Problem A: Ion Thrusters To Saturn

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Team 227

Abstract

Electrical propulsions such as ion thrusters are becoming reality lately, being far more efficient in terms of carrying mass of payload than the typical chemical propulsion in space. They obviously have some limitations, as the thrust is low but continuous in nature, the time scales of missions are tens of years which imply those missions can not be manned. Previous missions have been successful in landing probes on asteroids with similar propulsion systems. Here we calculated the optimal path (in terms of fuel consumption) to Saturn with Ion Thruster as our propulsion system. Time was not considered to be the limiting factor. We assume that the path requiring a gravity assist from both Mars and Jupiter to reach Saturn will be the lowest in the consumption of fuel. Thus we concluded the optimal fuel required will be **2268.64 kg** and thus the total maximum payload can be **2731.36 kg**. The total duration of the mission will be **10.04 years**. The Ion Thruster will be used continuously from the given low earth orbit to reach Mars and then directly at the end to get on to the required stable orbit of Saturn.

Contents:

1. Introduction

In this paper, we formulate a project with an aim to send a spacecraft of mass 5000 kg (including fuel) from low Earth orbit to a stable circular orbit around Saturn **spending the least amount of fuel.** Spacecraft is initially orbiting around the Earth with a Time Period of 90 minutes. The goal is to be achieved by using Ion Thrusters which essentially work on the principle of creation of thrust by accelerating ions using electricity [1]. Given Ion thrusters provide a constant thrust of 400 millinewtons with a Specific Impulse of 4000 seconds to the spacecraft, because of this minimal amount of thrust compared to chemical engines, it requires a continuous operation for a long time to achieve the necessary change in velocity. This also results in longer duration of the space missions having ion thrusters as a propulsion mechanism. Therefore, we use the Gravity assist technique during flyby through Mars and Jupiter to boost the speed of spacecraft, essential for minimization of fuel as well as generating a practical time stretch for such a mission involving interplanetary travel. So, we divide the trajectory into four sub-trajectories each accounting for an inherently different type of journey viz. From Earth's Orbit to Earth's Sphere of Influence(SOI), From Earth's SOI to Mars's Orbit, From Mars's Orbit to Jupiter's Orbit, and From Jupiter's Orbit to circular orbit around Saturn. We analyze the path taken by the spacecraft during each of the trajectories and calculate the fuel consumption as well as the time taken in the corresponding flight path. Here, SOI is a shorthand for the spherical region around a celestial body where the gravitational Field by the body itself is large enough to neglect the forces due to other celestial objects.

In 1^{ST} Sub Trajectory, we turn on the ion thrusters at a particular time in the orbit of the earth which can be considered as the launch time. Since these thrusters are able to generate a minimal thrust but for a continuous period of time, we arrive at the result that the path taken by the spacecraft would be a spiral around Earth till the point we keep the thrusters turned on. This process is continued till the time Spacecraft is out of Earth's SOI and enters the SOI of the Sun. Now on we call the trajectory to be 2^{nd} which is inside SOI of the Sun and hence continues on a similar type of spiral but around the Sun, Note that until this point we are constantly providing energy to the spacecraft and hence its orbital radius in the Sun's frame will increase. Uninterrupting the process we now reach a point when the spacecraft's orbit will coincide with Mars's Orbit and this would be the first time we turn the power off and use the Gravity assist technique from Mars to boost the speed to a required value.
We name the path after slingshot through Mars as sub - trajectory 3^{rd}. Moreover, after the slingshot from Mars, we assume the path of the sub-trajectory to be a straight line (as it is a considerably small part of a big bounded trajectory around the sun) and the same goes for the path from the interplanetary region between Jupiter and Saturn wherein we again use the gravity assist of Jupiter to boost and orient the speed in the required direction (This is the 4^{th} trajectory).

Note that this model is completely based on the assumption that the orientation of the planets Mars, Jupiter and Saturn required for the trajectory to happen is generalized which means that it can be obtained by defining a corresponding launch time which will result in the least fuel spent. Alternatively, the parameters of gravity assist (assist angle as given in Eq.(5)) can also be altered to achieve a trajectory of a similar kind given some fixed initial condition i.e. orientation of analysed planets. Moreover the orbits of planets are considered to be almost circular and all the trajectories to be planer. Another assumption which is considerable enough to note is that the radius of SOI of planets is negligible as compared to the scale of interplanetary distances.

