Whiteboard: 1/60, f5.6, iso 1000, **WB Use Grey Card**, Manual Audio at level 11, focal length ½ way to ∞. H4n **INPUT @ 95%** gain.

BBB 1/60, f5.6, iso 320, **WB Use Grey Card**, Manual Audio at level 11, focal length ½ way to ∞. H4n **INPUT @ 95%** gain.

BBB Check Horizontal Level & Billy's Lights!! Middle line at bottom 1/3 of desk & remember Billy and Bo chair locations!

Title: Introductory Rotational Equilibrium Problem **[GoPro on ceiling & Sony close up]**

Domino's holding meterstick with 250g masses on it. Switch to force sensors at end.

Mr.p: Good morning. Bobby, please read the problem, and Bo, please translate.

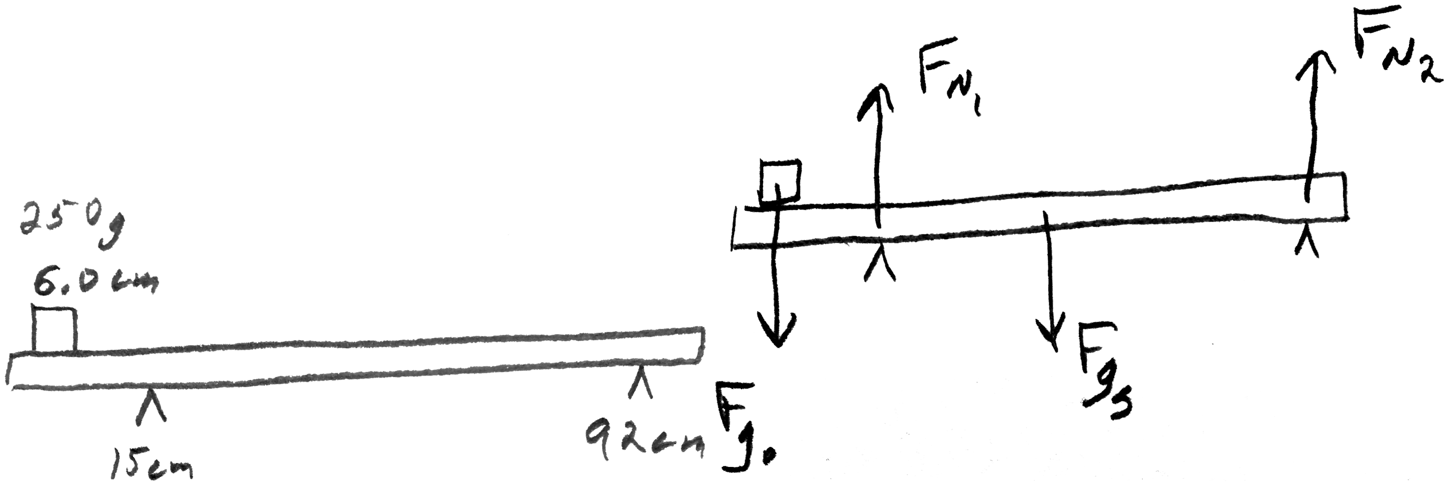
- Bobby: [intro] A uniform 0.093 kg meterstick is supported at the 15 cm and 92 cm marks. When a 0.250 kg object is placed at the 6.0 cm mark, what are the magnitudes of the forces supporting the meterstick?
- Bo: … The mass of the meterstick is 0.093 kg

and the mass of the object is 0.250 kg. … I suppose we should draw a picture of the meterstick supported at the 15 and 92 cm marks with the object on the meterstick at the 6.0 cm mark. I'm not sure what to do with those distances right now. … We are solving for the forces caused by the supports, which are both normal forces. So force normal 1 and 2 each equal question mark. … That's all I've got right now.

- Billy: We could draw a free body diagram of all the forces acting on the meterstick?
- Bo: Yeah. The force of gravity on the meterstick acts down at, well, the meterstick is "uniform" so

the force of gravity acts at the center of mass of the stick, so right in the middle at the 50 cm mark. The force of gravity on the object pushes down on the meterstick at the 6 cm mark. Let's say force normal 1 acts up on the support at the 15 cm mark and force normal 2 acts up on the support at the 92 cm mark.

