Roll No…….. **Total No. of Printed Pages: 2**
Total No. of Questions: [09]

B. Tech (EE, ECE, ETE, EIE, EEE) (Semester – 1st)
MATHEMATICS – I (CALCULUS AND DIFFERENTIAL EQUATIONS)
Subject Code: BMATH3101
Paper ID: [19110007]

Time: 03 Hours **Maximum Marks: 60**
Instruction for candidates:
1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:
a) Define Rolle's theorem. Explain with an example.
b) Define Beta function. Evaluate $B(1,\ 1)$.
c) Define a sequence and series with suitable examples. What are different tests for convergence of a series?.
d) Explain saddle points of a function. Find the same for the function $f(x, y) = x^2 - y^2$.
e) Define a curl of vector function. If the curl of a vector field is zero, what can be said about the function?
f) Evaluate $\int\int\int e^x y z^2 \, dV$ over the rectangular box $\{(x,\ y,\ z)\colon 0 \le x \le 1,\ 1 \le y \le 2,\ -\ 1 \le z \le 1\}$.
g) State Gauss's divergence theorem.
h) Define a differential equation mathematically. Explain the features necessary for the classification of a differential equation.
i) Define exact differential equation. Under what condition, the differential equation $xy^3\, dx + a\, x^2 y^2 dy = 0$ is exact?
j) Prove that e^{-x}, e^x and their linear combination are solutions of the differential equation $\frac{d^2y}{dx^2} - y = 0$.

Section – B **(5 marks each)**

Q2. What are indeterminate forms, and why do they require special techniques such as L'Hopital's rule to evaluate? List the common types of indeterminate forms.

Q3. Examine the convergence of the following sequences:
(i) $a_n = \left(\frac{3n+1}{3n-1}\right)^n$, (ii) $a_n = \frac{\left(\frac{10}{11}\right)^n}{\left(\frac{9}{10}\right)^n + \left(\frac{11}{12}\right)^n}$.

Q4. Prove that the function $f(x,\ y) = \{\frac{xy}{x^2+2y^2},\ (x,\ y) \neq (0,\ 0)\ 0,\ (x,\ y) = (0,\ 0)$ is not continuous at $(0,\ 0)$ but its partial derivatives f_x and f_y exist at $(0,\ 0)$.

Q5. Using Green's theorem, evaluate $\int\left(e^{-x}siny\,dx + e^{-x}cos\,y\,dy\right)$ along the rectangle with vertices $(0,\ 0)$, $(\pi,\ 0)$, $\left(\pi,\ \frac{\pi}{2}\right)$ and $\left(0,\ \frac{\pi}{2}\right)$.

Q6. Solve the differential equation: $\left(5\,x^3 + 12 + 6y^2\right)dx + 6\,xy\,dy = 0$.

Section – C **(10 marks each)**

Q7. (a) Find the surface area of the solid generated by revolving the circle: $x^2 + (y - 4)^2 = 4$ about the x-axis.

(b) Discuss the convergence or divergence of the series: $\left(\sqrt[3]{n^3 + 1} - n\right)$.

Q8. (a) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$.

(b) Find the volume of the ellipsoid: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

Q9. (a) What is the general form of a Clairaut's differential equation? Hence, solve $e^{4x}(p - 1) + e^{2y}p^2 = 0$.

(b) Find two linearly independent solutions of the differential equation: $2\,x^2\,y'' + x\,y' - \left(x^2 + 1\right)y = 0$ using the Frobenius method.