is the directed graph given below strongly connected? why or why not?**



Solution:

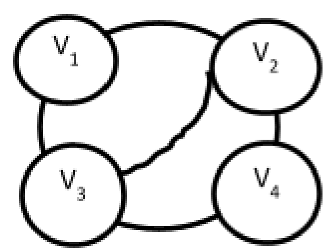
It is strongly connected because for any two vertices u and v there is a path from u to v from u to v.

9 **Draw a graph represented by the given adjaceny matrix**

0 1 1 0 0 1 1 0 0 1 0 1 1 0 1 0

Solution: The given adjacently matrix is 4x4 ans so the graph has 4 vertices $v_{1,}v_2,v_3,v_4$ say. Then A=

0 1 1 0 0 1 1 0 0 1 0 1 1 0 1 0



Type equation here.

10. PT identity element in a group is unique

Solution : property 1: The identity of a group is unique (or) if (G,*) is a group and e is an identity of G, then no other element of G is an identity of G.

Proof : suppose that e_1 and e_2 are two identities of the group (G,*)

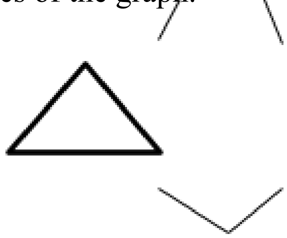
Now e_1 is the identity then $e_1 * e_2 = e_2 * e_1 = e_1$

Again e_2 is then identity then $e_2 * e_2 = e_1 * e_1 = e_1$

The identity is unique.

11. Define a connected and disconnected graph with example.

Solution: A graph G is connected if there is a path between any two of its vertices. otherwise it is disconnected. An undirected graph is connected graph is connected if there is a path between every pair of distinct vertices of the graph.



Connected graph

Disconnected graph

12. **. How many edges are there in a graph with 10 vertices each of degree 5?**

Solu:

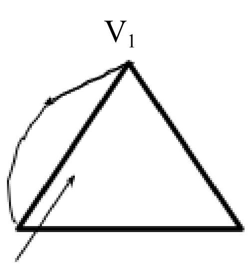
The sum of the degrees of the vertices is $10 \cdot 5 = 50$.

The handshaking theorem says $2m = 50$.

So the number of edges is $m = 25$.

13. **Draw a graph with the following adjacency matrix** 0 1 0 1 0 1 0 1 0

Solu:



25.How many edges are there in a graph with 10 vertices each of degree 3?

Solu:

The sum of the degrees of the vertices is $10 \cdot 3 = 30$.

The handshaking theorem says $2m = 30$.

So the number of edges is $m=15$.

UNIT IV

ALGEBRAIC STRUCTURES

1. **Show that every cyclic group is abelian.**

   Solution:

   Let (G, *) be a cyclic group generated by an element $a \in G$.

   (ie) G = (a)

   Then for any two elements $x, y \in G$

   We have $x = a^n , y = a^m$, where m,n are integer.

$\therefore \quad x * y = a^n * a^m = a^{n+m} = a^{m+n} = a^m * a^n = y * x$

Thus, (G,*)is abelian.

2. **Find the idempotent elements of** $G = \{1, -1, i, -i\}$ **under the binary operation multiplication.**

Solution: **(N/D 2016)**

⊠	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	I
I	i	-i	-1	1
-i	-i	i	1	-1

Here 1 is the identity element.

3. **Prove that identity element in a group is unique.** **(N/D 2015,M/J2014)**

Solution:

Let e_1 and e_2 be two identity elements of G.

$e_1 * e_2 = e_2$ (if e_1 as identity) and

$e_1 * e_2 = e_1$ (if e_2 as identity)

$\therefore e_1 = e_2$

4. **Prove or disprove, "Every subgroup of an abelian group is normal".** **(N/D 2013)**

Solution:

If G is a abelian, then every subgroup of G is normal in G,

$(as\ Ha = \{ha/h \in H\} = \{ah/h \in H\}$ since ha=ah= aH, for all $a \in G)$.

