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Total No. of Printed Pages: 3

Total No. of Questions: [09]

B. Tech Civil Engg. (Semester – 3rd)
MATHEMATICS-III (TRANSFORM & DISCRETE MATHEMATICS)
Subject Code: BMATH4301
Paper ID: [19110705]

Time: 03 Hours

Maximum Marks: 60

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A

(2 marks each)

Q1. Attempt the following:

- a. Find the Z-transform of $n \sin \sin (n\theta)$.
- b. Evaluate the integral $\int_0^{\infty} \frac{2^{-2t} - e^{-3t}}{t} dt$ using the Laplace Transform.
- c. Define an equivalence relation with the help of examples.
- d. Consider the set Q of rational numbers, and let $*$ be the operation on Q defined by $a * b = a + b - ab$. Show that $(Q, *)$ is a commutative semi group.
- e. Find the number of relations from $A = \{a, b, c\}$ to $B = \{1, 2\}$.
- f. Verify that the proposition $p \rightarrow q \leftrightarrow (\neg p \vee q)$ is a tautology.
- g. Write the contrapositive statement of the following:
 - (i) If Sadiq is a singer, then he is poor.
 - (ii) Only if Hassan studies will he pass the test.
- h. Show that $(x \wedge y) \vee (x' \wedge z) = (x' \vee y) \wedge (x \vee z)$.
- i. Find the minimum number of students needed to guarantee that 4 of them were born on the same day of the week.
- j. Find the Set $V(G)$ of vertices of G and the set $E(G)$ of edges of G in Figure 1 by defining a graph, vertices and edges.

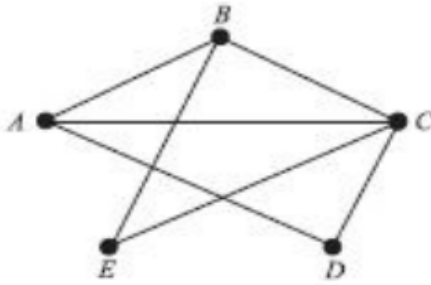


Figure 1:

Section – B

(5 marks each)

Q2. Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$. Hence evaluate

$$\int_0^{\infty} \frac{x \cos x \cos \frac{x}{2} - \sin x \sin \frac{x}{2}}{x^3} \cos \cos \frac{x}{2} dx.$$

Q3. Determine the validity of the following argument: $p \rightarrow \neg q, r \rightarrow q, r \vdash \neg q$.

Q4. Consider the third-order homogeneous recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$.

Find the solution with initial conditions $a_0 = 3, a_1 = 4, a_2 = 12$.

Q5. Consider the group $G = \{1, 2, 3, 4, 5, 6\}$ under multiplication modulo 7. Find the orders and subgroups generated by 2 and 3. Is G a cyclic?

Q6. Define Lattice. Is the Figure 2 a distributive and complemented lattice? Also, find the non-zero join irreducible elements and atoms of it.

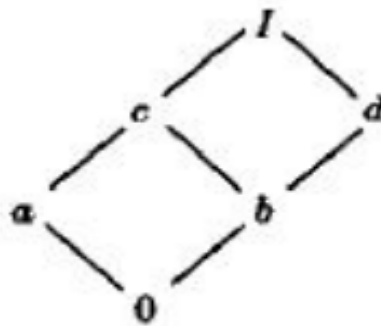


Figure 2:

Section – C

(10 marks each)

Q7. A cantilever beam is clamped at the end $x = 0$ and is free at the end $x = l$. It carries a uniform load per unit length from $x = 0$ to $x = \frac{l}{2}$ and zero otherwise. Calculate the deflection at any point using Laplace transformation.

- Q8. (i) Let H be a normal subgroup G . Then the cosets of H in G form a group under coset multiplication defined by $(aH)(bH) = abH$.
- (ii) A class has 10 male students and 8 female students. Find the number of ways the class can elect:
- (a) A class representative;
 - (b) 2 class representatives , one male and one female;
 - (c) A class president and vice president.
- Q9. Draw the graph $K_5, K_{3,3}, K_{2,3}$. Find an Euler path and a Hamiltonian path of each graph, if they exist. If it does not, why not?