2. Notations Used

Symbol	Definition	Numerical Value
μ_E	Standard Gravitational Parameter For Earth	$3.98 \times 10^{14}\ m^3 s^{-2}$
μ_S	Standard Gravitational Parameter For Sun	$1.32 \times 10^{20}\ m^3 s^{-2}$
G	Gravitational Constant	$6.67 \times 10^{-11}\ Kg^{-1} m^3 s^{-2}$
$\mathbf{m_0}$	Initial Mass of Spacecraft	$5000\ Kg$
$\mathbf{M_S}$	Mass of Sun	$1.98 \times 10^{30}\ Kg$
$\mathbf{M_E}$	Mass of Earth	$5.97 \times 10^{24}\ Kg$
$\mathbf{M_{Mars}}$	Mass of Mars	$0.642 \times 10^{24}\ Kg$
$\mathbf{M_{Jupiter}}$	Mass of Jupiter	$1.898 \times 10^{27}\ Kg$
$\mathbf{M_{Saturn}}$	Mass of Saturn	$5.68 \times 10^{26} Kg$
c	Exhaust velocity of the ion thruster	$39.2\ Km/s$
τ	Time Constant	----
$\mathbf{R_{Mars}}$	Orbital Radius of Mars	$2.27 \times 10^8\ Km$
$\mathbf{R_{Jupiter}}$	Orbital Radius of Jupiter	$7.78 \times 10^8\ Km$
$\mathbf{R_{Saturn}}$	Orbital Radius of Saturn	$1.43 \times 10^9\ Km$
$\mathbf{R_{ESOI}}$	Radius of Sphere of Influence of Earth	$0.929 \times 10^6\ Km$
$\mathbf{R_{SatSOI}}$	Radius of Sphere of Influence of Saturn	$5.45 \times 10^7\ Km$
$\mathbf{m_1}$	Mass of Spacecraft at the end of SOI of Earth	$4175.27\ Kg$
$\mathbf{m_2}$	Mass of Spacecraft after gravity assist from Mars	$3615.14\ Kg$
$\mathbf{m_3}$	Mass of Spacecraft in required orbit of Saturn	$\mathbf{2731.36\ Kg}$

3. Essential Equations of Motion

3.1 The Tsiolkovsky Rocket Equation

We desire to find an expression of fuel spent to change the speed of a spacecraft by Δ*v.* Assuming ideal conditions, such that there are no external forces on the spacecraft we arrive at the result known as Tsiolkovsky Rocket Equation.[7]

Using the Principle of Conservation of Momentum (COM) on the spacecraft- fuel system,

Let the initial mass of the system be m_0, and the speed of the mass ejected by the spacecraft be u relative to the spacecraft. Considering v to be the speed of the spacecraft(probe) at a particular instant, it's momentum will be given as mv.

Furthermore, assuming that the exhaust is ejected at a constant rate, after a given time δt, the mass of the probe would become $m - \delta m$ and the speed will be $v + \delta v$. Now, the eject mass will be δm, and its speed would be given by $u - v$.

By COM,

$$mv = (m - \delta m)(v + \delta v) + (\delta m)(v - u)$$

Since $\delta t \to 0$, $\delta m \to dm$ & $\delta v \to dv$. and , we solve the following equation.

$$mv = mv + mdv + vdm - dmdv - vdm + udm$$

The term $dmdv$ can be neglected. So,

$$-mdv = udm$$

$$-\frac{dv}{u} = \frac{dm}{m}$$

Integrating with the limits, we get

$$\int_{v_0}^{v} -\left(\frac{1}{u}\right)\mathrm{d}v = \int_{m_0}^{m}\left(\frac{1}{m}\right)\mathrm{d}m$$

$$-\frac{v}{u} - \frac{v_0}{u} = \ln(m) - \ln(m_0)$$

$$dv = u \ln\left(\frac{m_0}{m}\right)$$

Specific Impulse (I_{sp})