5. **Prove that if G is abelian group, then for all** $a,b \in G\ (a*b)^2 = a^2*b^2$ **(M/J 2013,A/M 2011)**

Solution:

Let as assume that G is abelian. Hence, for $a,b \in G$. We have $a*b = b*a$

Now $a^2*b^2 = (a*a)*(b*b)$

$= a*[a*(b*b)]$

$= a*[(a*b)*b]$

$= a*[(b*a)*b]$

$= a*[b*(a*b)]$

$= (a*b)*(a*b)$

$= (a*b)^2$

$\therefore (a*b)^2 = a^2*b^2$

Conversely, assume that $(a*b)^2 = a^2*b^2$

$\Rightarrow (a*b)*(a*b) = (a*a)*(b*b)$

$\Rightarrow a*[b*(a*b)] = a*[a*(b*b)]$

$\Rightarrow b*(a*b)=a*(b*b)$ (left cancellation law)

$\Rightarrow (b*a)*b=(a*b)*b$

$\Rightarrow b*a=a*b$ (right cancellation law)

$\Rightarrow$ G is abelian.

6. **Prove that the identity of a subgroup is the same as that of the group. (N/D 2012)**

Solution:

Let e be the identity element of a group G and

Let e’ be the identity element of a sub group H of G

Let $a \in H \Rightarrow a*e'=a$ ....(1)

Let $a \in H \Rightarrow a \in G \Rightarrow a*e=a$ ....(2)

From (1) and (2) we get

$a*e'=a*e \Rightarrow e'=e$

Therefore, the identity of a subgroup is the same as that of the group.

7. **If ‘a’ is a generator of a cyclic group G, then show that $'a^{-1}'$ is also a generator of G.**

Solution: **(M/J 2012)**

Let G=(a) be a cyclic generated by ‘a’

If $x \in G$, then $x = a^n$ for some $n \in Z$

$\therefore x = a^n = (a^{-1})^{-n}, (-n \in Z)$

$\therefore 'a^{-1}'$ is also a generator of G.

8. **Define homomorphism and isomorphism between two algebraic systems. (N/D 2011)**

Solution:

Homomorphism:

If $\{X,\bullet\}$ and $\{Y,*\}$ are two algebraic systems, where $\bullet$ and * are binary (n-ary) operations, then a mapping $g:X\to Y$ is called a homomorphism if for any $x_1,x_2\in X$.

$$g(x_1\bullet x_2)=g(x_1)*g(x_2)$$

If a function g satisfying the above condition exists, then $\{Y,*\}$ is called the homomorphic image of $\{X,\bullet\}$, even though $g(X)\subseteq Y$.

Isomorphism:

If $g:\{X,*\}\to(Y,*\}$ is one to one, onto, then g is called an isomorphism. In this case the algebraic systems $\{X,\bullet\}$ and $\{Y,*\}$ are said to be isomorphic.

9. **Obtain all the distinct left-cosets of {[0], [3]} in the group $(Z_6,+_6)$ and find their union.**

Solution: **(N/D 2010)**

Let $Z_6=\{[0],[1],[2],[3],[4],[5],[6]\}$ be a group and $H=\{[0],[3]\}$ be a sub group of Z_6 under $+_6$ (addition mod 6)

The left cosets of H are

$[0]+H=\{[0],[3]\}=H$

$[1]+H=\{[1],[4]\}$

$[2]+H=\{[2],[5]\}$

$[3]+H=\{[3],[6]\}=\{[3],[0]\}=\{[0],[3]\}=H$

$[4]+H=\{[4],[7]\}=\{[4],[1]\}=[1]+H$

$[5]+H=\{[5],[8]\}=\{[5],[2]\}=[2]+H$

$\therefore [0]+H=[3]+H=H$

And $[1]+H=[4]+H, [2]+H=[5]+H$ are the distinct left cosets of H in Z_6

10. Show that the set of all elements a of a group (G,*)such that a*x=x*a for every $x \in G$ is a subgroup of G. **(N/D2010)**

Solution:

Let $H=\{a \in G \setminus ax = xa, \forall x \in G\}$

As $ey=ye=y, \forall y \in G, e \in G,$ H is a non empty.

