Probability

1) A fair coin is flipped 4 times. We are interested in the number of heads flipped.

a) List the sample space, Ω

b) Find $n(\Omega)$, the number of events in the sample space.

b) Let *A* be the event of flipping 2 heads. Find *P*(*A*}, the probability of A occurring.

c) Let *B* be the event of flipping more than one head. Find $P(B)$.

2a) Two fair 6-sided dice are rolled. List the sample space Ω and find $n(\Omega)$

b) Find the probability that the maximum of the two numbers rolled is 4.

c) Find the probability that the minimum of the two numbers rolled is 4.

d) Find the probability that the sum of the two numbers is 7.

e) Find the probability that the sum of the two numbers is less than 7.

3) Roll a fair 6-sided fair die. Let *A* be the event of rolling an even number, i.e, $A = \{2,\ 4,\ 6\}$, and *B* is the event of rolling a multiple of 3, so $B = \{3,\ 6\}$

a) Venn Diagram

b) $P(A)$

c) $P(B)$

d) $P(A \cap B)$

e) $P(A \cup B)$

f) $P(A \cup 5)$

g) $P\left(B^{C}\right)$

4) Assume $\Omega = \{1,\ 2,\ 3,\ 4,\ 5\}$, $P(1) = 0.21$, $P(2) = 0.16$, $P(3) = P(4) = 0.1$, $A = \{1,\ 3,\ 5\}$ and $B = \{2,\ 3,\ 4\}$. Find the following:

a) $P(5)$

b) $P(A)$

c) $P(B)$

d) $P(A \cap B)$

e) $P(A \cup B)$

f) $P(A^c)$

g) $P\left((A \cap B)^c\right)$

5) Given $P\left(A \cap B^{c}\right) = 0.23$, $P\left(B \cap A^{c}\right) = 0.22$. $P((A \cup B)^{c}) = 0.1$, find $P(A \cap B)$.

6) Given $P(A) = 0.25$, $P(B) = 0.36$, $P(A \cup B) = 0.56$, find:

a) $P(A \cap B)$

b) $P\left(A^{c} \cap B^{c}\right)$

7) A card is chosen from a standard deck of 52. Find the probability the card is:

a) An ace or a face card

b) A heart or a queen

c) A red card or a king

d) A red card or a black card

8) Five cards are chosen from a standard deck of 52. Find the probability that:

a) All 5 are hearts

b) All 5 are from the same suit

c) 3 are red cards and 2 are black

d) At least 2 hearts

9) A jar contains 7 blue and 8 red marbles. You pick 5 marbles at once, without replacement. Find the following probabilities.

a) All 5 are red.

b) 3 are red and 2 are blue.

c) At least 4 are red.

d) At least 1 is red.

10) A family has 5 children. Find the probability of having at least one girl.

11) **The Birthday Problem** If there are 5 random people in a room, find the probability that 2 of them have the same birthday.

*How many people in a room would you guess you would need so that the probability that 2 people have the same birthday is 100%? (excluding leap day birthdays)

**How many people in a room would you guess you would need so that the probability that 2 people have the same birthday is 50%? (again, no leap day birthdays)

12) **Color Blindness** is an *X* - linked inherited disease. A woman who carries the color blindness gene on one of her *X* chromosomes but not the other has normal vision, but a man who carries the gene on his only *X* chromosome is color blind. If a woman with normal vision who carries the color blindness gene has a child with a man who is color blind, what is the probability the child will have normal vision?

13) A population of song sparrows contains 30 individuals, 10 of which carry gene *A*, another 12 carry gene *B*, and the rest carry gene *C*. DNA is taken from 4 individuals. What is the probability that all 4 will carry the same gene?

14) **Cancer Testing** All patients suspected of having a particular type of cancer are tested by two separate labs. The probability that lab A produces a positive test for any patient with cancer is 0.1, whereas for lab B it is 0.05. If the probability that both labs simultaneously produce a positive test for any cancer patient is 0.03, what is the probability that at least one lab will produce a positive test?

15) **Color Blindness** is a genetic disorder that results in a lack of color vision. Suppose we randomly choose an individual from the population. Let C be the event “individual is color-blind,” and M the event "individual is male.” If males make up 50% of the population, and it is estimated that $P(M \cup C) = 0.55$ and $P(M \cap C) = 0.05$, what is the probability that a randomly chosen individual will be color-blind?

16) Let $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}, A = \{1, 3, 5\}, B = \{2, 5, 6\}, C = \{1, 4, 6, 8, 9\}$

a) Venn Diagram

b) $A \cap B$

c) $(A \cup B)^c$

d) $B^c \cap C$

e) $\left(A^c \cap B\right)^c$

f) $(A \cap B) \cup C$

g) $A \cap (B \cup C)$

h) $A \cap (B \cap C)$