We use the given I_{sp} to calculate some useful quantities: Mathematically,

$$I_{sp} = \frac{T}{g_0 \alpha}$$

Where, $\alpha = \frac{dm}{dt}$ and $I_{sp} = \frac{c}{g_0}$

Where c is the exhaust velocity of the propellant and g_0 is the acceleration due to gravity on earth. Now, Tsiolkovsky Rocket Equation modifies into,

$$dv = I_{sp}\, c \ln\left(\frac{M_{initial}}{M_{final}}\right)$$

We also define an additional quantity τ (time constant) as,

$$\tau = \frac{M_{initial}}{\alpha} \tag{1}$$

3.2 The Spiral Path

The nature of our transfer orbit will be "quasi-circular" with a tendency of the outward spiral. [3] We desire to find out the exact equation of our path given the gravitational influence and the nature of our propulsion. Let us begin with Gauss' variational equation for eccentricity.

$$\frac{de}{dt} = \frac{1}{v}\left[2(e+\cos\theta)a_t + \frac{r\sin\theta}{a}a_n\right]$$

Where a_t (along velocity vector) and a_n (perpendicular to the velocity vector) are the perturbing acceleration components from our ion thrusters. We assume that the thrust is always along the direction of velocity and as we depart from a circular orbit we get.

$$\frac{de}{dt} = \frac{2\cos\theta}{v} a_t$$

Now we calculate the change in eccentricity over a single orbital orbit. Also, note for a circular orbit, angular velocity is constant, therefore we get.

$$\frac{d\theta}{dt} = \frac{v}{r} \qquad\qquad \frac{de}{d\theta} = \frac{2r\cos\theta}{v^2} a_t$$

Which after integration gives:

$$de = \frac{2r}{v^2} a_t \int_0^{2\pi} \cos\theta d\theta = 0$$

Now consider the Gauss' vibrational equation for the semi-major axis:

$$\frac{da}{dt} = \frac{2a^2 v}{\mu} a_t$$

By similar assumption for angular velocity and initial circular orbit we get

$$da = \frac{2a^3}{\mu} a_t \int_0^{2\pi} d\theta = \frac{4\pi a^3}{\mu} a_t$$

With this, we calculate the time required to perform this maneuver. Also, note the circular velocity is $v = \sqrt{\mu/a}$ therefore

$$\frac{da}{dt} = \frac{2a^{3/2}}{\sqrt{\mu}} a_t$$

So we get the transfer time as

$$t_f = \tau\left[1 - \exp\left(\frac{-\Delta v}{c}\right)\right] \tag{2}$$

Now as $a_t = \dfrac{T}{m}$; and integrate ,we get:

$$\sqrt{\frac{\mu}{a_0}} - \sqrt{\frac{\mu}{a_f}} = c\ln\left(\frac{\tau}{\tau - t_f}\right)$$

Note that the left-hand side is the initial circular orbital velocity, $v_0 = \sqrt{\mu/a_0}$, minus the final (target) circular velocity, $v_f = \sqrt{\mu/a_f}$

And solving for the major axis we get.

$$\text{Outward spiral:}\quad a(t) = \frac{\mu}{\left[v_0 - c\ln\left(\frac{\tau}{\tau - t}\right)\right]^2} \tag{3}$$

$$\text{Inward spiral:}\quad a(t) = \frac{\mu}{\left[v_0 + c\ln\left(\frac{\tau}{\tau - t}\right)\right]^2} \tag{4}$$

3.3 The Gravitational Assist

We desire to propel the spacecraft into higher velocity using the gravitational assist from the planets on the way. This maneuver increases the velocity without use of fuel, thus reduces the total time of the mission efficiently.
In this process both momentum and the kinetic energy will be conserved.

$$MU_1^2 + mv_1^2 = MU_2^2 + mv_2^2$$
$$MU_1 - mv_1 = MU_2 - mv_2$$

Here M,m are the masses of the planet and the spacecraft respectively. U_1, U_2,v_1,v_2 are the initial and final velocity of the planet and spacecraft respectively.
Solving for v_2 we get.

$$v_2 = \frac{\left(1 - \frac{m}{M}\right)v_1 + 2U_1}{1 + \frac{m}{M}}$$

Note that M is very large. So, $v_2 = v_1 + 2U_1$

Now consider this maneuver in Sun' frame and Planet's frame.

