DOT
EDUCATION
(KHDA APPROVED)

A IAL PURE MATHS 3 PAST PAPERS

Chapter 7 [MS]

1. JAN 2025 [3]

3.	$\frac{dy}{dx}=\frac{4(x+3)^2-2(4x+1)(x+3)}{(x+3)^4}$	M1 A1
	Solves their $4(x+3)^2-2(4x+1)(x+3)=0$ $\Rightarrow (x+3)(10-4x)=0 \Rightarrow x=...$	M1
	Critical value of $\frac{5}{2}$; Critical value of -3	A1; B1
	C increasing when $\frac{dy}{dx}>0 \Rightarrow -3<x<\frac{5}{2}$	A1
		(6 marks)

Note this appears as M1A1M1A1M1A1 on epen but is being marked as M1A1M1A1B1A1

M1: Attempts the quotient rule and achieves

$$\frac{dy}{dx}=\frac{a(x+3)^2-b(4x+1)(x+3)}{(x+3)^4} \quad a,b>0 \quad \text{condoning slips}$$

They may attempt to multiply out the $(x+3)^2$ first which is fine as long as they reach a quadratic

Product rule may be attempted, look for $\frac{dy}{dx}=A(x+3)^{-2}-B(4x+1)(x+3)^{-3}$, $A,B>0$

Note: Only the numerator is required for this problem, and this mark can be awarded if a student attempts just $u'v-uv'=0$. If the denominator is considered it must be treated correctly.

A1: For a correct (unsimplified) $\frac{dy}{dx}$ or $u'v-uv'$ (if this is all that is considered). By the Product rule $\frac{dy}{dx}=4(x+3)^{-2}-2(4x+1)(x+3)^{-3}$

M1: Solves their numerator/their $\frac{dy}{dx}=0$ (or with any inequality, and the "=0" may be implied) to obtain at least one critical value from a non-calculator method. If the numerator is expanded to a quadratic a method of solution must be shown (e.g. factorisation or completing the square or formula seen applied) They may be working with equalities or inequalities for this mark. Note this means students who cancel the $(x+3)$ and lose the second critical value can still score M1.

A1: Critical value of $\frac{5}{2}$ (allow equivalent fractions) found provided the previous M has been scored.

B1: Finds or identifies -3 as the other critical value (no need for method shown).

A1: Depends on first M. For $-3<x<\frac{5}{2}$ or $-3<x\leqslant\frac{5}{2}$ or equivalent set notation, e.g. $x\in\left(-3,\frac{5}{2}\right]$ Condone $-3\leqslant x\leqslant\frac{5}{2}$ or $-3\leqslant x<\frac{5}{2}$. Must be simplified fraction. Allow split inequalities $-3<x$ and $x<\frac{5}{2}$, condoning "or" or a comma between but if set notation used it must be intersection not union.

Note: If the denominator of $\frac{dy}{dx}$ is incorrectly treated but the numerator is correct, a correct answer may be achieved. In such cases award SC M0 A0 M1 A1 B1 A1 if the subsequent marks are earned.

Note: answers relying on calculator technology where roots appear from an unsimplified derivative can score maximum M1A1M0A0B1A1

2. JAN 2025 [9]

9. (a)	$\frac{1}{4}$	B1
		(1)
(b)	$f'(x)=\frac{3}{\sqrt{x}}\ln(4x)+6\sqrt{x}\times\frac{1}{x}$	M1 A1
	Sets $\frac{3}{\sqrt{x}}\ln(4x)+6\sqrt{x}\times\frac{1}{x}=0\Rightarrow\ln(4x)=-2$	dM1
	$Q\left(\frac{1}{4e^2},-\frac{6}{e}\right)$	A1 A1
		(5)
(c)	Attempts $-2\times y$ co-ordinate of Q	M1
	Range $g(x)\leqslant\frac{12}{e}$	A1ft
		(2)
		(8 marks)

(a)

B1: $\frac{1}{4}$ or exact equivalent. May be seen as the x coordinate in a coordinate pair.

(b)

M1: Attempts to differentiate using the product rule and achieves $f'(x)=\frac{a}{\sqrt{x}}\ln(4x)+b\sqrt{x}\times\frac{1}{x}$

A1: $f'(x)=\frac{3}{\sqrt{x}}\ln(4x)+6\sqrt{x}\times\frac{1}{x}$. Note the $\frac{1}{x}$ may be seen as $\frac{4}{4x}$

dM1: Sets $f'(x)=0$ and proceeds to $\ln(4x)=k$. The "=0" may be implied by the attempt to solve. Condone attempts where multiplying through by $\sqrt{x}$ occurs before setting equal to 0.

A1: $x=\frac{1}{4e^2}$ oe but allow awrt 0.034 following correct equation.

A1: $Q\left(\frac{1}{4e^2},-\frac{6}{e}\right)$ Both coordinates correct and simplified, but may be listed separately, $x=\ldots$, $y=\ldots$ and allow if y is found in (c). Accept negative powers of e, but must be simplified coefficients and simplified powers.

(c)

M1: Attempts $-2\times$ their y co-ordinate of Q. Also allow following decimal answer

A1ft: Range $g(x)\leqslant\frac{12}{e}$ following through on their negative y value for Q. Accept with y instead of g(x) and wtih negative powers or expression in e following their (b), e.g. $y\leqslant 12e^{-1}$ Accept decimals following a decimal answer to (b). Do not accept f(x).

3. OCT 2024 [7]

7 (a)	$\mathrm{f}(x)=x^3\sqrt{4x+7} \Rightarrow \mathrm{f}'(x)=3x^2\sqrt{4x+7}+2x^3(4x+7)^{-\frac{1}{2}}$	M1, A1
	$\Rightarrow \mathrm{f}'(x)=3x^2\sqrt{4x+7}+\dfrac{2x^3}{\sqrt{4x+7}}=\dfrac{3x^2(4x+7)+2x^3}{\sqrt{4x+7}}$	dM1
	$\Rightarrow \mathrm{f}'(x)=\dfrac{7x^2(2x+3)}{\sqrt{4x+7}}$	A1
		(4)
(b)	Substitutes $x=-\frac{3}{2}$ into $x^3\sqrt{4x+7} \Rightarrow y=\ldots$	M1
	$\left(-\frac{3}{2},-\frac{27}{8}\right)$	A1
		(2)
(c)	Attempts $-4\times$"$-\frac{27}{8}$"	M1
	y „ $\frac{27}{2}$	A1
		(2)
(d)	$(2,7)$	M1, A1
		(2)
		Total 10

(a)

M1: Attempts the product rule to achieve $f'(x) = Px^2\sqrt{4x+7} + Qx^3(4x+7)^{-\frac{1}{2}}$ where P and Q are positive constants.

Other methods are possible so look at each one carefully.

For example: $f(x) = x^3\sqrt{4x+7} \Rightarrow f(x) = \sqrt{4x^7 + 7x^6} \Rightarrow f'(x) = \frac{1}{2}\left(4x^7 + 7x^6\right)^{-\frac{1}{2}} \times \left(28x^6 + 42x^5\right)$

The same principles for the main mark scheme can be applied

A1: $f'(x) = 3x^2\sqrt{4x+7} + 2x^3(4x+7)^{-\frac{1}{2}}$ o.e. which may be left unsimplified

dM1: Dependent upon previous M1. It is for "correctly" producing a single fraction with a denominator of $\sqrt{4x+7}$.

Look for $f'(x) = Px^2\sqrt{4x+7} + Qx^3(4x+7)^{-\frac{1}{2}} \Rightarrow \dfrac{Px^2(4x+7) + Qx^3}{\sqrt{4x+7}}$ o.e

A1: $f'(x) = \dfrac{7x^2(2x+3)}{\sqrt{4x+7}}$

....

It is possible to attempt this by WMA14 methods using $[f(x)]^2 = x^6(4x+7) = 4x^7 + 7x^6$

Score

M1, A1: $2f(x)f'(x) = 28x^6 + 42x^5$

dM1, A1: $f'(x) = \dfrac{14x^5(2x+5)}{2f(x)} = \dfrac{14x^5(2x+5)}{2x^3\sqrt{4x+7}} = \dfrac{7x^2(2x+5)}{\sqrt{4x+7}}$

....
....

(b)

M1: Attempts to substitute $x = -\frac{3}{2}$ into $x^3\sqrt{4x+7}$ to find the y value of the minimum point .

This may be implied by the correct y value.
It can be attempted even if part (a) is missing or incorrect
If they ignore the given form of $f'(x)$ and use their own version, they must

- Set their $f'(x)$ which must be of the form $Px^2\sqrt{4x+7} + Qx^3(4x+7)^{-\frac{1}{2}} = 0$
- Solve $Px^2\sqrt{4x+7} + Qx^3(4x+7)^{-\frac{1}{2}} = 0$ by multiplying by $(4x+7)^{\frac{1}{2}}$ o.e. to achieve a non-zero value for x
- Substitute this non zero value for x in into $x^3\sqrt{4x+7}$ to find y

A1: $\left(-\frac{3}{2}, -\frac{27}{8}\right)$ which may be awarded separately as, for example, $x = -1.5$, $y = -3.375$

Ignore (0, 0) if also given.

(c)

M1: Attempts $\pm 4 \times "-\frac{27}{8}"$ which may be seen within an inequality.

A1: $y \leqslant \frac{27}{2}$ which must be correctly simplified.

Allow it to be written in other correct forms such as $g \leqslant \frac{27}{2}$, $g(x) \leqslant \frac{27}{2}$ or $(-\infty, 13.5]$

The original function is f so $f(x) \leqslant \frac{27}{2}$ would be M1 A0

(d)

M1: For one correct coordinate. Look for $(2, \ldots)$ or $(\ldots, 7)$. Allow for either $x = 2$ or $y = 7$

If the coordinates have been built up mark the final answer.

E.g. $\left(\frac{1}{2}, \frac{3}{8}\right) \rightarrow \left(2, \frac{3}{8}\right) \rightarrow (2, 15) \rightarrow (-6, 15)$ would score M0 A0

A1: $(2, 7)$. Allow $x = 2$ and $y = 7$ ISW after a correct answer

4. OCT 2024 [9]

9 (a)	$f'(x)=\dfrac{(2x+1)\times(12x+4)-2(6x^2+4x-2)}{(2x+1)^2}$	M1 A1
	$=\dfrac{12x^2+12x+8}{(2x+1)^2}$ o.e	A1
		(3)
(b)	At $x=2 \Rightarrow f'(x)=\dfrac{12\times2^2+12\times2+8}{(2\times2+1)^2}=\left(\dfrac{16}{5}\right)$	M1
	Full method of normal $y-6=-\dfrac{5}{16}(x-2)$	dM1
	$16y-96=-5x+10 \Rightarrow \quad 16y+5x=106 \quad *$	A1*
		(3)
(c)	Any correct value $A=3,\ B=\dfrac{1}{2}$ *or* $D=-\dfrac{5}{2}$	B1
	$6x^2+4x-2=(Ax+B)(2x+1)+D \Rightarrow$ Values of A, B and D	M1
	$3x+\dfrac{1}{2}+\dfrac{-5/2}{2x+1}$ o.e	A1
		(3)
(d)	$\displaystyle\int 3x+\frac{1}{2}+\frac{-5/2}{2x+1}\,dx=\frac{3}{2}x^2+\frac{1}{2}x-\frac{5}{4}\ln(2x+1)$	M1, A1 ft
	Area under curve = $\left[\frac{3}{2}x^2+\frac{1}{2}x-\frac{5}{4}\ln(2x+1)\right]_{\frac{1}{3}}^{2}=\left(\frac{3}{2}\times2^2+\frac{1}{2}\times2-\frac{5}{4}\ln(5)\right)-\left(\frac{3}{2}\times\left(\frac{1}{3}\right)^2+\frac{1}{2}\times\left(\frac{1}{3}\right)-\frac{5}{4}\ln\left(\frac{5}{3}\right)\right)$	dM1
	Area of $R=\displaystyle\int_{\frac{1}{3}}^{2} 3x+\frac{1}{2}+\frac{-5/2}{2x+1}dx+\frac{1}{2}\times6\times\left(\frac{106}{5}-2\right)$	M1
	Area of $R=\dfrac{964}{15}-\dfrac{5}{4}\ln 3$	A1
		(5)
		Total 14

(a)
M1: Condoning slips, this is for an attempt using

- the quotient rule to obtain $f'(x)=\dfrac{(2x+1)\times(Ax+B)-C\left(6x^2+4x-2\right)}{(2x+1)^2}$ Must be '–' on the numerator
- the chain rule on $3x+A+\dfrac{B}{2x+1}\Rightarrow f'(x)=3\pm D(2x+1)^{-2}$
- the product rule on $\left(6x^2+4x-2\right)(2x+1)^{-1}\Rightarrow f'(x)=(2x+1)^{-1}\times(Ax+B)-C\left(6x^2+4x-2\right)(2x+1)^{-2}$
  where A, B and C are >0

A1: Correct (unsimplified) expression for $f'(x)$. Look for

- $f'(x)=\dfrac{(2x+1)\times(12x+4)-2\left(6x^2+4x-2\right)}{(2x+1)^2}$ via the quotient rule
- $f'(x)=(2x+1)^{-1}\times(12x+4)+\left(6x^2+4x-2\right)\times-2(2x+1)^{-2}$ via the product rule
- $3+5(2x+1)^{-2}$ following division. Condone an incorrect value for A.

