

# CS 3500 - Programming Languages and Translators

## Axiomatic Semantics : Hoare Logic

**Axiom of Assignment:**

$$\frac{}{\{P_{x \rightarrow E}\} x := E \{P\}}$$

**Rule of consequence:**

$$\frac{P' \subseteq P, \{P\} S \{Q\}, Q \subseteq Q'}{\{P'\} S \{Q'\}}$$

**Rule of composition:**

$$\frac{\{P\} S1 \{Q\}, \{Q\} S2 \{R\}}{\{P\} S1; S2 \{R\}}$$

**Conditional Rule:**

$$\frac{\{C \wedge P\} S1 \{Q\}, \{\neg C \wedge P\} S2 \{Q\}}{\{P\} \text{if } C \text{ then } S1 \text{ else } S2 \{Q\}}$$

**While Rule:**

$$\frac{\{C \wedge P\} S \{P\}}{\{P\} \text{while } C \text{ do } S \text{ end } \{P \wedge \neg C\}}$$

## Applying the Axioms of Assignment and Consequence

### Example:

Compute the precondition of the following assignment:

$$\{ ? \} a := 2 * (b - 5) \{ a < 10 \}$$

Applying the Axiom of Assignment we get  $\{ 2 * (b-5) < 10 \}$ , which can be simplified to  $\{ b < 10 \}$

### Example:

Compute the precondition of the following sequence of statements:

$$\{ ? \} a := 2*b+1; \quad b := 4*a - 2 \{ b > 10 \}$$

Applying the Axiom of Assignment, the precondition of  $b := 4*a - 2$  is  $\{ 4*a - 2 > 10 \}$ , which can be simplified to  $\{ a > 3 \}$ .

Applying that postcondition to  $a := 2*b+1$  yields  $\{ 2*b+1 > 3 \}$ , which can be simplified to  $\{ b > 1 \}$ .

So, by the composition rule the precondition for this sequence of statements is  $\{ b > 1 \}$ .

**END.**