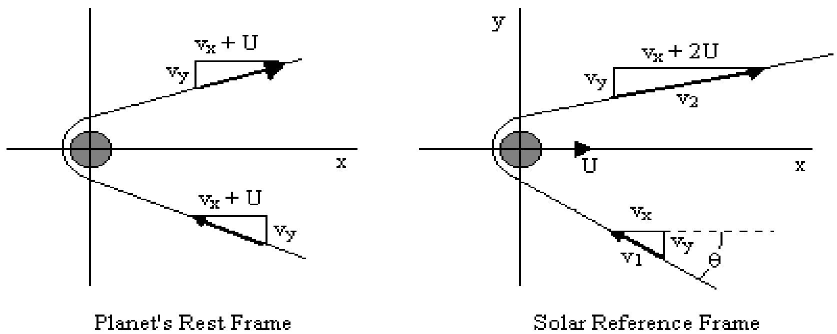

Figure. 0. Schematics of Gravity Assist

Similarly after solving for v_2 in both frames we get. [2]

$$v_2 = \left(v_1 + 2U\right)\sqrt{1 - \frac{4Uv_1(1 - \cos(\theta))}{\left(v_1 + 2U\right)^2}} \tag{5}$$

4. From Initial Orbit to the end of Earth's SOI

As ion thrusters produce a low thrust but continuous orbital maneuvers, the resultant path of our spacecraft will be an outward spiral. We consider the Earth's Sphere Of Influence(SOI) as a region outside of which spacecraft's path will be entirely decided by the Sun's gravitational influence and the Ion Thrusters will remain to be functioning until Mars.

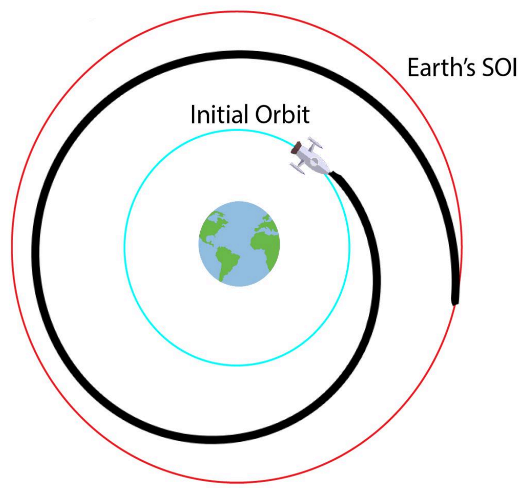

Figure 1. The Spiral Orbit of Spacecraft from Initial orbit to Earth's SOI
(Not to Scale)

We start by calculating the velocity of the spacecraft in the initial orbit around earth with time period **T = 90 mins.**
Using Kepler's Equation,

$$R_{orbit} = \left(\frac{\mu_E T^2}{4\pi^2}\right)^{\frac{1}{3}}$$

For T = 90 min = 5400 s

$$R_{orbit} = \left(3.98\times 10^{14}\times \frac{(5400)^2}{4\pi^2}\right)^{\frac{1}{3}} = 6649.21\ km$$

$$v_{orbit} = \sqrt{\frac{\mu_E}{R_{orbit}}}$$

$$v_{orbit} = \sqrt{\frac{\left(3.98\times 10^{14}\right)}{6649.21\times 10^{3}}} = 7.736\left(\frac{Km}{s}\right)$$

Using the standard value of the radius of influence for earth

$$R_{ESOI} = 0.909 \times 10^{9}\, m$$

$$v_{ESOI} = \sqrt{\frac{3.98\times 10^{14}}{0.909\times 10^{9}}} = 661.69\left(\frac{m}{s}\right)$$

The time required to complete this path can be obtained from Eq.(2) derived in Section 3.2.