A1: Simplified answers such as $\dfrac{4\left(3x^2+3x+2\right)}{(2x+1)^2}$, $\dfrac{12x^2+12x+8}{4x^2+4x+1}$ or $3+5(2x+1)^{-2}$ following correct A.

ISW following a correct answer

(b)
M1: Substitutes $x=2$ into their attempt at $f'(x)$.
If $f'(x)$ is incorrect then look for correctly embedded 2's or a correct value.

dM1: Full method for finding the equation of the normal at $(2,6)$. Look for $y-6=-\dfrac{1}{f'(2)}(x-2)$

A1*: $y-6=-\dfrac{5}{16}(x-2)$ o.e with at least one correct intermediate line leading to $16y+5x=106$

If $y=mx+c$ is used look for $6=-\dfrac{5}{16}\times2+c\Rightarrow c=\dfrac{106}{16}$ You can allow $y=-\dfrac{5}{16}x+\dfrac{106}{16}$ going straight to the given answer. ISW following sight of correct answer. It must follow a correct derivative in (a)

(c)
B1: Any correct value for $A=3,\ B=\dfrac{1}{2}\ or\ D=-\dfrac{5}{2}$.
They may be embedded within an expression e.g. $3x$ in the quotient of a division sum
M1: Correct method for finding all three constants.
Via comparison/inspection look for $6x^2+4x-2=(Ax+B)(2x+1)+D\Rightarrow$ Values of A, B and D
Via division look for a linear quotient and a constant remainder
A1: $3x+\dfrac{1}{2}+\dfrac{-5/2}{2x+1}$ o.e such as $3x+\dfrac{1}{2}-\dfrac{5}{2(2x+1)}$ which may be seen within an integral in (d).
It is not just for values of A, B and D.
ISW following a correct answer (e.g. they may go on to $\times2$ but then the A1 ft in (d) would be unavailable)

(d)

M1: Attempts to integrate their $Ax + B + \dfrac{D}{2x+1}$. Look for two terms in the correct form, one of which must be the ln term. Award for $...x^2 + ... + k\ln(2x+1)$ or $... + ...x + k\ln(2x+1)$
Also note that $k\ln(4x+2)$ and any multiple of $(2x+1)$ within the ln is also correct

A1ft: $\dfrac{3}{2}x^2 + \dfrac{1}{2}x - \dfrac{5}{4}\ln(2x+1)$ o.e. but follow through on their values of A, B and D

dM1: A correct method of finding the area under the curve using limits of $\dfrac{1}{3}$ and 2 either way around.
Dependent upon the previous M mark. Award if the intention is clear.

M1: For a correct method of finding the area of R. It is scored for adding the area under the curve to the correct calculation for the area of the triangle. The integration of f(x) need not be correct but the limits must be applied the correct way around. The area of the triangle may be done by integration. Look for

$$\int_{\frac{1}{3}}^{2} \mathrm{f}(x)\,\mathrm{d}x + \frac{1}{2}\times 6\times\left(\frac{106}{5}-2\right) = [I(x)]_{\frac{1}{3}}^{2} + \frac{1}{2}\times 6\times\left(\frac{106}{5}-2\right) = I(2) - I\left(\frac{1}{3}\right) + \frac{1}{2}\times 6\times\left(\frac{106}{5}-2\right) = ...$$

where $I(x)$ is their attempt at $\int \mathrm{f}(x)\,\mathrm{d}x$

A1: $\dfrac{964}{15} - \dfrac{5}{4}\ln 3$

5. JAN 2024 [4]

4(a)	$\mathrm{f}(x)=\dfrac{2x^2-32}{(3x-5)(x+4)}+\dfrac{8}{3x-5}$	B1
	$=\dfrac{2x^2-32}{(3x-5)(x+4)}+\dfrac{8}{3x-5}=\dfrac{2x^2-32+8(x+4)}{(3x-5)(x+4)}$ or e.g. $=\dfrac{2(x-4)(x+4)}{(3x-5)(x+4)}+\dfrac{8}{3x-5}=\dfrac{2(x-4)+8}{(3x-5)}$	M1
	$=\dfrac{2x\,\cancel{(x+4)}}{(3x-5)\,\cancel{(x+4)}}=\dfrac{2x}{3x-5}$ *	A1*
		(3)
(b)	$\mathrm{f}'(x)=\dfrac{2(3x-5)-3\times 2x}{(3x-5)^2}$	M1A1
	$\mathrm{f}'(x)=\dfrac{-10}{(3x-5)^2}$ As $(3x-5)^2>0$ (for $x>2$) then $\mathrm{f}'(x)<0$ Hence f is a decreasing function *	A1cso*
		(3)
(c)	$\mathrm{g}^{-1}(x)=\mathrm{e}^{\frac{x-3}{2}}$	M1A1
	$x \ldots 3$	B1
		(3)
(d)	$\mathrm{g}\left(\dfrac{2a}{3a-5}\right)=5\Rightarrow 3+2\ln\left(\dfrac{2a}{3a-5}\right)=5$	B1
	$\ln\left(\dfrac{2a}{3a-5}\right)=1\Rightarrow\dfrac{2a}{3a-5}=\mathrm{e}$	M1
	$\dfrac{2a}{3a-5}=\mathrm{e}\Rightarrow a=\ldots$	**d**M1
	$a=\dfrac{5\mathrm{e}}{3\mathrm{e}-2}$	A1
		(4)
(d) Way 2	$\mathrm{gf}(a)=5\Rightarrow\dfrac{2a}{3a-5}=\mathrm{g}^{-1}(5)$	B1
	$\dfrac{2a}{3a-5}=\mathrm{g}^{-1}(5)\Rightarrow\dfrac{2a}{3a-5}=\mathrm{e}$	M1
	$\dfrac{2a}{3a-5}=\mathrm{e}\Rightarrow a=\ldots$	**d**M1
	$a=\dfrac{5\mathrm{e}}{3\mathrm{e}-2}$	A1
		(13 marks)

(a)

B1: $3x^2+7x-20=(3x-5)(x+4)$ seen or used

M1: Combines fractions with a correct common denominator and the order of terms in the numerator consistent with their common denominator.

A1*: Achieves the given answer with no errors seen but condone e.g. a missing trailing bracket as long as it is "recovered".

Sufficient working should be shown but allow to go from e.g. $\frac{2(x-4)+8}{3x-5}$ to $\frac{2x}{3x-5}$

Note that candidates may take a longer route:

$$\frac{2x^2-32}{3x^2+7x-20}+\frac{8}{3x-5}=\frac{(2x^2-32)(3x-5)+8(3x^2+7x-20)}{(3x^2+7x-20)(3x-5)} \quad \textbf{M1}$$

$$=\frac{6x^3+14x^2-40x}{(3x^2+7x-20)(3x-5)}=\frac{2x(3x^2+7x-20)}{(3x-5)(x+4)(3x-5)} \quad \textbf{B1}$$

$$=\frac{2x(3x-5)(x+4)}{(3x-5)(x+4)(3x-5)}=\frac{2x}{3x-5}* \quad \textbf{A1*}$$

Or

$$\frac{2x^2-32}{3x^2+7x-20}+\frac{8}{3x-5}=\frac{(2x^2-32)(3x-5)+8(3x^2+7x-20)}{(3x^2+7x-20)(3x-5)} \quad \textbf{M1}$$

$$=\frac{6x^3+14x^2-40x}{(3x^2+7x-20)(3x-5)}=\frac{2x(3x^2+7x-20)}{(3x^2+7x-20)(3x-5)} \quad \textbf{B1}$$

In this case the B1 is scored for obtaining a factor of $3x^2+7x-20$ in the numerator and denominator

$$=\frac{2x}{3x-5}* \quad \textbf{A1*}$$

(b)

M1: Attempts to differentiate using the product or quotient rule.

Award for an expression of the form $\dfrac{\alpha(3x-5)-\beta x}{(3x-5)^2}$ or $\alpha(3x-5)^{-1}-\beta x(3x-5)^{-2},\ \alpha,\beta>0$

Condone attempts where $(3x-5)^2$ is expanded.

Alternatively, attempts to write $\dfrac{2x}{3x-5}$ as $A+\dfrac{B}{3x-5}$ before differentiating using the chain rule to obtain $\dfrac{\pm k}{(3x-5)^2}$

A1: Correct derivative simplified or unsimplified.

A1cso*: Requires a correct simplified derivative $\dfrac{-10}{(3x-5)^2}$ and then statements that convey:

- $(3x-5)^2>0$ oe e.g. denominator is positive (condone $(3x-5)^2\ \dots 0$)
- so $\mathrm{f}'(x)<0$
- function is decreasing

Some candidates may attempt to differentiate the original f(x) e.g.:

$$\mathrm{f}'(x)=\frac{4x\left(3x^2+7x-20\right)-\left(2x^2-32\right)(6x+7)}{(3x^2+7x-20)^2}-\frac{24}{(3x-5)^2}$$

Score M1 for:

$$\mathrm{f}'(x)=\frac{Ax\left(3x^2+7x-20\right)-\left(2x^2-32\right)(Cx+D)}{(3x^2+7x-20)^2}-\frac{E}{(3x-5)^2}$$

and (<u>first</u>) A1 if they reach $\dfrac{-10}{(3x-5)^2}$

(c)

M1: Rearranges $y=3+2\ln x$ to $x=\mathrm{e}^{\mathrm{f}(y)}$ or $x=3+2\ln y$ to $y=\mathrm{e}^{\mathrm{f}(x)}$

A1: Obtains $\mathrm{g}^{-1}(x)=\mathrm{e}^{\frac{x-3}{2}}$ or equivalent e.g. $\mathrm{g}^{-1}(x)=\sqrt{\mathrm{e}^{x-3}}$, $\mathrm{g}^{-1}(x)=\mathrm{e}^{\frac{3-x}{-2}}$ and isw.

Accept $\mathrm{g}^{-1}(x)=...$, $\mathrm{g}^{-1}=...$, $y=...$ but not e.g. $\mathrm{f}^{-1}(x)=...$

B1: Correct domain: $x\ \dots 3$ oe using correct notation e.g. $[3,\infty)$

(d) Condone the miscopy of f(x) for the M marks as long as it has the correct form e.g. $\frac{...a}{...a \pm ...}$

and allow use of x instead of a for all marks in (d)

B1: Sets up a valid equation in a e.g. $3+2\ln\left(\frac{2a}{3a-5}\right)=5$

M1: Rearranges to obtain $\ln\left(\frac{2a}{3a-5}\right)=k$ and uses the inverse law of logarithms to remove the ln

correctly to reach $\frac{2a}{3a-5}=\text{e}^k$ oe

dM1: Rearranges an equation of the form $\frac{2a}{3a-5}=\text{e}^k$ to obtain $a=\frac{...\text{e}^k}{...\text{e}^k \pm ...}$ or equivalent.

Depends on the previous method mark.

A1: $(a=)\ \frac{5\text{e}}{3\text{e}-2}$ oe e.g. $(a=)\ \frac{-5\text{e}}{2-3\text{e}}$ and apply isw once the correct exact answer is seen.

Accept e^1 for e.

(d) Way 2

B1: Sets up a valid equation in a e.g. $\frac{2a}{3a-5}=\text{g}^{-1}(5)$ seen or implied.

M1: Attempts $\text{g}^{-1}(5)$ and reaches $\frac{2a}{3a-5}=\text{e}^k$ oe

dM1: Rearranges an equation of the form $\frac{2a}{3a-5}=\text{e}^k$ to obtain $a=\frac{...\text{e}^k}{...\text{e}^k \pm ...}$ or equivalent.

Depends on the previous method mark.

A1: $(a=)\ \frac{5\text{e}}{3\text{e}-2}$ oe e.g. $(a=)\ \frac{-5\text{e}}{2-3\text{e}}$ and apply isw once the correct exact answer is seen.

Accept e^1 for e.

