The Graph is Lava - A Fun Introduction to Domain & Range
Unit 4, Lesson 10: Domain and Range (Part 1)

Abstract:

- This lesson is a fun, engaging introduction to the concept of domain and range. Students will learn the basic setup of the game while refreshing their knowledge of inequalities during a warm up round, and then discover domains and ranges in subsequent rounds. This game requires thoughtful participation while leaving room for silliness and play.

Keywords:

- Algebra
- Domain
- Range
- Inequalities
- Discover learning
- Math games
- Moving
- Interactive
- 8th-9th grade
- 13-14 yrs old

Goals and Learning Objectives:

- Learn the concept of domain and range
- Demonstrate their understanding through games and activities
- Refresh and practice inequalities
- Have fun!

Assessment:

- Students will demonstrate their understanding throughout the activity by responding to teaching instructions and moving to the appropriate place
- Invite students at the end to come up and explain their key takeaways in front of the class

- An exit ticket at the end of class will apply what they learned in the game to paper assignments

Purpose/Rationale:

- Discovery learning builds confidence in students that they have the tools they need to figure out new problems in math
- New concepts are less intimidating when they are introduced as a continuation of what students already know, instead of as a huge leap ahead. And the more fun, the less scary!
- This lesson aligns with Unit 4, Lesson of the Kendall Hunt, Illustrative Math Algebra I textbook used in Cambridge public schools

Materials/Resources:

- At minimum it requires a grid or graph on the ground. If access to a graph mat is not feasible, a graph can be made with tape or other materials
- If you'd like room to make it goofier, you can use miscellaneous items from the gym or other equipment room. As an example, we chose potato sacks to add a mobility obstacle while they are trying to locate and move to the safe areas

#1 Introduction: "Set Up for Success"

- The following is a suggestion for how to introduce the activity to the classroom:
    - Today we're going to learn about the concept of domain and range, and we'll do this by playing a game called the Graph is Lava!
    - This is like the Floor Is Lava, but we are playing on a graph. The graph will have some area or areas that are "safe" and some that are not, which we call lava. Your objective will be to get to the safe area before I say "freeze".
    - The safe area will be determined by an equation. For our warm up round, we'll do it with inequalities. This will be an excellent opportunity to practice those skills we learned back in Unit 2 and keep them fresh and active in your mind.
    - I'll say and write an expression for an inequality, give you a moment to think about it, and then say "go". After I say go, you need to move quickly and safely to a number that satisfies the inequality — everywhere else is lava. Your goal is to make it to safety before I say freeze.
    - After a few rounds, we'll shift the rule.

    - Any questions before we begin?

#2 Exploration: *"Do it"*

- Play the inequality version of the game. Suggested problem progressions are at the bottom of this document
- Continued suggested explanations:
    - Now that you're warmed up, we're going to switch up the rules for how we determine what part of the ground is lava.
    - I'm going to say and write a function, and all the possible y values of that function will be considered safe. Everything that is not a possible y value is lava.
    - For this version of the game, we will think of each axis as its own, independent number line. As long as you are standing in a spot whose y coordinate is a possible output of the function, it doesn't actually matter what your x coordinate is.
    - For example, if I say y is equal to x, then what are the possible values of y? (They answer: any number)
    - You might be tempted to be on a spot on the line x = y, like (3,3) or (2,2), but in some cases there might not be enough spots for everyone to be on the line. I encourage you to get comfortable just focusing on your corresponding y value.
    - As another example, if I say that y is equal to the *absolute value* of x, how does that restrict the possible values of y? (they answer: now they can't be negative numbers. If they need additional prompting to answer this question, give them a few examples like x = -5 and x = 5 and ask what the corresponding y value is. If they need more, ask "is it possible for y to be a negative number?")
    - Great, now let's practice this example with the ground is lava: go! … freeze!
- See later in this document for suggested problem progression.
- If they're really getting the hang of it, you can add in a silly curveball. Have them hop around in potato sacks to the safe areas! This will require them to think faster to compensate for the slower movement
- Play this for a while, then tell them that the concept they were just applying has a name!
    - What you just did was find the *range* of these functions. The *range* is the set of all possible output values

    - Now we're going to switch to finding all the possible inputs of these same functions. This is called the *domain* of a function.
- Then play again for domain! (Suggested problem progression also below)

 # 3 Sharing: *"What Happened"*

- You can pause to ask these questions after going through range, domain, or both
    - What did you notice?
    - Were there any patterns?
    - Which problems did you find easier?
    - More challenging?
    - What was your thought process to figure out the answers to the challenging questions?