$$T_1 = t_f = 80856749.6461\ seconds$$

$$T_1 = 1.43\ years$$

Now, we calculate the total fuel consumed during the flight path which is given by

$$m_{fuel1} = \alpha t$$

$$m_{fuel1} = 0.0102\times 10^{-3} \times 80856749.6461 = 824.73\ Kg$$

5. From Earth's SOI to Mars' Orbit

5.1 Interplanetary path

We continue on the spiral orbit just changing our frame of reference from Earth's frame to Sun's frame. After reaching Mars' SOI the ion thrusters are turned off. Further along, we only gain velocity from gravitational assist until reaching the destination. Thus the fuel is saved substantially.

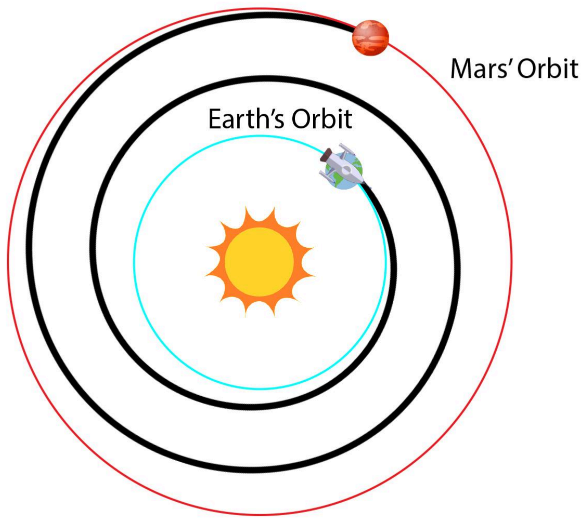

Figure 2. The Spiral Orbit of Spacecraft from Earth's SOI to Mars
(Not to Scale)

Note the mass of the spacecraft is reduced by 824.74 kg now.
Also note the velocity of the spacecraft after leaving Earth's SOI in Sun's frame is;

$$v_i = v_{Earth} + v_{ESOI}$$

And the final velocity would be determined by the resultant effect of the Sun's gravity and the thrusters . Therefore.

$$\Delta v = 30.29 + 0.66 - 24.85 = 5.64 \left(\frac{km}{s}\right)$$

Now we calculate time required to complete this path from Eq.(2) derived in Section 3.2.

$$T_2 = t_f = 54893569.34\ s$$

$$T_2 = 1.74\ years$$

Now, we calculate the total fuel consumed during the flight path which is given by

$$m_{fuel2} = \alpha t$$

$$m_{fuel2} = 0.0102 \cdot 10^{-3} \cdot 54893569.34 = 560.13\ kg$$

5.2 The Gravitational Assist from Mars

Let the velocity of the spacecraft entering and leaving Mars' SOI be v_i and v_{S1} respectively. See Fig. 3 We know $\mathbf{v_i}$ **= 24.8 km/s** and we found numerically $\mathbf{v_{S1}}$ **= 32 km/s** (see appendix) for our path to be optimal. The relative angle will be accordingly given the Eq.(5) in Section 3.3. This assumption does not change our fuel consumption but only affects the time of the mission.

6. From Mars's Orbit to Jupiter's orbit for Gravity Assist

6.1 The Interplanetary Path

As per our assumptions , we consider the path of the probe right after the gravity assist by Mars. Also according to the proposition to shut down the ion thrusters right before the gravitational slingshot of spacecraft by Mars, we can apply the principle of conservation of total mechanical energy of the spacecraft at any two positions between Mars and Jupiter's orbit in a frame of reference fixed at sun .

Furthermore, Gravitational effect of the nearby planets viz. Jupiter and Mars is considered to be negligible as compared to that of the sun in the interplanetary region .

Using the principle of conservation of energy at Two points A and B where A is the position of spacecraft just after gravitational slingshot from mars and B is the position of the spacecraft when it is nearly at the orbit of Jupiter around sun ($\mathbf{R_{Jupiter}}$).