Alternative for (d) which has been seen:

$$g\left(\frac{2a}{3a-5}\right)=5 \Rightarrow 3+2\ln\left(\frac{2a}{3a-5}\right)=5:$$ **B1 as above**

$$\Rightarrow \ln\left(\frac{2a}{3a-5}\right)^2=2 \Rightarrow \left(\frac{2a}{3a-5}\right)^2=e^2:$$ **M1 rearranges to reach** $\left(\frac{2a}{3a-5}\right)^2=e^{\cdots}$

$$\Rightarrow \left(9e^2-4\right)a^2-30e^2a+25e^2=0$$

$$\Rightarrow a=\frac{30e^2 \pm \sqrt{900e^4-100e^2\left(9e^2-4\right)}}{2\left(9e^2-4\right)}:$$ **dM1 forms and solves 3TQ in *a* (usual rules)**

$$=\frac{5e}{3e-2}$$ oe: **A1 as above**

6. JUNE 2024 [6]

6 (a)	$y=(4x-7)^{\frac{1}{2}} \Rightarrow \left(\frac{dy}{dx}=\right)2(4x-7)^{-\frac{1}{2}}$ (see notes)	M1 A1
	At $(8,5)$ gradient of tangent is $2(4\times8-7)^{-\frac{1}{2}}\left(=\frac{2}{5}\right)$	dM1
	Equation for l is $y-5=-\frac{5}{2}(x-8)$	ddM1
	$2y-10=-5x+40 \Rightarrow 5x+2y-50=0$ *	A1*
		(5)
(b)	$\int (4x-7)^{\frac{1}{2}}\,dx=\left[\frac{(4x-7)^{\frac{3}{2}}}{6}\right]$ (see notes)	M1, A1
	Complete area $=\int_{\frac{7}{4}}^{8}(4x-7)^{\frac{1}{2}}\,dx=\left[\frac{(4x-7)^{\frac{3}{2}}}{6}\right]_{\frac{7}{4}}^{8}+5$	dM1
	$=\frac{155}{6}$	A1
		(4)
		(9 marks)

(a)

M1: Attempts to differentiate to achieve $a(4x\pm 7)^{-\frac{1}{2}}$ or equivalent. The index does not need to be processed.

Alternatively, attempts to differentiate implicitly $y^2 = 4x - 7 \Rightarrow Ay\frac{dy}{dx} = B$

A1: Achieves $2(4x-7)^{-\frac{1}{2}}$ o.e. (which may be unsimplified but the index processed) or using implicit differentiation achieves $2y\frac{dy}{dx} = 4$

dM1: Substitutes $x = 8$ into their $a(4x\pm 7)^{-\frac{1}{2}}$ or substitutes $y = 5$ into their $Ay\frac{dy}{dx} = B$. It does not need to be evaluated. It is dependent on the first method mark but allow this mark to be scored if they have incorrectly manipulated their derivative before substituting in, or made transcription errors including if they lose the $-\frac{1}{2}$ index from the bracket.

ddM1: A full method for the equation of *l*. Look for

- substitution of $x = 8$ (or $y = 5$ if implicit differentiation) into an attempt at a derivative
- the application of the negative reciprocal rule
- the use of $(8,5)$ with a correct gradient for their value of *m* to form an equation for the normal.

$y - 5 = "-\frac{5}{2}"(x-8)$ with the coordinates in the correct positions. If $y = -\frac{1}{m}x + c$ is used they must proceed as far as $c = ...$

It is dependent on both of the previous two method marks.

A1*: Correctly achieves $5x + 2y - 50 = 0$ (in any order on the same side of the equation).
There must be a correct intermediate stage of working following their initial equation for *l* before achieving the given answer, e.g. $y - 5 = -\frac{5}{2}(x-8) \Rightarrow 2y - 10 = -5x + 40 \Rightarrow 5x + 2y - 50 = 0$

(b) **Note that if no integration is seen then 0 marks.**

M1: Integrates to achieve a correct form $b(4x\pm7)^{\frac{3}{2}}$ o.e. (the index does not need to be processed)

May use substitution e.g. $u=4x\pm7 \Rightarrow \int \frac{u^{\frac{1}{2}}}{4}\,du \Rightarrow Qu^{\frac{3}{2}}$ (the integral must be correct but allow a slip on the coefficient for the integrated expression)

A1: Correct integration $\frac{(4x-7)^{\frac{3}{2}}}{6}$ (which may be unsimplified) with or without a constant of integration. Condone poor notation e.g. if the integral sign and/or dx is still present. Index needs to be processed.

Using the substitution method score for e.g. $\frac{u^{\frac{3}{2}}}{6}$ where $u=4x-7$

dM1: Full method to find the shaded area. Note the area of the triangle is $\frac{1}{2}\times(10-8)\times5=5$

Look for $\left[b(4x\pm7)^{\frac{3}{2}}\right]_{\frac{7}{4}}^{8}+5$ o.e. (Provided this is seen then it is sufficient to proceed to the answer)

The limits must be correct for the integration

May see integration used for the triangle

e.g. $\int_{\frac{7}{4}}^{8}(4x\pm7)^{\frac{1}{2}}\,dx+\int_{8}^{10}\left(-\frac{5}{2}x+25\right)dx \Rightarrow \left[b(4x\pm7)^{\frac{3}{2}}\right]_{\frac{7}{4}}^{8}+\left[-\frac{5}{4}x^2+25x\right]_{8}^{10}$

It is dependent on the previous method mark, but you do not need to see the limits substituted in. If they do not use the given equation of the straight line l then this mark cannot be scored. Condone poor notation provided the intention is clear.

If using the substitution you may see e.g. $\left[\frac{u^{\frac{3}{2}}}{6}\right]_{0}^{25}+5$ (send to review if unsure)

A1: $\frac{155}{6}$ or exact equivalent e.g. $25\frac{5}{6}$ or $25.8\dot{3}$ provided all the previous marks have been scored.

7. JAN 2023 [3]

3(a)	$\log_{10} y = \frac{5}{16}x + 1.5$	M1A1
		(2)
(b)	$\log_{10} y = "\frac{5}{16}"x + "1.5" \Rightarrow y = 10^{"\frac{5}{16}"x + "1.5"}$	M1
	$\Rightarrow y = 10^{"\frac{5}{16}"x} \times 10^{"1.5"}$	M1
	$y = 31.6 \times 2.05^x$	A1
		(3)
		Total 5

(a)

M1: Scored for a complete attempt to get the equation of the line condoning $\log_{10} y \leftrightarrow y \leftrightarrow l$ and an incorrect sign on the gradient. So, allow for $(\log_{10} y) = \pm\frac{1.5}{4.8}x + 1.5$ o.e. and for $\frac{y-0}{x+4.8} = \pm\frac{1.5}{4.8}$

If this is attempted via simultaneous equations the mark is scored when the candidate reaches $m = \pm 0.3125$ $c = 1.5$

A1: Correct equation. e.g. $\log_{10} y = \frac{5}{16}x + 1.5$ or equivalent such as $16\log_{10} y = 5x + 24$

The $\log_{10}$ must not appear as "ln" but allow as "log" or "lg"

(b)

Main Method: Starting with their $\log_{10} y = mx + c$

M1: "Removes" the logs in their equation. e.g. $\log_{10} y = "m"x + "c" \Rightarrow y = 10^{"m"x + "c"}$

M1: "Correct" strategy to obtain values of k and b or else proceeding correctly to a form $y = 10^{"\frac{5}{16}"x} \times 10^{"1.5"}$

Allow for $k = 10^{"1.5"} (= 31.6)$ and $b = 10^{"\frac{5}{16}"} = (2.05)$. Note that you may see $10^{1.5}$ as $10\sqrt{10}$

A1: Correct equation produced, $y = 31.6 \times 2.05^x$, and no errors seen.

Condone correct working followed by $k = 31.6, b = 2.05$ with the equation being implied

cso ....the values must be 31.6 and 2.05, not values rounding to these numbers or exact values like $10\sqrt{10}$.

Note: A solution may be fudged from working similar to the following

$\log_{10} y = mx + c \Rightarrow y = 10^{mx} + 10^{c} \Rightarrow y = 31.6 + 2.05^x \Rightarrow y = 31.6 \times 2.05^x$

This will score Special case: M0 M1 A0

Alternative Method: Starting with $y = kb^x$

M1: Takes logs of both sides and applies at least one correct log law

e.g. $\log_{(10)} kb^x = \log_{(10)} k + \log_{(10)} b^x$, $\log_{(10)} b^x = x\log_{(10)} b$

M1: "Correct" strategy to obtain values for k and b from their $y = mx + c$

So $\log_{10} k = "c" \Rightarrow k = 10^c$ and $\log_{10} b = "m" \Rightarrow b = 10^m$

A1: Correct equation produced $y = 31.6 \times 2.05^x$ and no errors seen.
Condone correct working followed by $k = 31.6, b = 2.05$ with the equation being implied
cso ....the values must be 31.6 and 2.05, not values rounding to these numbers.
Correct answers for k and b without any working scores M0 M1 A0. Instructions on the paper state that they should show sufficient working to make their method clear.

8. JAN 2023 [4]

4(a)	Any correct constant, so for $A=2$ or $B=3$ or $C=-1$ or $D=5$	B1
	$2x^4+15x^3+35x^2+21x-4=Ax^2(x+3)^2+Bx(x+3)^2+C(x+3)^2+D$ $\Rightarrow A=...,B=...,C=...,D=...$ or $2x^4+15x^3+35x^2+21x-4\div(x^2+6x+9)=...x^2+...x+...+\dfrac{...}{(x+3)^2}$	M1
	2 correct of $A=2,B=3,C=-1,D=5$	A1
	$A=2,B=3,C=-1,D=5$	A1
		(4)
(b)	$\int f(x)\,dx=\int\left(2x^2+3x-1+\dfrac{5}{(x+3)^2}\right)dx=\dfrac{2x^3}{3}+\dfrac{3x^2}{2}-x-\dfrac{5}{x+3}(+c)$	M1A1ftA1
		(3)
		Total 7

(a)

B1: One correct constant or one correct term in $Ax^2+Bx+C+\dfrac{D}{(x+3)^2}$

M1: Complete method for finding A, B, C and D
For example substitution/comparing coefficients/long division
Via substitution/comparing coefficients the minimum required is an identity of the correct form (condoning slips) followed by values for A, B, C and D.
See scheme but there are other versions including

$2x^4+15x^3+35x^2+21x-4=(Ax^2+Bx+C)(x+3)^2+D\Rightarrow A=...,B=...,C=...,D=...$

Via division look for a divisor of x^2+6x+9, a quotient that is a quadratic and a remainder that is either linear or a constant term.
It could be attempted by dividing by $(x+3)$ twice.
FYI, the first division gives $2x^3+9x^2+8x-3$ with a remainder of 5
A1: 2 correct constants following the award of M1
For division, when the remainder is a linear term, it would be scored for two correct of Ax^2+Bx+C
A1: All correct following the award of M1

(b)

M1: $\int\dfrac{D}{(x+3)^2}dx\rightarrow\dfrac{k}{x+3}$ where k is a constant.

This may be awarded following a term of $\int\dfrac{\alpha x+D}{(x+3)^2}dx$ following division.

Look for $\int\dfrac{\alpha x+D}{(x+3)^2}dx$ being correctly split and $\rightarrow\int\dfrac{\alpha x}{(x+3)^2}dx+\int\dfrac{D}{(x+3)^2}dx\rightarrow$ something $+\dfrac{k}{x+3}$

A1ft: $\int\left(Ax^2+Bx+C+\frac{D}{(x+3)^2}\right)\mathrm{d}x=\frac{Ax^3}{3}+\frac{Bx^2}{2}+Cx-\frac{D}{x+3}(+c)$.

Correct integration following through on their non-zero constants.
Allow this to be scored with A, B, C and D as above or with made up values
A1: All correct with or without "+ c". Allow $-1x$ for $-x$

9. JAN 2023 [8]

8	$\int (2\cos x - \sin x)^2\, dx = \int (4\cos^2 x - 4\sin x\cos x + \sin^2 x)\, dx$	M1
	$\int 4\sin x\cos x\, dx = \int 2\sin 2x\, dx = -\cos 2x$ or $\int 4\sin x\cos x\, dx = -2\cos^2 x$ or $2\sin^2 x$	M1
	$\int (4\cos^2 x + \sin^2 x)\, dx = \int (1 + 3\cos^2 x)\, dx = \int \left(1 + 3\left(\frac{\cos 2x + 1}{2}\right)\right) dx$ or $\int (4\cos^2 x + \sin^2 x)\, dx = \int \left(4\left(\frac{\cos 2x + 1}{2}\right) + \frac{1 - \cos 2x}{2}\right) dx$	M1
	$\int (2\cos x - \sin x)^2\, dx = \frac{3}{4}\sin 2x + \cos 2x + \frac{5}{2}x (+c)$ or $\int (2\cos x - \sin x)^2\, dx = \frac{3}{4}\sin 2x + 2\cos^2 x + \frac{5}{2}x (+c)$ or $\int (2\cos x - \sin x)^2\, dx = \frac{3}{4}\sin 2x - 2\sin^2 x + \frac{5}{2}x (+c)$	A1A1
		(5)
		Total 5

Condone changes in variables throughout this solution as long as the answer is given in terms of x

M1: Expands to the form $p\cos^2 x + q\sin x\cos x + r\sin^2 x$

M1: Correct strategy for **integrating** $q\sin x\cos x$ (i.e. obtains $k\cos 2x$ or $k\sin^2 x$ or $k\cos^2 x$)

M1: Correct strategy for rewriting $p\cos^2 x + r\sin^2 x$ **into a form that can be integrated**.

