Proof the collatz conjecture never loops outside 1,2,4.

1)definition of x

Let's call the smallest number in a loop, x.

2)Traits of x

x can be odd or even,
If x is even,
They you would divide it by two to get,
x/2,
Which is smaller than x,
Which contradicts the definition of x.

3)traits of 3x+1

If a number is odd,
You times it by 3 and add 1,
All odd numbers can be written in the form of 2p-1,
Thus it can be written as,

$$3(2p-1)+1$$

$$6p-3+1$$

$$6p-2$$

$$2(3p-1)$$

As shown above,
The outcome of 3x+1 will always be even.

4)Premise of proof

If you keep transforming x,
Eventually you'll get an equation equal to x,
When the loop completes itself.

5)Transforming x
As shown in section 3,
x is odd,
So we transform it by timesing by three and adding 1 to get,

$$3x+1$$

And as shown in section 3,
We know the expression is even so we can divide it by two,
But we don't know how many times we can divide by two,
So we shall use the variable B[1]
to denote the amount of times times you can divide by two until you get an odd number,

$$\frac{3x+1}{2^{B[1]}}$$

And since we know that the expression is odd,
Since we divided it by as many times as possible,
We now times it by 3 and add 1,

$$3\frac{3x+1}{2^{B[1]}}+1$$

And as shown in section 3,
We know the expression is even so we can divide it by two,
But we don't know how many times we can divide by two,
So we shall use the variable B[2]
to denote the amount of times times you can divide by two until you get an odd number,

$$\frac{3\frac{3x+1}{2^{B[1]}}+1}{2^{B[2]}}$$

And since we know that the expression is odd,
Since we divided it by as many times as possible,
We now times it by 3 and add 1,

$$3\frac{3\frac{3x+1}{2^{B[1]}}+1}{2^{B[2]}}+1$$

And as shown in section 3,
We know the expression is even so we can divide it by two,

But we don't know how many times we can divide by two,
So we shall use the variable B[3]
to denote the amount of times times you can divide by two until you get an odd number,

$$\frac{3\frac{3\frac{3x+1}{2^{B[1]}}+1}{2^{B[2]}}+1}{2^{B[3]}}$$

If you haven’t noticed, the pattern is iterative,
Thus we can derive a formula for the transformation of x,

$$\frac{3\frac{3\frac{3\frac{3x+1}{2^{B[1]}}+1}{2^{B[2]}}+1}{2^{B[3]}}+1}{\dots}{2^{B[n]}}$$

Where n is the number of iterations,

$$\frac{3\,\frac{3\,\frac{3\,\frac{3\,\frac{3\,\frac{3x+1}{2^{B[1]}}+1}{2^{B[2]}}+1}{2^{B[3]}}+1}{\cdots}+1}{2^{B[n-2]}}+1}{2^{B[n-1]}}+1}{2^{B[n]}}$$

$$\frac{3}{2^{B[n]}}\,\frac{3\,\frac{3\,\frac{3\,\frac{3\,\frac{3x+1}{2^{B[1]}}+1}{2^{B[2]}}+1}{2^{B[3]}}+1}{\cdots}+1}{2^{B[n-2]}}+1}{2^{B[n-1]}}+\frac{1}{2^{B[n]}}$$

$$\frac{3}{2^{B[n]}}\,\frac{3\,\frac{3\,\frac{3\,\frac{3\,\frac{3x+1}{2^{B[1]}}+1}{2^{B[2]}}+1}{2^{B[3]}}+1}{\cdots}+1}{2^{B[n-2]}}+1}{2^{B[n-1]}}+\frac{1}{2^{B[n]}}$$

$$\frac{\frac{3^2}{2^{B[n]}}\,\frac{3\,\frac{3\,\frac{3\,\frac{3\,\frac{3x+1}{2^{B[1]}}+1}{2^{B[2]}}+1}{2^{B[3]}}+1}{\cdots}+1}{2^{B[n-2]}}+\frac{3}{2^{B[n]}}}{2^{B[n-1]}}+\frac{1}{2^{B[n]}}$$

$$\frac{\frac{3^2}{2^{B[n]}}}{2^{B[n-1]}}\,\frac{3\,\frac{3\,\frac{3\,\frac{3\,\frac{3x+1}{2^{B[1]}}+1}{2^{B[2]}}+1}{2^{B[3]}}+1}{\cdots}+1}{2^{B[n-2]}}+\frac{\frac{3}{2^{B[n]}}}{2^{B[n-1]}}+\frac{1}{2^{B[n]}}$$