Let v_{S1} be the velocity of the spacecraft just after the slingshot from Mars, we already calculated this velocity in the Section 6.2 and let the velocity of the spacecraft just before the slingshot from jupiter be v_{S2} so now let us apply energy conservation from point A to B.
Here we have assumed the Point A and B to be out of the sphere of influence of Mars and Jupiter.

$$\frac{1}{2}m_2 v_{S1}^{\ 2} - \frac{GM_S m_2}{R_{Mars}} = \frac{1}{2}m_2 v_{S2}^{\ 2} - \frac{GM_S m_2}{R_{Jupiter}}$$

Solving this equation for v_{S2} we get

$$v_{S2} = \sqrt{v_{S1}^{\ 2} + \frac{2GM_S}{R_{Jupiter}} - \frac{2GM_S}{R_{Mars}}}$$

Now using $v_{S1} = 32\ km/s$ we get,

$$v_{S2} = 11.48\ \frac{km}{s}$$

To calculate time we approximate our path into straight line originating tangentially to the velocity direction of Mars and calculate the tangential distance from Mars to Juptier so,
So the straight line distance D1 is our required distance as shown in Fig. 3.

$$t = 2 \cdot \frac{\left(\sqrt{R_{Jupiter}^{\ 2} - R_{Mars}^{\ 2}}\right)}{v_{S1} + v_{S2}}$$

$$T_3 = t = 1.09\ years$$

6.2 The Gravitational Assist from Jupiter

Let the velocity of the spacecraft entering and leaving Jupiter's SOI be v_{S2} and v_{S3} respectively. We found the values of this to be **11.48 km/s** and **16.39 km/s** respectively for our path to be optimal.(see appendix) The relative angle will be accordingly given by the Eq. (5) in Section 3.3. This assumption does not change our fuel consumption but only affects the time of mission.

Note v_{S3} is calculated in the next section.

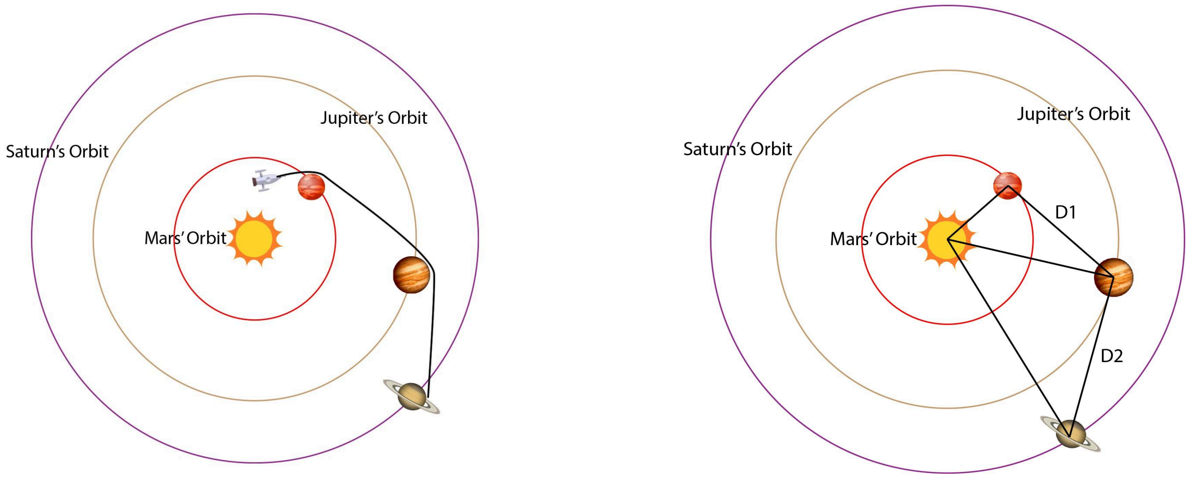

Figure 3. Path of spacecraft after leaving Mars through Gravity Assist (Left)
The Approximations of Distances Used to calculate Times T_3 and T_4
Between Interplanetary Travel (Right)
(Not to Scale)

7. From Jupiter's Orbit to Saturn's Orbit

Sustaining the assumptions of the previous section of space flight i.e from mars to saturn, in the subsequent path from Jupiter to just before Saturn's Sphere of Influence(SOI) we propose that the Energy of the Spacecraft in Sun's Frame would be conserved from any two point in between the journey after Gravitational slingshot from Jupiter to just before reaching the Saturn's SOI ($\mathbf{R_{SatSOI}}$) . We will again assume that the Gravitational Field of Sun will Dominate the path of the trajectory and other planetary forces are neglected.