#4 Generalizing: "*So What?"*

- There's a lot of information we can gather from a function. Domain and range are two examples of properties that can help us understand functions
- We only had one word problem today, but you can imagine that all sorts of word problems will have different restrictions or bounds on the possible inputs and output. How does knowing the domain and range help us analyze data?
- Can you think of other examples of real world data where the domain or range matter?

#5 Application and Wrap-Up: *"Now What?"*

- Ask if there are any questions
- Prompt students to summarize key takeaways
- Discuss what students can do with what they have learned.
- Prompt students to notice different examples of real-world problems that have domains and ranges
- See below for possible exit ticket

If there's any time remaining, play some more graph is lava!

This document has been adapted from:
https://experientiallearning.ucdavis.edu/toolbox/des-lessonplan.pdf

Possible problem progressions

Inequalities:

$y > 0$ $y < 5$

$y > 2x$ $y > x/3$ $y/2 < 2x$

$y < 2x - 3$ $y > 3x - 1$ $y/2 < x/3 + 5$

$y < x^2$ $y > x^2$ $y < x^3$ $y > x^3$

Domain & Range:

For these you can go through these equations first for range, and then you can go back through the same equations for domain

1. $y = 1/x$

   Range: all real numbers except $y = 0$

   Domain: all real numbers except $x = 0$

2. $y = 1/(x - 3)$

   Range: all real numbers except $y = 0$

   Domain: all real numbers except $x = 3$

3. $y = x^2 - 4$

   Range: $y >= -4$

   Domain: all real numbers

4. $y = 1/x^2$

   Range: $y > 0$

   Domain: all real numbers

5. $y = 1/(1+x^2)$

   Range: $0 < y < 1$

Domain: all real numbers

6. y = sqrt(x + 5)

   Range: y >= 0

   Domain: x >= -5

7. y = sqrt(1 - x) + 5

   Range: y >= 5

   Domain: x <= 1

8. y = sqrt(9 - x^2)

   Range: 0 <= y <= 3

   Domain: -3 <= x <= 3

9. y = abs(4 - x^2)

   Range: y >= 0

   Domain: all real numbers

10. x = number of students in this school who are absent today

    y = x - the number of students in this school who are absent today due to a common cold

    Hint: there are approx. ____ number of students in this school

    Range: WHOLE numbers 0 =< y =< ____ (this upper bound is likely outside the range of the graph on the ground)

    Domain: same as range

Videos:

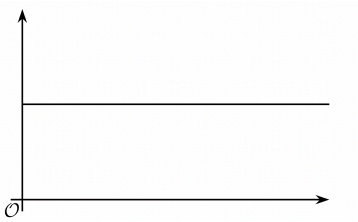
IMG_5832.mov

IMG_5830.mov

Possible Exit Ticket

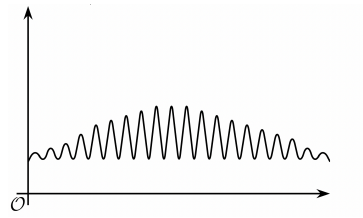
Selected from: Kendall Hunt, Illustrative Math, Algebra I

https://im.kendallhunt.com/HS/students/1/6/1/index.html



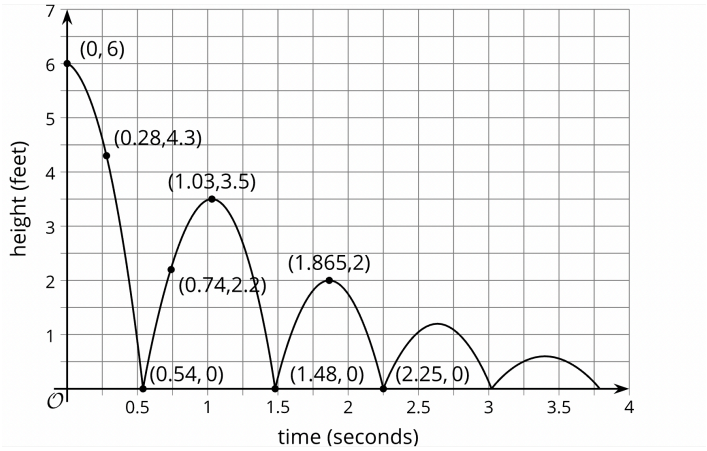
The following questions pertain to the two graphs on the left.

Do they have the same domain? Explain



Do they have the same range? Explain

A tennis ball was dropped from a certain height. It bounced several times, rolled along for a short period, and then stopped. Function *H* gives its height over time.



Here is a partial graph of *H*. Height is measured in feet. Time is measured in seconds.

Use the graph to help you answer the questions:

1. Find $H(0)$
2. Solve $H(x) = 0$
3. Describe the domain of the function
4. Describe the range of the function