$$\frac{\frac{\frac{3^3}{2^{B[n]}}}{2^{B[n-1]}}\,\frac{3\,\frac{3\,\frac{3\,\frac{3x+1}{2^{B[1]}}+1}{2^{B[2]}}+1}{2^{B[3]}}+1}{\cdots}+\frac{\frac{3^2}{2^{B[n]}}}{2^{B[n-1]}}}{2^{B[n-2]}}+\frac{\frac{3}{2^{B[n]}}}{2^{B[n-1]}}+\frac{1}{2^{B[n]}}$$

$$\frac{\frac{\frac{3^3}{2^{B[n]}}}{2^{B[n-1]}}}{2^{B[n-2]}} \; \frac{3\,\frac{3\,\frac{3\,\frac{3x+1}{2^{B[1]}}+1}{2^{B[2]}}+1}{2^{B[3]}}+1}{\cdots} + \frac{\frac{\frac{3^2}{2^{B[n]}}}{2^{B[n-1]}}}{2^{B[n-2]}} + \frac{\frac{3}{2^{B[n]}}}{2^{B[n-1]}} + \frac{1}{2^{B[n]}}$$

...

$$\frac{3^n}{\prod_{k=1}^{n} 2^{B[k]}}x + \sum_{k_1=1}^{n} \frac{3^{k_1-1}}{\prod_{k_2=1}^{k_1} 2^{B[k_2]}}$$

Since we know that the sequence loops,
We know that the equation above must eventually equal x,
When the pattern loops around,

$$\frac{3^{n}}{\prod_{k=1}^{n} 2^{B[k]}} x + \sum_{k_1=1}^{n} \frac{3^{k_1-1}}{\prod_{k_2=1}^{k_1} 2^{B[k_2]}} = x$$

$$\frac{3^{n}}{\prod_{k=1}^{n} 2^{B[k]}} x - x + \sum_{k_1=1}^{n} \frac{3^{k_1-1}}{\prod_{k_2=1}^{k_1} 2^{B[k_2]}} = 0$$

$$\left(\frac{3^{n}}{\prod_{k=1}^{n} 2^{B[k]}} - 1\right) x + \sum_{k_1=1}^{n} \frac{3^{k_1-1}}{\prod_{k_2=1}^{k_1} 2^{B[k_2]}} = 0$$

$$\left(\frac{3^{n}}{\prod_{k=1}^{n} 2^{B[k]}} - 1\right) x = -\sum_{k_1=1}^{n} \frac{3^{k_1-1}}{\prod_{k_2=1}^{k_1} 2^{B[k_2]}}$$

$$x = \frac{-\sum_{k_1=1}^{n} \frac{3^{k_1-1}}{\prod_{k_2=1}^{k_1} 2^{B[k_2]}}}{\frac{3^{n}}{\prod_{k=1}^{n} 2^{B[k]}} - 1}$$

Now let's simplify,

Let's first look at the product functions,
Because of the multiplication of powers rule,
We can write the equation as,

$$x = \frac{-\displaystyle\sum_{k_1=1}^{n} \frac{3^{k_1-1}}{2^{\sum_{k_1=1}^{k_2} B[k_2]}}}{\dfrac{3^n}{2^{\sum_{k=1}^{n} B[k]}} - 1}$$

$$x = \frac{-\displaystyle\sum_{k_1=1}^{n} \frac{3^{k_1-1}}{2^{\sum_{k_1=1}^{k_2} B[k_2]}}}{\dfrac{3^n - 2^{\sum_{k=1}^{n} B[k]}}{2^{\sum_{k=1}^{n} B[k]}}}$$

$$x = -\sum_{k_1=1}^{n} \frac{3^{k_1-1}}{2^{\sum_{k_1=1}^{k_2} B[k_2]}} \cdot \frac{2^{\sum_{k=1}^{n} B[k]}}{3^n - 2^{\sum_{k=1}^{n} B[k]}}$$

$$x = \frac{-2^{\sum_{k=1}^{n} B[k]} \cdot \sum_{k_1=1}^{n} \frac{3^{k_1-1}}{2^{\sum_{k_1=1}^{k_2} B[k_2]}}}{3^n - 2^{\sum_{k=1}^{n} B[k]}}$$

