Pendulum

A. Purpose

Study the relation between the oscillation period of a pendulum bob and its string length, using a smart phone with related apps to acquire experimental data.

B. Theory

When an object (the bob) is suspended with a string fixed at a pivot point above, it can switng back and forth when being slightly pushed away and released. Assuming the bob can be treated as a particle with mass m, the string has a length l and negligible mass, as shown in Fig. 1, then the total force $\Sigma\vec{F}$ acting on the bob can be written as

$$\Sigma\vec{F} = m\vec{g} + \vec{T}, \quad (1)$$

where $\vec{g}$ is the gravitational acceleraton (with a value of around 9.80 m/s^2 near the Earth surace), pointing towards the center of the Earth; $\vec{T}$ is the tension in the string, pointing towards the pivot point. If the string length remains unchanged during the swing, then the bob shall have no motion along the direction of the string, meaning its acceleration, or equivalently, its total force along the string direction shall be zero:

$$-\ mg\cos cos\,\theta\ +\ T\ =\ 0,$$

where θ is the angle between the string and the vertical. From this we obtain the magnitude of string tension

$$T\ =\ mg\cos cos\,\theta\,.$$

In addition, the total force in the direction perpendicular to the string shall be

$$\Sigma F\ =\ mg\sin sin\,\theta\,.$$

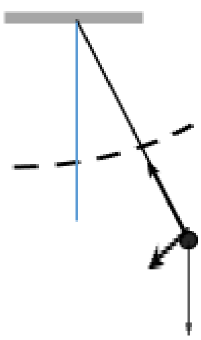

Fig. 1 Diagram showing the forces acting on the bob of a pendulum.

From Newton's second law of motion $\vec{F} = m\vec{a}$ we have

$$\Sigma F = mg \sin sin\, \theta = ma,$$

where a is the acceleration of the bob along the direction perpendicular to the string. This direction is also tengential to the motion trajectory, hence this acceleration is usually called the *tengential acceleration*. The above equation can be rewritten as

$$m\frac{d^2}{dt^2}s - mg \sin sin\, \theta = 0, \quad (2)$$

where t is the time, $s = \theta l$ is the circular trajectory of the bob measured from the lowest point. Therefore

$$m\frac{d^2}{dt^2}(\theta l) - mg \sin sin\, \theta = 0, \quad (3)$$

or

$$\frac{d^2}{dt^2}\theta - \frac{g}{l}\sin sin\, \theta = 0. \quad (4)$$

Assuming the amplitude of oscillaton is small, such as θ within $\pm 5°$, we have the approximation

$$\sin sin\, \theta \approx \theta,$$

Hence eq(4) is further smplifed as

$$\frac{d^2}{dt^2}\theta - \frac{g}{l}\theta = 0. \quad (5)$$

The general solution to this differential equation can be written as

$$\theta(t) = A \sin sin\, (\omega t + \phi), \quad (6)$$

where A is the amplitude of swing, ϕ is the initial phase angle at $t = 0$, while $\omega = \frac{2\pi}{T}$ is the angular frequency, T is the period of oscillation. Substitute eq(6) into eq(5) we will have $\omega^2 = \frac{g}{l}$, or $\frac{2\pi}{T} = \sqrt{\frac{g}{l}}$, hence the result

$$T = 2\pi\sqrt{\frac{l}{g}}. \quad (7)$$

This result shows that, under the conditions/assumptions of particle-like bob, small oscillation, negligible string mass and air drag, the period of a pendulum T depends only on string length l and gravitational acceleration g, and *not* on the mass *m* of the bob.