Let the velocity of spacecraft just after the slingshot from jupiter be v_{S3} and now we want to put the spacecraft into an orbit around Saturn, which is just inside the Saturn's SOI.

So now we want to have the velocity of spacecraft to be equal to the orbital velocity w.r.t. Saturn at $\mathbf{R_{SatSOI}}$ when the spacecraft reaches the SOI of Saturn So let that velocity be v_{SatSOI}. Notice that this velocity v_{SatSOI} is w.r.t. Saturn.

We know that the velocity of an object orbiting saturn at a distance R is given below and thus v_{SatSOI} is given by the equation below.

$$v = \sqrt{\frac{GM_{Sat}}{R}} \Rightarrow v_{SatSOI} = \sqrt{\frac{GM_{Sat}}{R_{SatSOI}}} = 0.834\ km/sec$$

Now let's apply Energy Conservation from point C to point D where point C is the point where the spacecraft starts its journey from Jupiter's Orbit around the Sun after slingshot and D is the point on the orbit around Saturn at a distance of $\mathbf{R_{SatSOI}}$ from Saturn. The velocity of particle at point C is v_{S3} and at the point D is v_{SatSOI} (As we discussed earlier).

Here we have to assume that the spacecraft enters the orbit of Saturn at $\mathbf{R_{SatSOI}}$ tangentially.
Using this assumption we can write the velocity of spacecraft at point D w.r.t. Sun to be $v_{SatSOI} + v_{saturn}$. We already know that the velocity of saturn v_{saturn} is the orbital velocity of Saturn w.r.t. Sun. And it is given by

$$v_{saturn} = \sqrt{\frac{GM_S}{R_{Saturn}}} = 9.45\ km/sec$$

Therefore by applying conservation of Energy from point C to point D we get.

$$\frac{1}{2}m_2 {v_{S3}}^2 - \frac{GM_S m_2}{R_{Jupiter}} = \frac{1}{2}m_2\left(v_{SatSOI} + v_{Saturn}\right)^2 - \frac{GM_s m_2}{R_{Saturn}} - \frac{GM_{Saturn} m_2}{R_{SatSOI}}$$

From the above equation we can calculate v_{S3} to be

$$v_{S3} = \sqrt{\left(v_{SatSOI} + v_{Saturn}\right)^2 - \frac{2GM_s}{R_{Saturn}} - \frac{2GM_{Saturn}}{R_{SatSOI}} + \frac{2GM_S}{R_{Jupiter}}}$$

Now putting the values in the above equation we get

$$v_{S3} = 16.39\ km/sec$$

To calculate time we approximate our path into a straight line originating tangentially to the velocity direction of Jupiter and calculate the tangential distance from Juptier to Saturn.
So the straight line distance D2 is our required distance as shown in Fig. 3.

To calculate the time we calculate velocity of spacecraft in Sun's frame approaching Saturn, i.e $v_{Saturn} + v_{SatSOI}$ therefore the time is

$$t = 2 \cdot \frac{\left(\sqrt{{R_{Saturn}}^2 - {R_{Jupiter}}^2}\right)}{v_{S3} + v_{Saturn} + v_{SatSOI}} \qquad T_4 = t = 3.08\ years$$

8. From Saturn's SOI to Final Orbit

Now that we have our spacecraft inserted into an orbit of Saturn at $\mathbf{R}_{\mathbf{SatSOI}}$ distance from Saturn with a velocity v_{SatSOI} relative to Saturn. So now we are going to use the inward spiral as derived in Eq.(5) in Section 3.2 to change the orbit of spacecraft from $\mathbf{R}_{\mathbf{SatSOI}}$ to the final orbit with a time period of **40 hours** and orbital velocity w.r.t. Saturn to be **11.823 km/s.**(see appendix).

