Assignment#2

MTH601 (FALL 2023)

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Question#1

Consider a company with four employees (E1, E2, E3, and E4) and four tasks (T1, T2, T3, and T4). The company wants to minimize the total cost of assigning each employee to a task based on their skill sets. The cost matrix is given below, where the entry in the ith row and j^{th} column represents the cost of assigning employee i to task j

	T_1	T_2	T_3	T_4
E_1	5	8	3	6
E_2	7	2	9	1
E_3	4	8	2	6
E_4	3	5	7	4

Formulate this problem as an assignment model and find the optimal assignment.

Solution:

The Matrix is given that:

	T_1	T_2	T_3	T_4
E_1	5	8	3	6
E_2	7	2	9	1
E_3	4	8	2	6
E_4	3	5	7	4

The matrix is in form of $n \times n$, where n is order of matrix and n=4.

Hence the matrix is balanced.

So, the following steps are followed to find optimal assignment solution.

Step-1

We subtract the minimum value in each row from all the elements in its row, we obtain:

As in 1st row minimum value is “3”in 2nd row minimum value is “1”,so on…..

	T_1	T_2	T_3	T_4
E_1	2	5	0	3
E_2	6	1	8	0
E_3	2	6	0	4
E_4	0	2	4	1

Step-2

We subtract the minimum value in each column from all the elements in its column, we obtain:

As in 1st column minimum value is “0”in 2nd column minimum value is “1”,so on…..

	T_1	T_2	T_3	T_4
E_1	2	4	0	3
E_2	6	0	8	0
E_3	2	5	0	4
E_4	0	1	4	1

Step-3

In this way make sure that in the matrix each row and each column has at-least one zero elements.

So, now draw minimum number of horizontal (n) and vertical lines (N) to cover all zero.

(a) $\quad If\ N = n, then\ the\ solution\ is\ optimal$

(b) $\quad if\ N < n, go\ to\ the\ next\ step.$

	T_1	T_2	T_3	T_4
E_1	2	4	0	3
E_2	6	0	8	0
E_3	2	5	0	4
E_4	0	1	4	1

So, here minimum number of lines (N) =3 which is less than n.

$N < n \qquad \therefore n = 4\ and\ 3 < 4$

Now from the step 3 the part (b) is held, so, we go to next step.

Step-4

Find the smallest element of uncovered element (x).

a. **Uncovered element –x**
b. **Intersection element +x**
c. **Lines elements as same**

So, here uncovered minimum value is "1". Hence x=1

	T_1	T_2	T_3	T_4

E_1	2	3	0	2
E_2	7	0	9	0
E_3	2	4	0	3
E_4	0	0	4	0

Step-4

Now again repeat the step-3, so we obtain

	T_1	T_2	T_3	T_4
E_1	2	3	0	2
E_2	7	0	9	0
E_3	2	4	0	3
E_4	0	0	4	0

Step-5

Now again repeat the step-3, so we obtain

So, here x=2

	T_1	T_2	T_3	T_4
E_1	0	1	0	0
E_2	7	0	11	0

E_3	0	2	0	1
E_4	0	0	6	0

Number of lines drawn to cover all the zero (N=4) and the order of matrix is $n \times n$ where n=4

Hence the matrix is optimal than we make an assignment.

	T_1	T_2	T_3	T_4
E_1	$\boxed{0}$	1	0 (×)	0 (×)
E_2	7	$\boxed{0}$	11	0 (×)
E_3	0 (×)	2	$\boxed{0}$	1 $\square$
E_4	0 (×)	0 (×)	6	0

Optimal assignment:

$E_1 \to T_1,\ E_2 \to T_2\ ,E_3 \to T_3,\ E_4 \to T_4$

$Minimum\ \cos t\ = 5+2+2+4=13$

	T_1	T_2	T_3 $\square$	T_4
E_1	0 (×)	1	0	0 (×)
E_2	7	$\boxed{0}$	11	0 (×)
E_3	$\boxed{0}$	2	0 (×)	1 $\square$
E_4	0 (×)	0 (×)	6	0

$E_1 \rightarrow T_3,\ E_2 \rightarrow T_2\ ,E_3 \rightarrow T_1, E_4 \rightarrow T_4$

$Minimum\ \cos t\ = 3+2+4+4=13$