In this lab we will practice using a smart phone as part of the experimental instrument. We will use an app called "phyphox" to read the signal of the built-in gyroscope sensor, which can give us the temporal change of instantaneous angular speed $\omega(t)$. From the instantaneous angular speed,

$$\omega(t) = \frac{d}{dt}\theta(t) = A\omega \cos cos\,(\omega t + \phi),$$

we see that it also oscillates in the same frequency as the angle $\theta(t)$, meaning the period of its oscillation is the same $T = 2\pi\sqrt{\frac{l}{g}}$, therefore we can obtain the period T by measuring the oscillation of $\omega(t)$. We shall also verify the validity of eq(7) with different string length l, using a smart phone (that is obviously not a particle).

C. Devices/Instruments

Smart phone (your own), string, rod, clips, measureing tape.

Fig. 2 A smart phone pendulum. Smart phone can be put in a plastic bag, which can be suspended with two parallel strings fixed to a rod above.

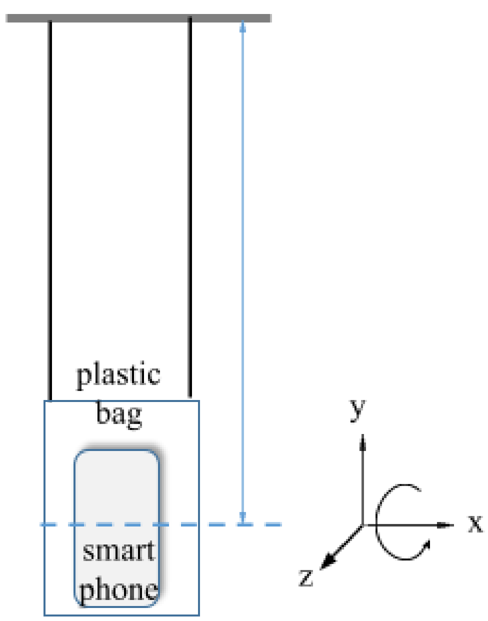

Fig. 3 Schematic of a smart phone pendulum. The phone's built-in gyroscope can detect the its angular speed about the x-, y-, and z-axis. If the phone is arranged as shown above, the forward-backward swing corresponds to a rotation about x-axis.

D. Experimental Steps

I. Install and use the “phyphox” app.

> Note: You will use your own smart phone in the experiment. Use it with care or you will be responsible for your own careless actions.

1. Download and install “phyphox” app at https://phyphox.org/

2. Configure the wifi connection of your phone such that it connects to the same access point (AP) as the computer at your desk (for faster data transmission). Password of wifi AP can be found on the black board.
3. Start phyphox, select “Gyroscope”. Tap the settings menu (⋮) at the top-right corner, check the option “Allow remote access”. A URL for connetion will be shown at the bottom of the screen (something like 192.168.x.xx:8080), as shown in Fig. 4. Coneect to this URL with the computer at your desk, you shall see the same scree in your browser.

Fig. 4 Menu items in the "phyphox" app. (Left) Menu items in Chinese. (Middle) Menu items in English. (Right) Tap the settings menu (⁝) at the top-right corner of screen, check the option "Allow remote access", a URL for connection will be shown at the bottom.

4. Fix the rod at the edge of the table. *Note: Watch for the rods when moving around the tables.*
5. Obtain two strings of about 1.5m long, tie each of them to on of the top corners of a plastic bag, and fix the other end of the strings to the rod. Adjust the string such that the length of swing is about 60cm. Try to adjust the two strings such that they appear parallel after putting your smart phone in the plastic bag, and that the smart phone in the bag can swing freely, as shown in Fig. 2 and Fig. 3. Measure the actual length of swing l from the center of rod to the center of phone.
6. Tap or click the "Run" button either from the phone or the computer to start gyroscope measurements, as shown in Fig. 5, to measure the phone's angular speed of free swing. Obtain average period T for at least 30 stable periods.
7. Capture the data figure on the computer screen, paste it to a Word document to keep records. Tap or click the settings menu (⁝) at the top-right corner and select "Export Data" to save the data into a computer file. You might want to pause (||) the acquisition before saving the data, to avoid slowdown or (perhaps) errors in data transmission. <u>*Note: Connection between the computer and the phone can be broken in a few seconds if acquition is paused without any further actions. Re-establishing the connection will erase the existing, unsaved data.*</u>