6) traits of the numerator of the equation

$$-2^{\sum_{k=1}^{n} (B[k])} \cdot \sum_{k=1}^{n} \left(\frac{3^k}{2^{\sum_{k_1=1}^{k} B[k_1]}} \right)$$

Let's look at the second sum function,
Any positive number to the power of any number,
Is always positive,
So the numerator and denominator in the sun function,
Is always positive,
A positive number divided by a positive number is always positive,
So the contents of the sum function are always positive,
The sum of positive numbers is always positive,
So the sum function is always positive,
Let's look at the other section of the numerator,
Which is made of a -1 multiplied by 2 to the power of another number,

A positive number to the power of any number is always positive,
And a positive number multiplied by -1 is always negative,
So the other section of the numerator is always negative,
A negative number multiplied by a positive number is always negative,
So the numerator is always negative.

6) x must be positive

x must always be positive,
Since the numerator is always negative,
The only way x can be positive is if the denominator is negative as well,

$$3^n - 2^{\sum_{k=1}^{n} (B[k])} < 0$$

$$3^n < 2^{\sum_{k=1}^{n} (B[k])}$$

$$n \ln(3) < \sum_{k=1}^{n} (B[k]) \ln(2)$$

$$\frac{n \ln(3)}{\ln(2)} < \sum_{k=1}^{n} (B[k])$$

7) the first iteration

Let's look at iteration one,
Which is equivalent of saying when n = 1,

$$\frac{n\ln(3)}{\ln(2)} \leq \sum_{k=1}^{n} B[k]$$

$$\frac{1\ln(3)}{\ln(2)} \leq \sum_{k=1}^{1} B[k]$$

$$\frac{\ln(3)}{\ln(2)} \leq B[1]$$

Since we know B[1]'s lower bound,
We can say,

$$\frac{\ln(3)}{\ln(2)} + C = B[1]$$

Where C is a positive number,
Now let's look at iteration one,

$$\frac{3x+1}{2^{B[1]}}$$

$$\frac{3x+1}{2^{\frac{\ln(3)}{\ln(2)}+C}}$$

We know that the expression must be greater or equal to x,
So we can write,

$$\frac{3x+1}{2^{\frac{\ln(3)}{\ln(2)}+C}} \geq x$$

$$3x+1 \geq 2^{\frac{\ln(3)}{\ln(2)}+C}x$$

$$3x-2^{\frac{\ln(3)}{\ln(2)}+C}x+1 \geq 0$$

$$\left(3-2^{\frac{\ln(3)}{\ln(2)}+C}\right)x+1 \geq 0$$

$$1 \geq -\left(3-2^{\frac{\ln(3)}{\ln(2)}+C}\right)x$$

$$1 \geq \left(2^{\frac{\ln(3)}{\ln(2)}+C}-3\right)x$$

$$\frac{1}{x} \geq 2^{\frac{\ln(3)}{\ln(2)}+C}-3$$

$$\frac{3}{x} \geq 2^{\frac{\ln(3)}{\ln(2)}+C}$$

$$\ln\left(\frac{3}{x}\right) \geq \left(\frac{\ln(3)}{\ln(2)}+C\right)\ln(2)$$

$$\frac{\ln\left(\frac{3}{x}\right)}{\ln(2)} \geq \frac{\ln(3)}{\ln(2)}+C$$

$$\frac{\ln\left(\frac{3}{x}\right)}{\ln(2)} - \frac{\ln(3)}{\ln(2)} \geq C$$

Since C is positive,
Which is the equivalent of saying C >= 0,
We know,

$$\frac{\ln\left(\frac{3}{x}\right)}{\ln(2)} - \frac{\ln(3)}{\ln(2)} \geq 0$$

$$\frac{\ln\left(\frac{3}{x}\right)}{\ln(2)} \geq \frac{\ln(3)}{\ln(2)}$$

$$\ln\left(\frac{3}{x}\right) \geq \ln(3)$$

$$\frac{3}{x} \geq 3$$

$$3 \geq 3x$$

$$1 \geq x$$

The only value of x where C can be a positive number is 1,
Any other value of x will mean C must be less than 0,
Which is a contradiction since C must be positive,

For iteration 1 to work,

$$x = 1$$

8) the nth iteration
Let's look at iteration n,

Proof the collatz conjecture must eventually loop

Assumption 1
If the sequence does not loop,
Then it must tend to infinity.

Assumption 2
And for all values which don't loop,

There must be a smallest value.