Score for one of

- writes in terms of just $\sin^2 x$ and then uses $\sin^2 x = \frac{\pm 1 \pm \cos 2x}{2}$
- writes in terms of just $\cos^2 x$ and then uses $\cos^2 x = \frac{\pm 1 \pm \cos 2x}{2}$
- writes both $\sin^2 x$ and $\cos^2 x$ in terms of $\cos 2x$ with **at least one** of these via use of a correct/allowable form. That is $\sin^2 x = \frac{\pm 1 \pm \cos 2x}{2}$ or $\cos^2 x = \frac{\pm 1 \pm \cos 2x}{2}$

A1: Integrates and achieves 2 correct terms (of the 3 required terms)

NB: An unsimplified expression is acceptable for this mark so please check carefully. e.g. $2x + ... + \frac{1}{2}x$ counts as one correct term.

A1: Correct simplified integration (+ c not required).

An alternative solution is via "R cos" or "R sin" but the last two marks are unlikely to be achieved due to the fact that an exact answer is difficult to arrive at.

8	$\int (2\cos x - \sin x)^2\,dx = \int 5\cos^2(x+0.464)\,dx$	M1
	$= \int \frac{5\{\cos(2x+0.927)+1\}}{2}\,dx$	M1
	$\frac{5\sin(2x+0.927)}{4} + \frac{5}{2}x$	M1
	$\frac{3\sin 2x}{4} + \cos 2x + \frac{5}{2}x + c$	A1A1
		(5)
		Total 5

M1: Writes $2\cos x - \sin x$ in the form $R\cos(x \pm \alpha)$ o.e. and squares.

Requires a full method so for the form $R\cos(x \pm \alpha)$

- requires $R^2 = 2^2 + 1^2$
- requires $\tan\alpha = \pm\frac{1}{2} \Rightarrow \alpha = ...$ in radians but condone in degrees

M1: Correct strategy for writing $\cos^2(x \pm \alpha)$ into a form that can be integrated using the double angle formula

$\cos^2(x \pm \alpha) \to \frac{\pm 1 \pm \cos(2x \pm 2\alpha)}{2}$ but condone $\cos^2(x \pm \alpha) \to \frac{\pm 1 \pm \cos(2x \pm \alpha)}{2}$

α should be in radians but condone in degrees

M1: Correct strategy for writing $\cos^2(x \pm \alpha)$ into a form that can be integrated using the double angle formula

$\cos^2(x \pm \alpha) \to \frac{\pm 1 \pm \cos(2x \pm 2\alpha)}{2}$ but condone $\cos^2(x \pm \alpha) \to \frac{\pm 1 \pm \cos(2x \pm \alpha)}{2}$

α should be in radians but condone in degrees

M1: It is for integrating $\cos(2x \pm \delta) \to \pm\sin(2x \pm \delta)$ following use of an acceptable double angle formula

A1: 2 correct terms of $\frac{3\sin 2x}{4} + \cos 2x + \frac{5}{2}x + c$

A1: Fully correct and simplified (+ c not required) or $\frac{5\sin(2x + 2\arctan 0.5)}{4} + \frac{5}{2}x$

8	$\int (2\cos x - \sin x)^2\,dx = \int 5\sin^2(x-1.107)\,dx$	M1
	$= \int \frac{5\{1-\cos(2x-2.21)\}}{2}\,dx$	M1
	$= \frac{5}{2}x - \frac{5\sin(2x-2.21)}{4}$	M1
	$= \frac{5}{2}x + \frac{3\sin 2x}{4} + \cos 2x + c$	A1A1
		(5)
		Total 5

10. JAN 2023 [10]

10(a)	$(F=)35$	B1
		(1)
(b)	$200=\dfrac{350e^{15k}}{9+e^{15k}} \Rightarrow 1800+200e^{15k}=350e^{15k} \Rightarrow 150e^{15k}=1800$	M1
	$e^{15k}=\dfrac{1800}{150} \Rightarrow 15k=\ln 12 \Rightarrow k=\dfrac{1}{15}\ln 12\,*$	dM1A1*
		(3)
(c)	$F=\dfrac{350e^{kt}}{9+e^{kt}} \Rightarrow \dfrac{dF}{dt}=\dfrac{350ke^{kt}(9+e^{kt})-350e^{kt}(ke^{kt})}{(9+e^{kt})^2}$	M1A1
	$\dfrac{3150ke^{kt}}{(9+e^{kt})^2}=10 \Rightarrow 315ke^{kt}=81+18e^{kt}+e^{2kt} \Rightarrow e^{2kt}+(18-315k)e^{kt}+81=0$	M1
	$e^{2kt}+(18-315k)e^{kt}+81=0 \Rightarrow e^{kt}=\dfrac{315k-18\pm\sqrt{(18-315k)^2-4\times 81}}{2} \Rightarrow kt=...$	M1
	$T=$ awrt 5.7, 20.8	A1
		(5)
		Total 9

(a)
B1: Correct value, 35
(b)
M1: Uses $F=200$ and $t=15$ and reaches $Ae^{15k}=B$ where $A\times B>0$
dM1: Proceeds using correct order of operations to obtain a value for k

Look for $Ae^{15k}=B \Rightarrow e^{15k}=\dfrac{B}{A} \Rightarrow 15k=\ln\dfrac{B}{A} \Rightarrow k=...$

or Alternatively $Ae^{15k}=B \Rightarrow \ln A+15k=\ln B \Rightarrow 15k=\ln B-\ln A \Rightarrow k=...$
A1*: Correct proof with all necessary steps shown.

Minimum acceptable solution $200=\dfrac{350e^{15k}}{9+e^{15k}} \Rightarrow 150e^{15k}=1800 \Rightarrow e^{15k}=12 \Rightarrow k=\dfrac{1}{15}\ln 12$

(c) Allow the whole of part (c) to be done with the letter k, the exact value for k or using $k=$ awrt 0.166
M1: Correct attempt of the quotient (product or chain) rule.

For the Quotient Rule look for $\dfrac{dF}{dt}=\dfrac{Pe^{kt}(9+e^{kt})-Qe^{kt}(e^{kt})}{(9+e^{kt})^2}$ $\quad P,Q>0$

For the Product Rule look for $\dfrac{dF}{dt}=Pe^{kt}(9+e^{kt})^{-1}\pm Qe^{kt}e^{kt}(9+e^{kt})^{-2}$ $\quad P,Q>0$

For Chain Rule look for $F=A\pm\dfrac{B}{9+e^{kt}} \Rightarrow \dfrac{dF}{dt}=Qe^{kt}(9+e^{kt})^{-2}$ $\quad Q$ is a constant

A1: Correct differentiation, which may be unsimplified.

Allow for an expression in k, with exact $k=\frac{1}{15}\ln 12$, or using $k=$ awrt 0.166

M1: Sets their derivative = 10 and obtains a 3TQ in e^{kt}
It is dependent upon a reasonable attempt to differentiate.

In almost all cases the M1 will have been awarded but condone an attempt following $\frac{dF}{dt}=\frac{vu'+uv'}{v^2}$

M1: Scored for

- solving a 3TQ in e^{kt} by any method including a calculator (you may need to check with accuracy to 2sf rounded or truncated). Note that the equation may be a quadratic in $12^{\frac{1}{15}t}$ instead of $e^{\left(\frac{1}{15}\ln 12\right)t}$
- then taking ln's to obtain at least one value for kt
  FYI the correct quadratics are;
  $e^{\left(\frac{2}{15}\ln 12\right)t}+(18-21\ln 12)e^{\left(\frac{1}{15}\ln 12\right)t}+81=0,$
  or $\left(e^{0.166t}\right)^2-34.18e^{0.166t}+81=0\Rightarrow e^{0.166t}=2.56, 31.62\Rightarrow 0.166t=0.94, 3.45$

A1: awrt 5.7, 20.8

11. JUNE 2023 [3]

3 (i)	$\frac{d}{dx}\ln\left(\sin^2 3x\right) = \frac{1}{\sin^{\not 2} 3x} \times 2\not{\sin 3x} \times 3\cos 3x = 6\cot 3x$	M1 A1 **(2)**
(ii) (a)	$\frac{d}{dx}\left(3x^2-4\right)^6 = 36x\left(3x^2-4\right)^5$	M1 A1 **(2)**
(b)	$\int x\left(3x^2-4\right)^5 dx = \frac{1}{36}\left(3x^2-4\right)^6$	B1ft
	$\int_0^{\sqrt{2}} x\left(3x^2-4\right)^5 dx = \left[\frac{1}{36}\left(3x^2-4\right)^6\right]_0^{\sqrt{2}} = \frac{1}{36}(2)^6 - \frac{1}{36}(-4)^6 = -112$	M1 A1cso **(3)** **(7 marks)**

Notes

(i)

M1 Attempts to differentiate a ln function. Award for $\frac{d}{dx}\ln\left(\sin^2 3x\right) = \frac{1}{\sin^2 3x} \times \ldots$ where ... could be 1

An alternative could be $\frac{d}{dx}\ln\left(\sin^2 3x\right) = \frac{d}{dx}2\ln\left(\sin 3x\right) = (2\times)\frac{1}{\sin 3x}\times\ldots$ or $\frac{d}{dx}\ln\left(\frac{1-\cos 6x}{2}\right) = \frac{2}{1-\cos 6x}\times\ldots$

A1 $6\cot 3x$ o.e. such as $\frac{6\cos 3x}{\sin 3x}$ or $\frac{6}{\tan 3x}$ or $6(\tan 3x)^{-1}$ but not $6\tan^{-1}3x$ Accept also $\frac{6\sin 6x}{1-\cos 6x}$ or $\frac{3\sin 6x}{\sin^2 3x}$ and isw after a suitably simplified answer.

Constant terms must be gathered and no uncancelled common factors in numerator and denominator.

(ii) (a)

M1 Achieves $\frac{d}{dx}\left(3x^2-4\right)^6 = Ax\left(3x^2-4\right)^5$ where A is a constant which may be 1.

A1 $\frac{d}{dx}\left(3x^2-4\right)^6 = 36x\left(3x^2-4\right)^5$ oe. Need not be simplified. Isw after a correct answer.

(ii) (b)

B1ft $\int x\left(3x^2-4\right)^5 dx = \frac{1}{36}\left(3x^2-4\right)^6$ or $\frac{1}{A}\left(3x^2-4\right)^6$ following through on their (a) provided it is of the form $\frac{d}{dx}\left(3x^2-4\right)^6 = Ax\left(3x^2-4\right)^5$ This may arise from attempts via substitution and can be scored from a restart if (ii)(a) was incorrect. Need not be simplified and isw if simplified incorrectly. Condone notation errors such as unneeded integral signs - mark the expression that is their attempt at the integration.

M1 Substitutes in both limits and subtracts (either way round) into an expression of the form $D\left(3x^2-4\right)^6$ where D is a constant but allow slips such as a missing power if the intention is clear. Sight of the subtraction is sufficient. Implied by the correct answer for their integral if substitution not seen. If using integration by substitution they must be substituting the **correct** limits for their variable.

A1cso $(R=)\ -112$ and isw if they make the answer positive after a correct answer seen.
Note: Answer only with no working at all shown scores no marks. Correct integral must be seen.
Note: Attempts at integration by parts are unlikely to succeed, but if done correctly and achieve the correct form of the answer may score the relevant marks.

Note (ii) may be completed by expansion.

(a)

M1 Requires expansion to form $ax^{12}+bx^{10}+cx^8+dx^6+ex^4+fx^2+g$ followed by an attempt to integrate each term (power decreased by 1)

A1 Requires correct derivative. $8748x^{11}-58320x^9+155520x^7-207360x^5+138240x^3-36864x$

(b)

B1ft Correct answer from a restart, which may be via expansion

$$\frac{81x^{12}}{4}-162x^{10}+540x^8-960x^6+960x^4-512x^2$$

M1 Substitutes both limits and subtracts into an expression of the form $ax^{12}+bx^{10}+cx^8+dx^6+ex^4+fx^2$

A1cso As main scheme.

12. OCT 2023 [3]

3(a)	$(\cos 2A \equiv)\cos A\cos A - \sin A\sin A \equiv \cos^2 A - (1-\cos^2 A)$	M1
	$\Rightarrow \cos 2A \equiv 2\cos^2 A - 1$ *	A1*
		(2)
(b)	$\int (3-2\cos 6x)\,\mathrm{d}x = 3x - \frac{\sin 6x}{3} \quad (+c)$	M1A1
	$\left[3x - \frac{\sin 6x}{3}\right]_{\frac{\pi}{12}}^{\frac{\pi}{8}} = \left(3\left(\frac{\pi}{8}\right) - \frac{\sin\left(6\times\frac{\pi}{8}\right)}{3}\right) - \left(3\left(\frac{\pi}{12}\right) - \frac{\sin\left(6\times\frac{\pi}{12}\right)}{3}\right) = \frac{1}{8}\pi + \frac{2-\sqrt{2}}{6}$	dM1A1
		(4)
		(6 marks)

Notes
(a)
M1: Writes $(\cos 2A \equiv)\cos A\cos A - \sin A\sin A$ and attempts to **use** (not just stated separately) the identity $\pm\sin^2 A \pm \cos^2 A = \pm 1$ Note going directly to $\cos 2A \equiv \cos^2 A - \sin^2 A$ is allowed M1 as long as a use of the Pythagorean identity is clear. NB Watch out for answers which use double angle identities in the proof (circular reasoning) and score M0 if no Pythagorean identity is used.
A1*: Achieves the given answer with no errors seen (and no circular reasoning). Must see $\cos 2A \equiv \cos A\cos A - \sin A\sin A$ and clear attempt at substitution of $\sin^2 A + \cos^2 A = 1$ but allow "LHS" for "cos 2*A*". Going directly to $\cos 2A \equiv \cos^2 A - \sin^2 A$ in the first step would score A0. Correct notation must used throughout but condone a missing closing bracket.
Note : $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1$ with no further evidence scores M0A0 as they are just writing out known formulae.
(b)
M1: Attempts to substitute in the given result in part (a) and integrates to the form $...x \pm ...\sin 6x$ Must be a full substitution in terms of *x*, $\cos 2A \to k\sin 2A$ is M0 unless recovered, e.g by later substitution may be implied by the limits substituted).
A1: $3x - \frac{\sin 6x}{3}$
dM1: Shows evidence of substituting in both limits into an expression of the form $...x \pm ...\sin 6x$ and subtracts either way round to achieve an exact (not necessarily simplified but trig must be evaluated) answer.
A1: Correct answer e.g as scheme or $\frac{1}{8}\pi + \frac{1}{3} - \frac{\sqrt{2}}{6}$ or exact equivalent (and isw after a correct answer).