Let x and z in H

Then $xy=yx \; and \; zy=yz \; \forall y \in G$

$(xz)y=x(yz) \Rightarrow (yx)z=y(xz), \forall y \in G$

$\therefore xz \in H, \forall x, z \in H$

$x \in H \Leftrightarrow xy=yx,$

$\Leftrightarrow xy=yx, \quad \forall y \in G$

$$\Leftrightarrow x^{-1}(xy)x^{-1} = x^{-1}(yx)x^{-1}, \qquad \forall y \in G$$

$$\Leftrightarrow (x^{-1}x)(yx^{-1}) = (x^{-1}y)(xx^{-1}),$$

$$\Leftrightarrow yx^{-1} = x^{-1}y,$$

$$\Leftrightarrow x^{-1} \in H$$

Therefore , H is a subgroup.

11. Let <M, *, e_M> be a monoid and $a \in M$. If a invertible, then show that its inverse is unique.

Solution: **(A/M 2011)**

Let b and c be elements of M such that $a*b = b*a = e$ and

$a*c = c*a = e$ since b=b*e = b*(a*c) = (b*a)*c = e*c =

UNIT V

LATTICE AND BOOLEAN ALGEBRA

1. Define lattices homomorphism

Solution:

Let $(L_1,\wedge,\vee)$ and $(L_2,*,\oplus)$ be given lattices.A mapping f: L_1 -> L_2 is called lattices homomorphism if for all a,b $\in$ L_1.

i) f(a ∧ b) =f(a).f(b)

ii) f(a v b) =f(a) ⊕f(b) a homomorphism which is 1-1 is called an isomorphism.

2. Prove the Boolean identity a.b +a.b' =a .

Solution:

To prove $a.b + a.b' = a$.

Consider LHS

$a.b + a.b' = a\,(b+b')$

$a\,(b+b') = 1.a$ (since complement laws $(b+b') = 1$)

$= a$ RHS

Hence proved.

3. **Is a Boolen algebra contains five elements?justify your answer.**

Solution:

No, there is no boolean algebra with five elements.

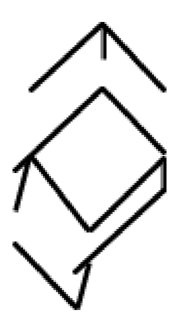
Stone's representation theorem state that any boolean algebra is isomorphic to power set algebra $\rho(s)$.

∴ The element is Boolean algebra should be of the form 2^n

4. **Let A ={ a,b,c} and P(A) be its power set.Draw a hasse diagram of < P(A) ,⊆ >.**

Solution:

Let A = {a,b,c} be a given set and P(A) be its power set.Let ⊆ be the inclusion relation on the elements of P(A) .Clearly (P(A),⊆) is a poset .The hasse diagram is given by



For the subset B = { {b,c},{b},{c}} the upper bounds are {b,c}and {a,b,c} and φ its lower bound

For the subset C = { {a,c} ,{c}} the upper bounds are {a,c} and {a,b,c} while the lower bounds are {c} and φ

5. PT X ={ 1,2,3,4,6,24} and R be a division relation. Find the hasse diagram of the poset < X,R >

6. L et X ={ 1,2,3,4,5,6} and R be a relation define as < X,Y> ε R. Iff x – y is divisible by 3. Find the elements of a relation R

(iv) R = {(x, y): $x - y$ is an integer}

Now, for every $x \in$ **Z**, (x, x) ∈R as $x - x = 0$ is an integer.

∴R is reflexive.

Now, for every x, $y \in$ **Z** if (x, y) ∈ R, then $x - y$ is an integer.

⇒ $-(x - y)$ is also an integer.

⇒ $(y - x)$ is an integer.

∴ (y, x) ∈ R

∴R is symmetric.

7.ST the absorption laws are valid a Boolean algebra .

Absorption Laws for Boolean Algebra

$A + A.B = A$ Proof from truth table,

Inputs			Output
A	B	AB	A+A.B
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

$$A + A.B = A(1 + B) = A$$

Both A and A+A.B column is same. $Similarly,\ A(A + B) = A$ Proof from truth table,

A B A+B A.X(A+B)

0 0 0	0
0 1 1	0
1 0 1	1
1 1 1	1

Both A and A.X or A(A+B) column are same.

$A(A+B) = A.A + AB = A + A.B = A(1+B) = A$ De Morgan's Therem,

$$\overline{A+B} = \overline{A}\ \overline{B}$$
$$and\ \ \overline{AB} = \overline{A} + \overline{B}$$