Assumption 3
All numbers can be written in one of the four forms,
Where x is a whole number,

$$4x$$

$$4x + 1$$

$$4x + 2$$

$$4x + 3$$

Proof basis
The smallest value which doesn't loop must be equal to one of the four expressions,
So if we transform the four expressions,
And they transform to a number smaller than the original expression,
Then the expression can't express the value of the smallest number.

Proof part 1 - 4x

$$4x$$

$$2x$$

2x < 4x,
Which is a contradiction since 4x is the smallest number.

Proof part 2 - 4x+1

$$4x + 1$$

$$12x + 4$$

$$6x + 2$$

$$3x + 1$$

3x+1 is less than 4x+1,
Which is a contradiction since 4x+1 is the smallest number.

Proof part 3 - 4x+2

$$4x + 2$$

$$2x + 1$$

2x+1 is less than 4x+2,
Which is a contradiction since 4x+2 is the smallest number.

Proof part 4 - 4x +3
4x+3 can’t be solved as simply as the rest,
So it has to be broken up into four expressions,

$$4(4p)+3 = 16p + 3$$

$$4(4p + 1)+3 = 16p + 7$$

$$4(4p + 2)+3 = 16p + 11$$

$$4(4p + 3)+3 = 16p + 15$$

Now one of these four expressions could be equal to the smallest value which does not loop.

Proof part 4a - 16p +3

$$16p + 3$$

$$48p + 10$$

$$24p + 5$$

$$72p + 16$$

$$36p + 8$$

$$18p + 4$$

$$9p + 2$$

$$9p + 2 < 16p + 3$$

This is a contradiction since 16p+3 is the smallest number,

Proof part 4b - 16p +7

$$16p + 7$$

$$48p + 22$$

$$24p + 11$$

$$72p + 34$$

$$36p + 17$$

$$108p + 52$$

$$54p + 26$$

$$27p + 13$$

The expression above can be odd or even,
Dependant on if p is odd or even

Proof part 4ba - if p is even

$$27p + 13$$

$$81.0p + 40$$

$$40.5p + 20$$

$$20.25p + 10$$

$$10.125p + 5$$

$$10.125p + 5 < 16p + 7$$

The last expression is less than 16p+7,
Which is a contradiction,
Since 16p+7 is the smallest value.

Proof part 4bb - if p is odd

$$27p + 13$$

$$13.5p + 6.5$$

$$13.5p + 6.5 < 16p + 7$$

The last expression is less than 16p+7,
Which is a contradiction,
Since 16p+7 is the smallest value.

Proof part 4c - 16p+11

$$16p + 11$$

$$48p + 34$$

$$24p + 17$$

$$72p + 52$$

$$36p + 26$$

$$18p + 13$$

$$54p + 40$$

The expression above can be odd or even,
Dependant on if p is odd or even

Proof part 4ca - if p is even

$$27p + 20$$

$$13.5x + 10 < 16p + 11$$

The last expression is less than 16p+11,
Which is a contradiction,
Since 16p+11 is the smallest value.

Proof part 4cb - if p is odd

$$27p + 20$$

$$81p + 61$$

$$40.5p + 30.5$$

$$20.25p + 15.25$$

$$10.125p + 7.625 < 16p + 11$$

The last expression is less than 16p+11,
Which is a contradiction,
Since 16p+11 is the smallest value.

Proof part 4d - 16p+15

$16p + 15$

$48p + 46$

$24p + 23$

$72p + 70$

$36p + 35$

$108p + 106$

$54p + 53$

$162p + 160$

$81p + 80$

The last expression is odd or even,
Dependant on if p is odd or even,

Proof part 4da - if p is even

$$81p + 80$$

$$40.5p + 40$$

$$20.25p + 20$$

$$10.125p + 10 < 16p + 15$$

The last expression is less than 16p+15,
Which is a contradiction,
Since 16p+15 is the smallest value.

Proof part 4db - if p is odd

$$81.0p + 80.0$$

$$243.0p + 241.0$$

$$121.5p + 120.5$$

$$60.75p + 60.25$$

$$30.375p + 30.125$$

$$15.1875p + 15.0625 < 16p + 15$$

The last expression is less than 16p+15,
Which is a contradiction,
Since 16p+15 is the smallest value.

Proof conclusion
Since all 4 possible expressions for all numbers,
Transform to become a number smaller than themselves,
We know their can’t be a smallest number which does not loop,
And the absence of a smallest number can only be when their are no numbers in a set,
Thus no number doesn't loop,
Thus all numbers must loop.