13. JAN 2022 [3]

3 (i)	$\int(2x-5)^7\,dx=\frac{(2x-5)^8}{16}+c$	M1 A1
		(2)
(ii)	$\int\frac{4\sin x}{1+2\cos x}\,dx=-2\ln(1+2\cos x)\quad(+c)$	M1 A1
	$\int_0^{\frac{\pi}{3}}\frac{4\sin x}{1+2\cos x}\,dx=\left[-2\ln(1+2\cos x)\right]_0^{\frac{\pi}{3}}=-2\ln 2+2\ln 3=\ln\frac{9}{4}$	dM1 A1
		(4)
		(6 marks)

(i)

M1: Achieves $a(2x-5)^8$ or equivalent where a is a constant. Alternatively achieves au^8 with $u=(2x-5)$

Allow this mark from a miscopy such as $\int(2x-3)^7\,dx=k(2x-3)^8$

A1: Achieves $\frac{(2x-5)^8}{16}+c$ or exact simplified equivalent such as $\frac{1}{16}(2x-5)^8+c$. **The +*c* (o.e) must be present.**

Any attempts that start by multiplying out $(2x-5)^7$ are likely to end in failure. They are unlikely to get an expression of the form $a(2x-5)^8$

FYI

$(2x-5)^7=128x^7-2240x^6+16800x^5-70000x^4+175000x^3-262500x^2+218750x-78125$

Score B1 SC for at least 5 out of 8 **correct** terms of

$\frac{128}{8}x^8-\frac{2240}{7}x^7+\frac{16800}{6}x^6-\frac{70000}{5}x^5+\frac{175000}{4}x^4-\frac{262500}{3}x^3+\frac{218750}{2}x^2-78125x$

(ii)

M1: Achieves $b\ln(1+2\cos x)$ or $b\ln|1+2\cos x|$ where b is a constant. Condone a missing bracket

Alternatively achieves $b\ln u$ with $u=1+2\cos x$ (You may see $b\ln ku$ which is also correct)

A1: Achieves $-2\ln(1+2\cos x)$, $-2\ln|1+2\cos x|$ or $-2\ln u$ o.e .with $u=1+2\cos x$ oe. There is no need for $+c$ This may be left unsimplified. Only condone a missing bracket if subsequent work implies one

dM1: Substitutes **both** 0 and $\frac{\pi}{3}$ into an expression of the form $b\ln(1+2\cos x)$ a or $b\ln|1+2\cos x|$ and subtracts either way around . There must have been some attempt to evaluate the trig terms

Alternatively substitutes both 3 and 2 into an expression of the form $k\ln u$ and subtracts

A1: $\ln\frac{9}{4}$

Note that algebraic integration must be seen here. Candidates using their calculators to just write down

$\int_0^{\frac{\pi}{3}}\frac{4\sin x}{1+2\cos x}\,dx = 0.81093... = \ln\frac{9}{4}$ should be awarded 0 marks

14. JUNE 2022 [3]

3(a)	$$\int \frac{9x}{3x^2+k}\,dx = \frac{3}{2}\ln\left(3x^2+k\right)+C$$	M1A1
		(2)
(b)	$$\frac{3}{2}\ln(75+k)-\frac{3}{2}\ln(12+k)=\ln 8$$	M1
	$$\frac{3}{2}\ln\left(\frac{75+k}{12+k}\right)=\ln 8 \quad \text{oe}$$	dM1
	$$\frac{75+k}{12+k}=4 \Rightarrow k=\ldots$$	ddM1
	$$(k=)\ 9$$	A1
		(4)
		(6 marks)

(a)

M1 Integrates to $A\ln(3x^2+k)$ with or without the + C. Allow A = 1 for this mark. Condone invisible brackets.

A1 $\frac{3}{2}\ln\left(3x^2+k\right)+C$ or $\frac{3}{2}\ln B\left(3x^2+k\right)$ oe Must include the constant of integration that is different from k. Must have brackets around $3x^2+k$

Also allow eg $\ln\left[B\left(3x^2+k\right)\right]^{\frac{3}{2}}$

(b) ***Be aware they can solve equations on the calculator which is not acceptable***

M1 Attempts to substitute in 5 and 2 into their changed expression which must include x and k, subtracting either way round and sets equal to $\ln 8$. The values embedded in an equation is sufficient to score this mark. Condone slips if they attempted to evaluate when substituting in. Ignore the $+C$.

dM1 Attempts to apply the subtraction (or addition) law of logarithms once to an equation of the form $\pm A\ln(B+k)\pm A\ln(C+k)=\ln D$ or equivalent (allow $A=1$).

Typically look for $A\ln(B+k)-A\ln(C+k)=\ln D \Rightarrow A\ln\left(\frac{B+k}{C+k}\right)=\ln D$ or equivalent

Do not be concerned with arithmetical errors and condone slips dealing with their A

Condone invisible brackets.

ddM1 Solves their equation of the form $\ln\left(\frac{B+k}{C+k}\right)=\ln D^{\frac{1}{A}}$ or $\ln\left(\frac{B+k}{C+k}\right)^{A}=\ln D$

where $A\neq 1$, and the A is correctly dealt with by
- removing lns correctly
- rearranging to form a linear equation in k
- proceeding to find a value for k.

eg $"\ln\left(\frac{"75"+k}{"12"+k}\right)=\ln 8^{"\frac{2}{3}"} \Rightarrow "75"+k="48"+"4k" \Rightarrow k=...$

It is dependent on both of the previous method marks.
Condone arithmetical slips in their rearrangement and condone invisible brackets as long as the intention is clear. Also condone this mark to be scored if k is negative.

A1 9 cao (with intermediate working seen)

They cannot proceed from $\frac{3}{2}\ln(75+k)-\frac{3}{2}\ln(12+k)=\ln 8$ **to** $(k=)\,9$ **without some intermediate working seen (eg an equation with the lns removed)**

Note an answer of $\ln(3x^2+k)\ (+C)$ in (a) will score a maximum of M1dM1ddM0A0 in (b)

15. JUNE 2022 [6]

6(a)	$(\text{f}'(x)=)\ 5(x^2-2)\times\frac{1}{2}\times 4\times(4x+9)^{-\frac{1}{2}}+10x(4x+9)^{\frac{1}{2}}$	M1A1
	$(\text{f}'(x)=)\ \frac{10(x^2-2)+10x(4x+9)}{(4x+9)^{\frac{1}{2}}}=\frac{50x^2+90x-20}{(4x+9)^{\frac{1}{2}}}$	dM1
	$(\text{f}'(x)=)\ \frac{10(5x^2+9x-2)}{(4x+9)^{\frac{1}{2}}}$	A1
		(4)
(b)	Sets $\text{f}'(x)=0$ to give $x=-2,\ \frac{1}{5}$	B1
		(1)
(c)	$y=5\left(("-2")^2-2\right)\left(4\times("-2")+9\right)^{\frac{1}{2}}=10$	M1
	$(-2, 10)$	A1cao
		(2)
(d)	$\text{f}(0)=-30$	B1
	Upper bound $=2\times"10"+4=...$ or Lower bound $=2\times"-30"+4=...$	M1
	$-56\leqslant \text{g}(x)\leqslant 24$	A1
		(3)
		(10 marks)

A1 $k>3.6$ or eg $k>\frac{18}{5}$ but not $k\geqslant\frac{18}{5}$ If $k<-2$ is found then it must be rejected

(a)

M1 Attempts the product rule. Award for $\pm A(5x^2-10)(4x+9)^{-\frac{1}{2}}\pm Bx(4x+9)^{\frac{1}{2}}$ or equivalent where $A, B \neq 0$ The indices do not need to be processed for this mark.

If they quote the product rule eg $uv'+u'v$ and state the expressions for u, u', v and v' then allow a miscopy when substituted. They must have eg $u = 5x^2 - 10 \to u' = Ax$ and $v = (4x+9)^{\frac{1}{2}} \to v' = B(4x+9)^{-\frac{1}{2}}$

May also attempt the product rule on $5x^2(4x+9)^{\frac{1}{2}} - 10(4x+9)^{\frac{1}{2}}$ so award for

$\pm Ax(4x+9)^{\frac{1}{2}} \pm Bx^2(4x+9)^{-\frac{1}{2}} \pm C(4x+9)^{-\frac{1}{2}}$ $A, B, C \neq 0$

A1 Correct unsimplified expression for $f'(x)$ eg $2(5x^2-10)(4x+9)^{-\frac{1}{2}} + 10x(4x+9)^{\frac{1}{2}}$ oe

dM1 Proceeds to a single fraction, multiplies out the brackets on the numerator and proceeds to the form $\dfrac{3TQ}{(4x+9)^{\frac{1}{2}}}$ or $3TQ(4x+9)^{-\frac{1}{2}}$. They cannot proceed straight to the given 3TQ or a multiple of it without some working seen to score this mark.

Alternatively, you may only see manipulation of the numerator and then $\dfrac{3TQ}{(4x+9)^{\frac{1}{2}}}$ which is acceptable: $10(x^2-2)+10x(4x+9) = 10(x^2-2+4x^2+9x) = 10(5x^2+9x-2) \Rightarrow \dfrac{10(5x^2+9x-2)}{(4x+9)^{\frac{1}{2}}}$

Condone arithmetical slips in their working but their method to proceeding to a single fraction must be correct.

It is dependent on the previous method mark.

A1 $\left(f'(x)=\right) \dfrac{10(5x^2+9x-2)}{(4x+9)^{\frac{1}{2}}}$ with no errors seen in the main body of their solution.

Accept $\dfrac{10(5x^2+9x-2)}{\sqrt{4x+9}}$

Condone invisible brackets as long as they are recovered, or their subsequent working makes their intention clear. They do not need to explicitly state $k = 10$ but if there is a contradiction between their expression and their stated value then the expression takes precedence. Do not be concerned with what they have before the =

(b) Mark (b) and (c) together

B1 Sets $f'(x) = 0$ to give $-2, \frac{1}{5}$ Withhold if one is rejected. Condone $(-2, 0)$ $\left(\frac{1}{5}, 0\right)$. May be seen in (c)

(c)

M1 Substitutes their smaller root from solving $5x^2+9x-2=0$ into $y=5(x^2-2)(4x+9)^{\frac{1}{2}}$ and proceeds to find a value for y. Condone bracket omissions, miscopying or arithmetical slips as long as it is clear that they are attempting to substitute their smaller root into $f(x)$

A1cao $(-2, 10)$ only (Allow $x=-2,\ y=10$)

(d)

B1 $(f(0)=)-30$ seen (which may be implied by sight of -56)

M1 A correct attempt at the lower or upper bound.

- Either attempts $2\times"10"+4$ and identifies this as the upper bound. Follow through on their part (c) y coordinate. $<"24"$ or $\leqslant"24"$ is acceptable
- Or attempts $2\times"-30"+4$ and identifies this as their lower bound. $>"-56"$ or $\geqslant"-56"$ is acceptable for this mark. It cannot be scored for using $2\times f\left(\frac{1}{5}\right)+4$ as their lower bound.

Note that $g>-56$ or $g\geqslant-56$ will score B1M1.
Allow use of max or min for upper and lower bound for this mark.

A1 $-56\leqslant g(x)\leqslant 24$ oe eg "$\{g(x)\in\mathbb{R}:-56\leqslant g(x)\cap g(x)\leqslant 24\}$" or "$-56\leqslant g(x)$ AND $g(x)\leqslant 24$" or $g\in[-56, 24]$ or other variations similar to these. Condone g for $g(x)$ and $-56\leqslant y\leqslant 24$ or $-56\leqslant 2f(x)+4\leqslant 24$ but not $-56\leqslant f\leqslant 24$

DO NOT ACCEPT "$-56\leqslant g(x)\cup g(x)\leqslant 24$" or "$-56\leqslant g(x)$ OR $g(x)\leqslant 24$"

16. OCT 2022 [1]

1(a)	$\left(\frac{dy}{dx}=\right) 6\times3(3x-2)^5 \quad (=18(3x-2)^5)$	M1A1
		(2)
(b)	$"18"(3\times\frac{1}{3}-2)^{"5"}=-18$	M1
	$-18\rightarrow\frac{1}{18}$	M1
	$y-1="\frac{1}{18}"\left(x-\frac{1}{3}\right)$	dM1
	$3x-54y+53=0$	A1
		(4)
		(6 marks)

(a)

M1 Applies the chain rule to achieve a form of $A(3x-2)^5$ which may be unsimplified.
The index does not need to be processed for this mark.

