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Total No. of Questions: [09]

B.Sc. (Hons.) Physics (Semester – 4th)
MATHEMATICS-IV
Subject Code: BMATH5401
Paper ID: [19131519]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. State Dirichlet's Conditions.

b. Evaluate$L\{\sin sin\, t\ \ \sin sin\, 2t\}$.

c. State Triangular wave form with its Fourier expansion.

d. Write the Fourier transforms of a function.

e. What do you mean by a periodic function?

f. State Modulation theorem of Fourier transforms.

g. Find Laplace transform of $\delta(t-a)$.

h. Define unit step function.

i. State Parseval's inequality.

j. State & prove first shifting property of Laplace transform.

Section – B **(5 marks each)**

Q2. Find the half range cosine series for $f(x) = x\sin x\ in\ (0,\pi)$ and prove that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ ...\infty = \frac{\pi-2}{4}$.

Q3. Prove that $\int_0^{\infty} \frac{t^2}{\left(t^2+\right)1^2} dt = \frac{\pi}{4}$.

Q4. Evaluate $\int_0^{\infty} te^{-2t} \sin sin\, 3t\, dt$.

Q5. State and Prove Convolution theorem of Fourier Transforms.

Q6. Find the inverse Laplace transform of $\frac{S}{\left(S^2+1\right)\left(S^2+4\right)}$.

Section – C **(10 marks each)**

Q7. Solve $\frac{dx}{dt} + y = \sin sin\, t$, $\frac{dy}{dt} + x = \cos cos\, t$, when $y(0) = 0$, $x(0) = 2$.

Q8. Use finite Fourier transform to solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$; given $u(0, t) = 0$, $u(4, t) = 0$, $and\ u(x, 0) = 2x$, where $0 < x < 4, t > 0$.

Q9. Solve If $f(x) = \{0; \ -\pi \leq x \leq 0\ \ \sin sin\, x; \ 0 \leq c \leq \pi\}$ prove that $f(x) = \frac{1}{\pi} + \frac{1}{2}\sin sin\, x - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\cos cos\, 2nx}{4n^2-1}$. Hence prove that (i) $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots = \frac{1}{2}$ (ii) $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \ldots = \frac{\pi-2}{4}$.