Using the Eq. (2) spiral with $v_i = v_{SatSOI}$ and $v_f = 11.8\ km/s$ we get the time to go from $\mathbf{R}_{\mathbf{SatSOI}}$ to final orbit of 40 hours to be T_5 which is

$$\Rightarrow \tau = \frac{m_2}{\alpha} = \frac{\left(m_0 - 824.73 - 560.13\right)}{\alpha} = 354283720\ sec$$

$$\Rightarrow T_5 = \frac{\tau}{a}\left(1 - e^{-\frac{\Delta v}{c}}\right)$$

$$\Rightarrow T_5 = 86610628.4201\ sec$$

$$\Rightarrow T_5 = 2.75\ years$$

Therefore the mass of fuel spent in putting the spacecraft into the final orbit of 40 hours time period is

$$\Rightarrow m_{fuel3} = \alpha T_5 = 883.78\ kg$$

9. Conclusion

In summary, the results of the previous sections indicate that by using gravity assist maneuvers multiple times we can minimize the fuel consumption in total. In order to enter Saturn's orbit of time period 40 hours from the initial conditions defined as ,orbiting Earth with time period 90 minutes, the spacecraft would consume the following amount of total fuel,

$$\Rightarrow M_{fuel} = m_{fuel1} + m_{fuel2} + m_{fuel3}$$

$$\Rightarrow M_{fuel} = 2268.64\ Kg$$

Which accounts to the efficiency of ,

$$\Rightarrow \eta = \frac{m_3}{m_0} \times 100$$

$$\Rightarrow \eta = 54.63\ \%$$

The Ion Thruster propulsion provides an efficiency of **54.63%** which when compared to the efficiency of chemical propulsion which is **5-8%,** performs substantially better in this regard.

We now look at the total time required for the path, which is given by following

$$\Rightarrow T_{total} = T_1 + T_2 + T_3 + T_4 + T_5$$

$$\Rightarrow T_{total} = 10.04\ years$$

Thus we can conclude that the massive efficiency for a sufficiently large interplanetary travel is somewhat compensated by the duration of Journey by using the ion propulsion system. Also , we can appreciate the boosts in speeds given by gravity assist maneuvers which ultimately contribute to both the fuel consumption as well as the duration of interplanetary travel.

Although , the results obtained are comprehensive enough under the assumptions made earlier but further research can be performed based on these concepts using much accurate astronomical data and relative positions of planets at a desired timeframe.We can also take into consideration the effects of the gravitational fields of other astronomical objects on the space probe while it is performing the gravity assist maneuver which is essentially removing the thin line of Sphere of Influence. This would increase the accuracy of the calculated fuel consumed.
Overall, the stated model indeed produces whopping efficiency with a practical timeframe and hence seems to be a promising idea to prospect the never ending space.

10. References

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[3] Craig A. Kluever *Space Flight Dynamics*(May 2018). retrieved from https://wodeshu.gitee.io/orbit/text00000.html

[4] Moran, Arom. Mann,John (May 1,2016) *Orbital Mechanics of Gravitational Slingshots* retrieved from http://symbolaris.com/course/fcps16/projects/amoran.pdf

[5] Sphere of Influence (8 November 2020). *Wikipedia* retrieved from https://en.wikipedia.org/wiki/Sphere_of_influence

[6] Planetary Fact Sheet - Metric https://nssdc.gsfc.nasa.gov/planetary/factsheet/

[7] Tsiolkovsky Rocket Equation(8 November 2020). *Wikipedia* retrieved from https://en.wikipedia.org/wiki/Tsiolkovsky_rocket_equation

[8] Some cliparts for making figures were taken from www.google.com/images

11. Appendix

Links to the Desmos graphs used for calculations are:

1. Part 1 - https://www.desmos.com/calculator/7gtsc8cp8n
2. Parr 2 - https://www.desmos.com/calculator/9zxo5kny4h
3. Part 3 - https://www.desmos.com/calculator/ntukjcstze
4. Part 4 - https://www.desmos.com/calculator/f5hooiiwr1