Alternatively, multiplies out the brackets to achieve an expression of the form $Ax^6+Bx^5+Cx^4+Dx^3+Ex^2+Fx+G$ (where all coefficients do not need to be simplified) and reduces the power by 1 on at least one of the terms. You will need to check this carefully to make sure it is the relevant term where the power has decreased
eg $Bx^5\rightarrow...\times Bx^4$
Condone slips in their expansion.

(The actual expansion is $64-576x+2160x^2-4320x^3+4860x^4-2916x^5+729x^6$)

A1 $\left(\frac{dy}{dx}=\right)18(3x-2)^5$ or unsimplified equivalent eg $6\times3(3x-2)^5$ but the index must be processed. If they have multiplied out then accept $-576+4320x-12960x^2+19440x^3-14580x^4+4374x^5$.
Accept x^1 for x

(b)

M1 Substitutes $x=\frac{1}{3}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ which must be of the form $A(3x-2)^5$ (or accept a polynomial of degree 5) and finds a value for $\frac{\mathrm{d}y}{\mathrm{d}x}$

M1 Finds the negative reciprocal of their gradient. May be implied by the equation of their line. It cannot be scored for setting $\frac{\mathrm{d}y}{\mathrm{d}x}=0$, solving to find x and then taking the negative reciprocal of this.

dM1 Attempts to find the equation of the normal. Look for $y-1=\text{"}\frac{1}{18}\text{"}\left(x-\frac{1}{3}\right)$ (must be a changed gradient from their tangent gradient). It is dependent on the first method mark. If they use $y=mx+c$ they must proceed as far as $c=\ldots\left(=\text{"}\frac{53}{54}\text{"}\right)$

A1 $3x-54y+53=0$ or any integer multiple of this with all terms on one side of the equation eg $-6x+108y-106=0$ You must see the $=0$
isw after a correct answer is seen. Eg if they proceed to state values for *a*, *b* and *c* which contradict their equation then take their equation as their final answer.

17. JAN 2021 [1]

1	$\int \frac{x^2-5}{2x^3}\mathrm{d}x = \int Ax^{-1} - Bx^{-3}\,\mathrm{d}x = C\ln x + Dx^{-2}\,(+c)$	M1 dM1
	$= \frac{1}{2}\ln x + \frac{5}{4}x^{-2} + c$	A1
		(3)
		Total 3

M1: Correct **attempt** to integrate.

Score for an attempt to divide by the x^3 term forming a sum of two terms and then integrating.
Award for achieving one term in the correct form. Either $C\ln x + ...$ **or** $... + Dx^{-2}$
Note that $C\ln ax$ and versions such as $k\ln 2x^3$ are also acceptable for $C\ln x$ so look at responses involving lns carefully . Ignore spurious notation e.g. $\int C\ln x$ for the M marks as long as integration has been attempted

dM1: Achieves both terms in the correct form. Score for $\pm C\ln x \pm Dx^{-2}$ or equivalent
Be aware that $C\ln ax \pm Dx^{-2}$ and other variations are also correct

A1: $\frac{1}{2}\ln x + \frac{5}{4}x^{-2} + c$ or equivalent simplest form **with the** $+c$. E.g $\ln\sqrt{x} + \frac{5}{4x^2} + c$
ISW after a correct answer.

Some candidates may incorporate the $+c$ within the log so $\frac{1}{2}\ln kx + \frac{5}{4}x^{-2}$ where k is an arbitrary constant is ok.

Note that $\frac{1}{2}\ln 2x + \frac{5}{4}x^{-2} + c$ is not the simplest form and is A0. $\int \frac{1}{2}\ln x + \frac{5}{4}x^{-2} + c$ would also be A0

....

Attempts via integration by parts can be scored in the same way

$$\int \frac{x^2-5}{2x^3}\mathrm{d}x = \int (x^2-5)\times\frac{1}{2}x^{-3}\,\mathrm{d}x = (x^2-5)\times -\frac{1}{4}x^{-2} - \int 2x\times -\frac{1}{4}x^{-2}\mathrm{d}x = (x^2-5)\times -\frac{1}{4}x^{-2} + \frac{1}{2}\ln x + c$$

M1: For an attempt to integrate by parts the correct way around and achieves $(x^2-5)\times px^{-2} \pm q\ln ax + c$
If the rule is quoted it must be correct.
It is possible to integrate by parts the other way around but unlikely. It can be scored in a similar way.

dM1: Score for

- either then simplifying to an expression of the form $\pm C\ln x \pm Dx^{-2}$ with or without "+ c" which could be numerical
- or integrating to a correct but unsimplified answer $(x^2-5)\times -\frac{1}{4}x^{-2} + \frac{1}{2}\ln ax$ with or without "+ c"

A1: $\frac{1}{2}\ln x + \frac{5}{4}x^{-2} + c$ NOT $\frac{1}{2}\ln x + \frac{5}{4}x^{-2} + \frac{1}{4} + c$ (The answer must be in simplest form and **with the** $+c$)

....

18. JAN 2021 [3]

3(a)	$2x^2-3x-5=(2x-5)(x+1)$	B1
	$3-\dfrac{x-2}{x+1}+\dfrac{5x+26}{2x^2-3x-5}=\dfrac{3(x+1)(2x-5)-(x-2)(2x-5)+5x+26}{(x+1)(2x-5)}$	M1 A1
	$=\dfrac{(4x+1)(x+1)}{(x+1)(2x-5)}=\dfrac{4x+1}{2x-5}$	A1
		(4)
(b)	Correct attempt at inverse $y=\dfrac{4x+1}{2x-5}\Rightarrow x=...$	M1
	$f^{-1}(x)=\dfrac{5x+1}{2x-4}$	A1
		(2)
(c)	$2<x<\dfrac{17}{3}$	M1 A1
		(2)
		Total 8

(a)

B1: Correct factorisation, can be scored anywhere. Sight of $2x^2-3x-5=(2x-5)(x+1)$ Condone $2(x-2.5)(x+1)$

M1: Attempts to combine **all three terms** using a common denominator. Allow the terms to be separate.

There must be an **attempt** to adapt the numerators of the first two terms, one of them must be adapted correctly.

So allow for example $3-\dfrac{x-2}{x+1}+\dfrac{5x+26}{2x^2-3x-5}=\dfrac{3(x+1)(2x-5)-(x-2)\times 2x-5+5x+26}{(x+1)(2x-5)}$

This may be done in stages but is only scored when all three terms are combined.

Condone a fraction where the denominator $(x+1)(2x^2-3x-5)$ is used. (In this case there must be an attempt to adapt the numerators of the all terms and two of the three numerators must be adapted correctly)

$$3-\frac{x-2}{x+1}+\frac{5x+26}{2x^2-3x-5}=\frac{3(x+1)(2x^2-3x-5)}{(x+1)(2x^2-3x-5)}-\frac{(x-2)(2x^2-3x-5)}{(x+1)(2x^2-3x-5)}+\frac{(5x+26)(x+1)}{(x+1)(2x^2-3x-5)} \quad **$$

A1: Correct fraction with denominator $(x+1)(2x-5)$ or equivalent such as $2x^2-3x-5$

Allow this to be given separately

$$3-\frac{x-2}{x+1}+\frac{5x+26}{2x^2-3x-5}=\frac{3(x+1)(2x-5)}{(x+1)(2x-5)}-\frac{(x-2)(2x-5)}{(x+1)(2x-5)}+\frac{5x+26}{(x+1)(2x-5)}$$

If ** was given then they must proceed to $\dfrac{(x+1)(4x^2+5x+1)}{(x+1)(2x^2-3x-5)}$

A1: Correct fraction (or correct values). Proceeds to $\dfrac{4x+1}{2x-5}$ via $\dfrac{(4x+1)(x+1)}{(x+1)(2x-5)}$ oe.

(b)
M1: Attempts at the method for finding the inverse.

Score for an attempt to change the subject for their $y=\frac{ax+b}{cx+d}$ or possibly $y=\frac{a}{c}\pm\frac{e}{cx+d}$

Look for a minimum of cross multiplying by $cx+d$ and proceeding to a form $x=\mathrm{g}(y)$

Some candidates may swap x and y first e.g $x=\frac{ay+b}{cy+d}$ and proceed to $y=$.. which is fine (same conditions)

Allow this to be scored if one (but not more) of a, b or $d=0$

Allow this to be scored for candidates who don't finish (a) and attempt to change the subject for $y=\frac{ax+b}{cx+d}$

A1: Correct inverse $\mathrm{f}^{-1}(x)=\frac{5x+1}{2x-4}$ but condone $y=\frac{5x+1}{2x-4}$ and $\mathrm{f}^{-1}=\frac{5x+1}{2x-4}$

Allow other equivalents such as $y=\frac{-5x-1}{4-2x}$, $y=\frac{-5x/2-1/2}{2-x}$ or $y=\frac{5}{2}+\frac{11}{2x-4}$

(c)
M1: For finding one "end" of the domain. Ignore any inequalities.

This must be numerical.....and you are just looking for the number, not the variable so $y=$ ... is OK

Sight of either $\frac{17}{3}$ or their $\mathrm{f}(4)$ which may need to be checked

or 2 or $\mathrm{f}(x)$ as $x\to\infty$ that is their $\frac{a}{c}$ (Can be scored for $x\neq 2$)

A1: Correct domain with allowable notation.

Allowable equivalent forms are e.g. $\left(2, \frac{17}{3}\right)$, $x>2$ and $x<\frac{17}{3}$. Condone "or"

19. JAN 2021 [9]

Question Number	Scheme	Marks
9(i)	$\int \frac{3x-2}{3x^2-4x+5}\,\mathrm{d}x = \frac{1}{2}\ln\left(3x^2-4x+5\right)(+c)$	M1, A1
		(2)
(ii)	$\int \frac{\mathrm{e}^{2x}}{\left(\mathrm{e}^{2x}-1\right)^3}\,\mathrm{d}x = -\frac{1}{4}\left(\mathrm{e}^{2x}-1\right)^{-2}(+c)$	M1, A1
		(2)
		Total 4

(i)

M1: Integrates to a form $\alpha\ln\left(3x^2-4x+5\right)$ where α is a constant . Condone a missing bracket.

Do not accept $\alpha\ln\left(3x^2-4x+5\right)+\mathrm{f}(x)$, e.g. $\ln\left(3x^2-4x+5\right)+2x$

If the substitution $u=3x^2-4x+5$ is attempted, the mark can be awarded for $k\ln u$

It is unlikely but $\alpha\ln\beta\left(3x^2-4x+5\right)$ and $\alpha\ln\left(3x^2-4x+5\right)^\beta$ are also correct

A1: $\frac{1}{2}\ln\left(3x^2-4x+5\right)$ o.e. with or without the + c. A bracket or modulus must be present.

ISW after a correct answer.

Do not penalise $\frac{\ln\left(3x^2-4x+5\right)}{2}$ or $\ln\left(3x^2-4x+5\right)/2$ if the intention is clear

Penalise spurious incorrect notation for the A mark only. So do not allow $\frac{\ln\left(3x^2-4x+5\right)\mathrm{d}x}{2}$

(ii)

M1: Integrates to a form $\beta\left(e^{2x}-1\right)^{-2}$ where β is a constant .

Do not accept $\beta\left(e^{2x}-1\right)^{-2}+g(x)$, e.g. $\beta\left(e^{2x}-1\right)^{-2}+e^{2x}$

Allow substitutions. So for example,

if the substitution $u=e^{2x}-1$ is attempted, the mark can be awarded for ku^{-2}

if the substitution $u=e^{x}$ is attempted, the mark can be awarded for $k\left(u^{2}-1\right)^{-2}$

if the substitution $u=e^{2x}$ is attempted, the mark can be awarded for $k(u-1)^{-2}$

A1: $-\frac{1}{4}\left(e^{2x}-1\right)^{-2}$ or exact equivalent with or without the $+c$

ISW after a correct answer. Need not be simplified

Penalise spurious incorrect notation for the A mark only. So do not allow $\int -\frac{1}{4}\left(e^{2x}-1\right)^{-2}$

20. OCT 2021 [5]

Question Number	Scheme	Marks
5 (i)	$\int \frac{8}{(2x-3)^3}\,dx = \frac{-2}{(2x-3)^2}(+c)$	M1 A1
	$\int_2^4 \frac{8}{(2x-3)^3}\,dx = \left[\frac{-2}{(2x-3)^2}\right]_2^4 = -\frac{2}{25}+2=\frac{48}{25}$	dM1 A1
		(4)
(ii)	$\int x(x^2+3)^7\,dx = \frac{1}{16}(x^2+3)^8 + c$	M1 A1
		(2)
		(6 marks)
Alt(i)	Let $u=2x-3$ $\int \frac{8}{u^3}\times\frac{1}{2}\,du = -\frac{2}{u^2}(+c)$ $\int_2^4 \frac{8}{(2x-3)^3}\,dx = \left[-\frac{2}{u^2}\right]_1^5 = -\frac{2}{25}+2=\frac{48}{25}$	M1 A1 dM1 A1
Alt(ii)	Let $u=x^2+3$ $\int x(x^2+3)^7\,dx = \int \frac{u^7}{2}\,du = \frac{u^8}{16}+c = \frac{1}{16}(x^2+3)^8+c$	M1A1

(i)

M1 Achieves $\frac{A}{(2x-3)^2}$ or equivalent or in the alternative method $\frac{A}{u^2}$

A1 Achieves $\frac{-2}{(2x-3)^2}$ or in the alternative method $-\frac{2}{u^2}$ (which may be unsimplified but the indices must be processed). There is no requirement for the $+c$

dM1 Substitutes 2 and 4 into $\frac{A}{(2x-3)^2}$ or equivalent or 1 and 5 into $\frac{A}{u^2}$ and subtracts either way round. May be implied but M1 must have been scored.

