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B.Sc. (Hons.) Physics (Semester – 5th)
MATHEMATICAL PHYSICS-I
Subject Code: BPHYS1501
Paper ID: [19131524]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. State Green's theorem.

b. Show that div$\left(r^n \vec{r}\right) = (n + 3)r^n$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

c. Define covariant and contra variant vectors (tensor of rank one)

d. Show that the Kronecker delta is a mixed tensor of rank two

e. Show that $grad\left(\frac{1}{r}\right) = -\frac{r}{r^3}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

f. Use Milne Thompson method to find an analytic function whose real part is u(x, y)=2xy+2x

g. State Cauchy Riemann Equations

h. State Einstein's summation convention

i. Find the volume of the parallelepiped formed by the vectors $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 4\hat{i} + 5\hat{j} + 6\hat{k}$, $\vec{c} = 7\hat{i} + 8\hat{j} + 9\hat{k}$

j. State Cauchy's integral formula

Section – B **(5 marks each)**

Q2. Find the directional derivative of f(x, y)=xy+yz+zx in the direction of vector $\hat{i} + 2\hat{j} + 2\hat{k}$ at the point (1,2,0)

Q3. Use Divergence theorem to calculate $\iint \vec{F}.\vec{n}dS$ where $\vec{F} = \left(x^2 - yz\right)\vec{i} + \left(y^2 - zx\right)\vec{j} + (z^2 - xy)\,\vec{k}$ taken over the rectangular parallelepiped S: $0 \leq x \leq a,\ 0 \leq\leq y \leq b,\ 0 \leq z \leq c$

Q4. Prove the sum (or difference) of two tensors which have same number of covariant and the same contravariant indices is again a tensor of the same rank and type as the given tensors.

Q5. Prove that the transformation of a contravariant vector is transitive.

Q6. Evaluate the integrals (i) $\int_C \frac{\cos\cos e^z}{z(z+2)}dz$ (ii) $\int_C \frac{\sinh sinh\, z}{z^2(z-2)}dz$, where $C = \{z: |z| = 1\}$ taken in the positive sense.

Section – C **(10 marks each)**

Q7. Find $\int_C \vec{F}.dr$ where $\vec{F} = (2x + y)\,\vec{i} + (3y - x)\,\vec{j}$ and C is the curve in the x-y plane consisting of the straight lines from (0,0) to (2, 0) and then to (3, 2)

Q8. If T_i be the component of a covariant vector show that $\left(\frac{\partial T_i}{\partial x^j} - \frac{\partial T_j}{\partial x^i}\right)$are component of a Anti- symmetric covariant tensor of rank two.

Q9. (a) Derive the Laurent series of the functions $\frac{1}{z^2(1-z)}$

(b) Determine the residue of the multivalued function $f(z) = \frac{z^{1/2}}{z^2+1}$ at each of its poles.