Name: __________
Period:__________
Date:____________

Calculus Day 5
Krishnan 2016-2017

Example 6

Find the gradient function of $f(x) = x^2 - \frac{4}{x}$ and hence find the gradient of the tangent to the function at the point where $x = 2$.

$$f(x) = x^2 - \frac{4}{x} = x^2 - 4x^{-1}$$

$$\therefore \quad f'(x) = 2x - 4(-1x^{-2}) = 2x + 4x^{-2} = 2x + \frac{4}{x^2}$$

If $x = 2$ in the gradient function we will get the gradient of the tangent at $x = 2$.
So, as $f'(2) = 4 + 1 = 5$, the tangent has gradient of 5.

Find the gradient of the tangent to:

a $y = x^2$ at $x = 2$

b $y = \frac{8}{x^2}$ at $x = 9$

c $y = 2x^2 - 3x + 7$ at $x = -1$

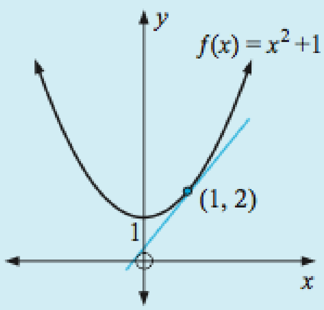
d $y = 2x - 5x^{-1}$ at $x = 2$

e $y = \frac{x^2 - 4}{x^2}$ at $x = 4$

f $y = \frac{x^3 - 4x - 8}{x^2}$ at $x = -1$

Example 8

Find the equation of the tangent to $f(x) = x^2 + 1$ at the point where $x = 1$.



Since $f(1) = 1 + 1 = 2$, the point of contact is (1, 2).

Now $f'(x) = 2x$

$\therefore$ $f'(1) = 2$

$\therefore$ the tangent has equation $\dfrac{y-2}{x-1} = 2$

i.e., $y - 2 = 2x - 2$

or $y = 2x$

Example 9

Find the coordinates of any points on the curve with equation $f(x) = x^3 + 3x^2 - 9x + 5$ where the tangent is horizontal.

$$f(x) = x^3 + 3x^2 - 9x + 5$$
$$f'(x) = 3x^2 + 6x - 9$$

Now tangents are horizontal when they have zero gradient i.e., when $f'(x) = 0$.

i.e., $3x^2 + 6x - 9 = 0$

i.e., $x^2 + 2x - 3 = 0$

i.e., $(x+3)(x-1) = 0$ $\therefore$ $x = -3$ or 1

When $x = -3$, $f(-3) = (-3)^3 + 3(-3)^2 - 9(-3) + 5 = 32$

When $x = 1$, $f(1) = 1^3 + 3 \times 1^2 - 9 \times 1 + 5 = 0$

Tangents are horizontal at the points (−3, 32) and (1, 0).