A1 $\frac{48}{25}$ or 1.92 isw after a correct answer

(ii)

M1 Achieves $k\left(x^2+3\right)^8$ or equivalent or in the alternative method ku^8. Alternatively multiplies out the expression and integrates achieving an expression of the form

$\pm\ldots x^{16}\pm\ldots x^{14}\pm\ldots x^{12}\pm\ldots x^{10}\pm\ldots x^8\pm\ldots x^6\pm\ldots x^4\pm\ldots x^2$

A1 $\frac{1}{16}\left(x^2+3\right)^8+c$ Must be in terms of x and the $+c$ must be present

Allow $\frac{x^{16}}{16}+\frac{3x^{14}}{2}+\frac{63x^{12}}{4}+\frac{189x^{10}}{2}+\frac{2835x^8}{8}+\frac{1701x^6}{2}+\frac{5103x^4}{4}+\frac{2187x^2}{2}+c$ or simplified equivalent

21. OCT 2021 [10]

Question Number	Scheme	Marks
10.(a)	$(1+2\cos 2x)^2 = 1+4\cos 2x+4\cos^2 2x$ Uses $\cos 4x = 2\cos^2 2x - 1 \Rightarrow (1+2\cos 2x)^2 = 1+4\cos 2x+2\cos 4x+2$ $= 3+4\cos 2x+2\cos 4x$	 M1 A1
		(2)
(b)	$a = \frac{2\pi}{3}$ $\int 3+4\cos 2x+2\cos 4x\,dx = 3x+2\sin 2x+\frac{1}{2}\sin 4x$ Area $= \left[3x+2\sin 2x+\frac{1}{2}\sin 4x\right]_0^{\frac{2\pi}{3}} = 2\pi - \frac{3}{4}\sqrt{3}$	B1 M1 A1 ft dM1 A1
		(5)
		(7 marks)

(a)

M1 Attempts to multiply out $(1+2\cos 2x)^2 = 1+\ldots\cos 2x+\ldots\cos^2 2x$ and use $\cos 4x = 2\cos^2 2x - 1$ to obtain $(1+2\cos 2x)^2$ in the form $p+q\cos 2x+r\cos 4x$.

Condone slips in the rearrangement of $\cos 4x = 2\cos^2 2x - 1$ but it must be clear that the identity was correct originally otherwise M0.

Beware of candidates who write $(1+2\cos 2x)^2 = 1+4\cos 2x+4\cos^2 4x$ which is M0A0

A1 $3+4\cos 2x+2\cos 4x$

(b)

B1 Deduces that $a=\frac{2\pi}{3}$ (allow 120° for this mark). If more than one angle is found, then look for which one is substituted into their integrated expression.

M1 Integrates $q\cos 2x + r\cos 4x \rightarrow \pm ...\sin 2x \pm ...\sin 4x$

A1ft Integrates $p + q\cos 2x + r\cos 4x \rightarrow px + \frac{q}{2}\sin 2x + \frac{r}{4}\sin 4x$ unsimplified where *p*, *q* and $r \neq 0$

dM1 Substitutes 0 and $a=\frac{2\pi}{3}$ (or awrt 2.09) (or equivalent) into a valid function (M1 must have been scored) and subtracts either way around. Note $q, r \neq 0$ but they do not need a *px* term.

Also allow $a=\frac{\pi}{3}$ (or awrt 1.05) or $a=\frac{4\pi}{3}$ (or awrt 4.19) *a* must be in radians to evaluate correctly.

This mark cannot be scored without a value for *a*. You do not have to explicitly see 0 substituted in and their answer may imply a correct substitution into their integrated expression.

A1 $2\pi - \frac{3}{4}\sqrt{3}$ or simplified equivalent

22. JAN 2020 [4]

4. (i) (a)	$f'(x)=\dfrac{4(x-3)(2x+5)-(2x+5)^2}{(x-3)^2}$ or $\dfrac{(x-3)(8x+20)-(4x^2+20x+25)}{(x-3)^2}$	M1 A1
	$=\dfrac{(2x+5)(2x-17)}{(x-3)^2}$	M1 A1
(b)	Attempts both critical values or finds one "correct" end	M1
	$x<-2.5,\ x>8.5$ (accept $x\leqslant -2.5,\ x\geqslant 8.5$)	A1
		(6)
(ii)	Attempts the chain rule on $(\sin 4x)^{\frac{1}{2}} \to A(\sin 4x)^{-\frac{1}{2}}\times\cos 4x$	M1
	$g(x)=x(\sin 4x)^{\frac{1}{2}} \Rightarrow g'(x)=(\sin 4x)^{\frac{1}{2}}+x\times\frac{1}{2}(\sin 4x)^{-\frac{1}{2}}4\cos 4x$	M1 A1
	Sets $g'(x)=0 \to (\sin 4x)^{\frac{1}{2}}+x\times\dfrac{2\cos 4x}{(\sin 4x)^{\frac{1}{2}}}=0$ and $\times\dfrac{(\sin 4x)^{\frac{1}{2}}}{\cos 4x}$ oe	M1
	$\to \tan 4x+2x=0$	A1
		(5)
		11 marks

(i)(a)

M1 Attempts the quotient rule and achieves

$$f'(x) = \frac{A(x-3)(2x+5) - B(2x+5)^2}{(x-3)^2} \quad A, B > 0 \text{ condoning slips}$$

Alternatively uses the product rule and achieves

$$\frac{d}{dx}\left\{(x-3)^{-1}(2x+5)^2\right\} = \pm A(2x+5)^2(x-3)^{-2} + B(x-3)^{-1}(2x+5) \quad A, B > 0$$

They may attempt to multiply out the $(2x+5)^2$ first which is fine as long as they reach a 3TQ.

A1 Score for correct unsimplified $f'(x)$

M1 Attempts to take out a factor of $(2x+5)$ or multiplies out and attempts to factorise the numerator.

The method must be seen $\frac{(x-3)4(2x+5) \pm (2x+5)^2}{\ldots} = \frac{(2x+5)\{(x-3)4 \pm (2x+5)\}}{\ldots}$ condoning slips.

If the method is not seen it may be implied by a correct result for their fraction

This can be achieved from an incorrect quotient or product rule. E.g. $\frac{vu'+uv'}{v^2}$ or $\frac{vu'-uv'}{v}$

It can be scored by candidates who multiply out their numerators and then factorise by taking out a factor of $(2x+5)$

If the product rule is used it would be for writing as a single fraction and taking out, from the numerator, a common factor of $(2x+5)$.

A1 $\frac{(2x+5)(2x-17)}{(x-3)^2}$ but accept expressions such as $\frac{4(x+2.5)(x-8.5)}{(x-3)^2}$ or $\frac{(2x+5)(2x-17)}{(x-3)(x-3)}$

Note the final two marks in (i)(a) may be scored in (i)(b), ONLY IF the correct work is done on the complete fraction, not just the numerator

(i)(b)

M1 Achieves the two critical values from the quadratic numerator of their $f'(x)$

Alternatively finds one correct end for their $(2x+5)(2x-17)>0$ or $(2x+5)(2x-17)\geqslant 0$

So award for either $x<-2.5$ or $x>$ their 8.5 which may be scored from an intermediate line.

A1 $x<-2.5, x>8.5$ (accept $x\leqslant -2.5,\ x\geqslant 8.5$).

Ignore any references to "and" or "or" so condone $x<-2.5$ and $x>8.5$

Mark the final response. This is not isw

It may follow working such as $(2x+5)(2x-17)>0 \Rightarrow x>-\frac{5}{2}, x>\frac{17}{2}$. So $x<-2.5, x>8.5$

Accept alternative forms such as $(-\infty,-2.5]\cup[8.5,\infty)$

(ii)

M1 Attempts the chain rule on $(\sin 4x)^{\frac{1}{2}} \rightarrow A(\sin 4x)^{-\frac{1}{2}}\times\cos 4x$

M1 For an attempt at the product rule.

If they state $u=x, v=(\sin 4x)^{\frac{1}{2}}, u'=1, v'=...$ award for $(\sin 4x)^{\frac{1}{2}}+x\times$ their v'

If this is not stated or implied by their $uv'+vu'$ then award for $(\sin 4x)^{\frac{1}{2}}+x\times(\sin 4x)^{-\frac{1}{2}}...$

A1 $(g'(x))=(\sin 4x)^{\frac{1}{2}}+2x(\sin 4x)^{-\frac{1}{2}}\cos 4x$ which may be unsimplified.

You may not see the lhs which is fine. Condone $\sin 4x^{\frac{1}{2}}$ for $(\sin 4x)^{\frac{1}{2}}$ if subsequent work is correct

M1 Sets their $g'(x)$ which must be of the form $(\sin 4x)^{\frac{1}{2}}+kx(\sin 4x)^{-\frac{1}{2}}\cos 4x$ equal to 0

and proceeds with correct work to an equation of the correct form. Allow $\tan 4x=kx$ here

A1 cso $\tan 4x+2x=0$

Alt to (a) via division which may not be very common

M1 Score for $\frac{(2x+5)^2}{x-3}\rightarrow Ax+B+\frac{C}{x-3}$ differentiating to $A\pm\frac{C}{(x-3)^2}$

A1 $4-\frac{121}{(x-3)^2}$

M1 Forms a single fraction and attempts to factorise out $(2x+5)$ from the numerator (which must be a 3TQ)

A1 $\frac{(2x+5)(2x-17)}{(x-3)^2}$

Alt to (b) via squaring

$$[g(x)]^2=x^2\sin 4x \Rightarrow 2g(x)g'(x)=2x\sin 4x+4x^2\cos 4x$$

M1 Correct form for the rhs. Apply the same rules as the main method. Condone slips on coefficients

dM1 Correct form for the left hand side as well as the right hand side. Condone a slip on the coefficient

A1 $2g(x)g'(x)=2x\sin 4x+4x^2\cos 4x$

ddM1 Sets $g'(x)=0$ and proceeds with correct work to an equation of the correct form.

A1 cso $\tan 4x+2x=0$

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23. JAN 2020 [8]

Question Number	Scheme	Marks
8.(i)	$\int \frac{2}{3x-1}\,dx = \frac{2}{3}\ln(3x-1)$	M1 A1
	$\int_3^{42} \frac{2}{3x-1}\,dx = \frac{2}{3}\ln(125) - \frac{2}{3}\ln(8) = \ln\frac{25}{4}$	dM1 A1
		(4)
(ii)	$\frac{2x^3-7x^2+8x+1}{(x-1)^2} = 2x + B + \frac{C}{(x-1)^2}$	B1
	Full method to find values of A, B and C	M1
	$\frac{2x^3-7x^2+8x+1}{(x-1)^2} = 2x - 3 + \frac{4}{(x-1)^2}$	A1
	$\int h(x)\,dx = \int Ax + B + \frac{C}{(x-1)^2}\,dx$	
	$= \frac{1}{2}Ax^2 + Bx - \frac{C}{(x-1)}$	M1 A1 ft
	$= x^2 - 3x - \frac{4}{(x-1)}\quad (+c)$	A1
		(6)
		10 marks

(ii) Alt I first 3 marks	$2x^3 - 7x^2 + 8x + 1 = Ax(x-1)^2 + B(x-1)^2 + C$ Any of $A = 2, B = -3,$ or $C = 4$ Either substitution or equating coefficients to get two values All values correct $A = 2, B = -3, C = 4$	 B1 M1 A1 **(3)**
(ii) Alt II first 3 marks	$x^2 - 2x + 1 \overline{)\, 2x^3 - 7x^2 + 8x + 1}$ with quotient $2x - 3$, remainder 4 Likely to be $2x$ For attempt at division Correct quotient and remainder	 B1 M1 A1 **(3)**

(i)

M1 Integrates to $k\ln(3x-1)$ condoning slips. Condone with a missing bracket.