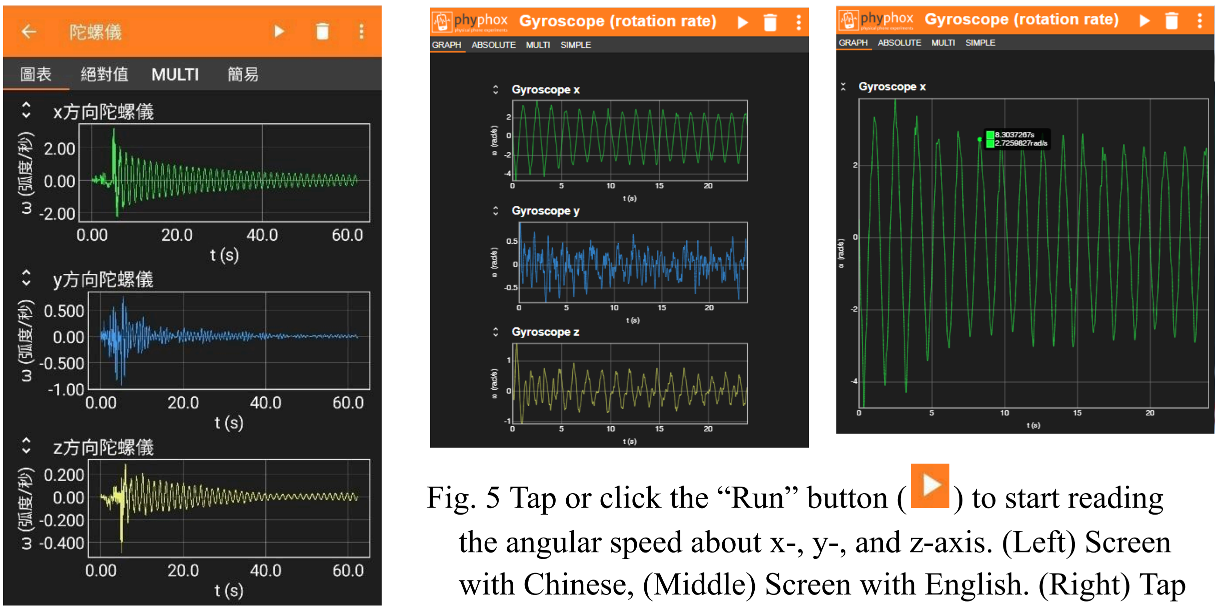

Fig. 5 Tap or click the "Run" button ( ) to start reading the angular speed about x-, y-, and z-axis. (Left) Screen with Chinese, (Middle) Screen with English. (Right) Tap

or click the zoom button () at the top-left corner to zoom in one of the data figures. Move the mouse pointer over any of the data points, its value will be shown on screen.

8. Change the string length, repeat steps 5-7, for at least three times to obtain the corresponding period T at each of the different lengths $\boldsymbol{l}$. You may try with $\boldsymbol{l}$ ~ 60 cm, 40 cm, and 20 cm. Actual lengths shall be measured and recorded.
9. Data Plots: From eq(7) we know $T^2 = \frac{4\pi^2}{g} l$. Make a plot of T^2 vs l (T^2 being the ordinate while l the abscissa). Do they seem to behave linearly (follow a straight line)? Fit the data points with a straight line and find its slope. Calculate the value of g from the slope (which should be equal to $\frac{4\pi^2}{g}$).
10. Errors: Calculate period T using eq(7) with the length l measured in step 5, compare it with the actual period T measured in step 6, find the discrepancy between them.

<u>Note:</u>

1. Make sure your phone is tied up tightly so it will not fly off to the floor.

E. Questions:

1. In step 10, the discrepancy in period T between your data and eq(7) could be different at different string length. Try to explain this.

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