Please note that $k\ln(3ax-a)$ where a is a positive constant is also correct

If a substitution is made, i.e $u = 3x-1$, then they must proceed to $k\ln u$ where k is a constant

A1 $\frac{2}{3}\ln(3x-1)$

Also accept $\frac{2}{3}\ln(3ax-a)$ where a is a positive constant is also correct or $\frac{2}{3}\ln u$ where $u = 3x-1$,

Do not allow with the missing bracket unless subsequent work implies that it is present

dM1 Substitutes in both limits and applies one ln law correctly. May be subtraction law or power law.

If a substitution has been made then the correct limits must be used. With u they are 125 and 8

A1 $\ln\frac{25}{4}$ or simplified equivalent such as $\ln 25 - \ln 4$, $2\ln 5 - 2\ln 2$ or $2\ln\frac{5}{2}$

ISW if followed by decimals

(ii)

B1 Any correct value of A, B or C seen or implied.

M1 A full method to find values of A, B and C.

If they attempt $2x^3-7x^2+8x+1=Ax(x-1)^2+B(x-1)^2+C$ via this route, this expression must be correct.

If they attempt by division then they must proceed to a linear quotient but may get a linear remainder.

A1 Correct A, B, C or correct expression. This may be implied by a correct quotient and remainder.

M1 $\int\frac{P}{(x-1)^2}\,\mathrm{d}x\to\frac{Q}{(x-1)^1}$ o.e. where P and Q could be the same

Award for $\int\frac{P}{(x-1)^2}\,\mathrm{d}x\to\frac{Q}{u}$ where they have previously set $u=x-1$

A1ft $\int Ax+B+\frac{C}{(x-1)^2}\,\mathrm{d}x=\frac{1}{2}Ax^2+Bx-\frac{C}{(x-1)}$ or unsimplified equivalent.

So allow $\frac{1}{2}Ax^2+Bx+\frac{C}{-1}(x-1)^{-1}$ with the indices processed

Also allow with non-numerical values.

Also score for $\int Ax+B+\frac{C}{(x-1)^2}\,\mathrm{d}x=\frac{1}{2}Ax^2+Bx-\frac{C}{u}$ where they have previously set $u=x-1$

A1 $x^2-3x-\frac{4}{(x-1)}\quad(+c)$ or exact simplified equivalent with or without the $+c$

So allow $x^2-3x-4(x-1)^{-1}\quad(+c)$

24. OCT 2020 [5]

5 (a)	$\sin 3x \equiv \sin(2x+x) \equiv \sin 2x\cos x + \cos 2x\sin x$	M1
	$\equiv 2\sin x\cos x\cos x + \left(1-2\sin^2 x\right)\sin x$	M1
	$\equiv 2\sin x\left(1-\sin^2 x\right) + \left(1-2\sin^2 x\right)\sin x$	ddM1
	$\equiv 3\sin x - 4\sin^3 x$ *	A1*
		(4)
(b)	$\int_0^{\frac{\pi}{3}} \sin^3 x \, dx = \int_0^{\frac{\pi}{3}} \frac{3}{4}\sin x - \frac{1}{4}\sin 3x \, dx$	M1
	$= \left[-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x\right]_0^{\frac{\pi}{3}}$	dM1 A1
	$= \frac{5}{24}$	A1
		(4)
		(8 marks)

(a)

M1 Uses $\sin 3x = \sin(2x+x) = \pm\sin 2x\cos x \pm \cos 2x\sin x$

M1 Uses the correct identity for $\sin 2x = 2\sin x\cos x$ and any correct identity for $\cos 2x$

ddM1 Dependent upon both previous M's. It is for using $\cos^2 x = 1-\sin^2 x$ to get an equation in only $\sin x$

A1* Fully correct solution with correct notation within their proof. Examples of incorrect notation include use of $\sin x^2$ instead of $\sin^2 x$ or use of sin instead of $\sin x$ and so on. Penalise in the A mark only for such.

Note: The ddM mark and final A mark may be score by substituting $\sin^2 x = 1-\cos^2 x$ into the right hand side of the equation to reach an identical expression to an expanded left hand side ("working from both sides"), with the A mark then awarded for correct work leading to identical expressions **and** an minimal conclusion given (e.g. //)

Note You may see use of De Moivre's Theorem from an FP3 candidate. This can score full credit if carried out correctly. If there are errors or you are unsure then send to review.

If attempted in reverse:

M1 $3\sin x - 4\sin^3 x = 3\sin x - 2\sin^2 x\sin x - 2\sin x(1-\cos^2 x) = \sin x - 2\sin x\frac{1}{2}\left(1-\cos 2x\right) + \sin 2x\cos x$

Uses $\sin^3 x = \sin x \times \sin^2 x$ with either $\sin^2 x = \frac{1}{2}(1-\cos 2x)$ OR $\sin^2 x = 1-\cos^2 x$ and $2\sin x\cos x = \sin 2x$

M1 Uses both steps above to get to an equation with sin2x and cos2x

ddM1 Gathers terms to reach $= \cos 2x\sin x + \sin 2x\cos x$

A1 Completes the proof uses $\cos 2x\sin x + \sin 2x\cos x = \sin 3x$ with no errors seen.

(b)

M1 Attempts to use part (a) to simplify. Accept $\int \sin^3 x \, dx = \int A\sin x + B\sin 3x \, dx$

dM1 $\int A\sin x + B\sin 3x \, dx \to a\cos x + b\cos 3x$

A1 $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$ oe (not a multiple of this (unless recovered))

A1 CSO $= \frac{5}{24}$

Note an answer of A1 $\frac{5}{24}$ with no supporting working scores no marks as algebraic integration is specified. But alternative methods of integration are permissible. Two alternatives are:

5 (b) Alt 1	$\int_0^{\frac{\pi}{3}} \sin^3 x \, dx = \int_0^{\frac{\pi}{3}} \sin x\left(1-\cos^2 x\right) dx = \int_0^{\frac{\pi}{3}} \sin x - \sin x\cos^2 x \, dx$	M1
	$= \left[-\cos x - \left(-\frac{\cos^3 x}{3}\right)\right]_0^{\frac{\pi}{3}}$	dM1A1
	$= \frac{5}{24}$	A1
		(4)
(b) Alt 2	$u = \cos x \Rightarrow \frac{du}{dx} = -\sin x \Rightarrow \int_0^{\frac{\pi}{3}} \sin^3 x \, dx = \int_1^{\frac{1}{2}} u^2 - 1 \, du$	M1
	$= \left[\frac{u^3}{3} - u\right]_1^{\frac{1}{2}}$	dM1 A1
	$= \frac{5}{24}$	A1
		(4) **(8 marks)**

Notes: First three marks as follows (final A is as main scheme).

Alt 1

M1: Splits as $\sin^3 x = \sin x \sin^2 x$ and applies $\sin^2 x = 1 - \cos^2 x$ to get the integrand into and integrable form.

dM1 for $\sin x \to \pm\cos x$ and $\sin x \cos^2 x \to K\cos^3 x$

A1 $-\cos x - \left(-\frac{\cos^3 x}{3}\right)$ oe (not a multiple of this (unless recovered))

Alt 2

M1 Sets $u = \cos x$, finds $\frac{du}{dx} = \pm\sin x$ and makes a full substitution using both of these to get an integral in terms of u only. (Limits not needed for this mark).

dM1 for $au^2 - b \to Au^3 - bu$

A1 For reaching $\left[\frac{u^3}{3} - u\right]_1^{\frac{1}{2}}$ including correct limits or for undoing the substitution and reaching $\frac{\cos^3 x}{3} - \cos x$

25. OCT 2020 [9]

9 (a)	$$\begin{array}{r} x^2+2 \\ x^2-x-12\,\overline{)\,x^4-x^3-10x^2+3x-9} \\ \underline{x^4-x^3-12x^2} \\ 2x^2+3x-9 \\ \underline{2x^2-2x-24} \\ 5x+15 \end{array}$$	M1A1	
	$\dfrac{x^4-x^3-10x^2+3x-9}{x^2+x-12} \equiv x^2+2+\dfrac{5(x+3)}{(x-4)(x+3)}$	M1	
	$\equiv x^2+2+\dfrac{5}{(x-4)}$ or stating $P=2, Q=5$	A1	
			(4)
(b)	$g'(x)=2x-\dfrac{5}{(x-4)^2}$	M1A1	
	Subs $x=2$ into $g'(2)=2\times2-\dfrac{5}{(2-4)^2}=\dfrac{11}{4}$	M1	
	Uses $m=g'(2)=\left(\dfrac{11}{4}\right)$ with $(2, g(2))=\left(2,\dfrac{7}{2}\right)$ to form equation of tangent		
	$y-\dfrac{7}{2}=\dfrac{11}{4}(x-2)\Rightarrow y=\dfrac{11}{4}x-2$	dM1A1	
			(5)
(c)	$\int x^2+2+\dfrac{5}{(x-4)}\,dx=\dfrac{1}{3}x^3+2x,+5\ln\lvert x-4\rvert$	M1 A1ft	
	Area $R=\left[\dfrac{1}{3}x^3+2x+5\ln\lvert x-4\rvert\right]_0^2=\left(\dfrac{8}{3}+4+5\ln 2\right)-(0+0+5\ln 4)$	dM1	
	$=\dfrac{20}{3}+5\ln 2-5\ln 4=\dfrac{20}{3}-5\ln 2$	ddM1 A1	
			(5)
		(14 marks)	
Alt (a)	$x^4-x^3-10x^2+3x-9\equiv(x^2+P)(x^2-x-12)+Q(x+3)$	M1	
	Compare terms (OR sub in values) and solve simultaneously to find P and/or Q ie $x^2\Rightarrow P-12=-10,\quad x\Rightarrow -P+Q=3,\quad$ const $\Rightarrow -12P+3Q=-9$ $\Rightarrow P=\ldots$ or $Q=\ldots$	M1	
	$P=2, Q=5$ *	A1,A1	
	Award in the order shown here on ePEN.		

(a)

M1: Divides to obtain a quadratic quotient and a linear remainder. May divide by $(x-4)$ and then by $(x+3)$ or vice versa to reach these but must be a full process.

FYI: By $x + 3$ first gives x^3-4x^2+2x-3 as quotient, followed by $x^2+2+\frac{5}{x-4}$

By $x-4$ first gives $x^3+3x^2+2x+11+\frac{35}{x-4}$ then $x^2+2+\frac{5}{x+3}+\frac{35}{(x+3)(x-4)}\rightarrow x^2+2+\frac{5}{x-4}$

A1: Obtains a quotient of x^2+2 and a remainder of $5x+15$

M1: Writes the given expression in the required form using $x^2-x-12=(x-4)(x+3)$ or divides $x+3$ into their remainder term.

A1: Correct answer. $(P=2, Q=5)$ May be awarded following an incorrect "ax^2+2" quadratic factor.

Alt:

M1: Multiplies though completely by the denominator and cancels the $x-4$ term.

M1: Complete process of comparing coefficients or substituting values to find a value for either P or Q

A1: Either $P = 2$ or for **showing** $Q = 5$* (must have seen a correct equation for Q)

A1: Both $P=2$ and **showing** $Q=5$* Note that $Q = 5$ is given so it must be shown from correct work, not just stated.

Note M0M1A1A0 is possible if Q is assumed and factorisation of x^2-x-12 is never seen.

(b)

M1: For $\frac{Q}{x-4}\rightarrow\frac{...}{(x-4)^2}$

A1: For $g'(x)=2x-\frac{5}{(x-4)^2}$. Note that this can be scored from an incorrect P.

M1: Attempts the gradient of C at the point where $x=2$

dM1: Depends on previous M. A complete method of finding the equation of the tangent. If $y=mx+c$ is used, they must proceed as far as finding c.

A1: $y=2.75x-2$ or exact equivalent and isw.

Note: This may be attempted from the original function $g(x)=\frac{x^4-x^3-10x^2+3x-9}{x^2+x-12}$

M1: Scored for an attempt at the quotient rule A1 if correct and so on.

$$\rightarrow\frac{(ax^3+bx^2+cx+d)(x^2+x-12)-(x^4-x^3-10x^2+3x-9)(px+q)}{(x^2+x-12)^2}$$

(c)

M1: Attempts to integrate with $\int \frac{..}{(x-4)}\,dx \to ..\ln|x-4|$ Condone $\ln(x-4)$

A1ft: $\int x^2 + P + \frac{5}{(x-4)}\,dx = \frac{1}{3}x^3 + Px + 5\ln|x-4|$ following through on their P.

dM1: Dependent on first M mark. Attempt the area of R using the limits 0 and 2 in their integrated function and subtracting the correct way round (or recovered).

ddM1: Depends on both previous M's. Scored for attempting to combine two log terms using correct log work

Allow the method and final accuracy if $\ln(-2) - \ln(-4) \to \ln\left(\frac{-2}{-4}\right) = -\ln 2$ is used (bod that modulus is meant)

Do not allow if $\ln(-a) \to -\ln a$ is used.

A1: cao $\frac{20}{3} - 5\ln 2$

Note: If a candidate gives correct values of m and c in (b) and of a and b in (c) but has not stated the answer in correct form, then penalise only the first instance.