Mr.p: [$m_s = 0.093kg; m_o = 0.250kg; F_{N_1} = ?; F_{N_2} = ?$] This is a good start. Billy, what should we do from here?



- Billy: Well, the net force in the y-direction on the meterstick equals the force of gravity acting on the object, which is negative because it is down, plus force normal number 1, which is positive because it is up, minus the force of gravity acting on the meterstick, minus because the force of gravity is down, plus force normal number 2, which is also positive because it is also up. The net force in the y-direction acting on the meterstick also equals the mass of the meterstick times the acceleration in the y-direction of the meterstick. However, the meterstick is at rest, so its acceleration in the y-direction equals zero. … Oh, it’s in *translational* equilibrium and we can substitute mass times acceleration due to gravity for both forces of gravity. … But we don’t know either force normal, so this is one equation with two unknowns, which we cannot currently solve.

- Bo: … Equation holster.
- Billy & Bobby: Equation holster!

Mr.p: [

$$\sum F_y = -F_{g_o} + F_{N_1} - F_{g_s} + F_{N_2} = ma_y = m(0) = 0 \Rightarrow -m_o g + F_{N_1} - m_s g + F_{N_2} = 0$$]

That is correct. We cannot currently do anything more with the net force equation, so we put it in our equation holster for when we need it later. Now, notice the meterstick is not moving, in fact, it is also not *rotating*. This means it has zero angular velocity and zero angular acceleration.

Billy: Oh, it is in *rotational* equilibrium. The meterstick is not only in *translational* equilibrium, it is also in *rotational* equilibrium!

Mr.p: That is correct Billy. Remember, this is a special case where the object is at rest and is in what we call *static* equilibrium. When an object is in *static* equilibrium, the net torque acting on the object equals zero about not just one axis of rotation, but *any* axis of rotation. Therefore, in this problem we can set the net torque equal to zero about any axis of rotation. Bobby, could you please do that?

Bobby: You want me to find the net torque about an axis of rotation? ... But the meterstick is not rotating?

Mr.p: Correct. Therefore, you can pick any axis of rotation and the net torque will always equal zero. Please, give it a try.

- Bobby: Okay. The net torque acting on the meterstick with an axis of rotation at … one end? I don't know, does it matter?

- Bo: … Put the axis of rotation at the location of one of the normal forces.
- Billy: Oh right, because then the torque caused by that force normal will be zero!
- Bobby: Okay. Let's pick force normal 1, or the 15 cm mark, as our axis of rotation. The net torque then equals the torque caused by the force of gravity on the object plus the torque caused by force normal one plus the torque caused by the force of gravity on the meterstick plus the torque caused by force normal 2.
- Billy: I thought the torque caused by force normal

number 1 was zero because torque equals r vector times force times the sine of the angle between the direction of “r” and the force. And because the axis of rotation is where force normal number 1 acts on the meter stick, the “r” value is zero which makes the torque caused by force normal number 1 equal to zero.

- Bo: And shouldn’t the torque caused by the force of gravity on the meterstick be negative because it would cause the meterstick to rotate clockwise or into the board, which is negative?
- Bobby: Right. But the torque caused by the force of gravity on the object and the torque caused by

force normal 2 are both positive because they would cause the meterstick to rotate counterclockwise or out of the board which is positive. … All of that equals rotational inertia times angular acceleration, but the angular acceleration of the meterstick is zero, so the net torque acting on the meterstick with the axis of rotation at force normal 2 equals zero.

Mr.p: [Add AoR to picture.

$$\sum \tau_{\substack{AoR\,@\\F_{N_1}}} = \tau_o + \tau_1 - \tau_s + \tau_2 = I\alpha = I\left(0\right) = 0$$

Cross off τ_1 w/

$\tau_1 = r_1 F_{N_1} \sin\theta_1 \Rightarrow r_1 = 0 \Rightarrow \tau_1 = 0$] Bo, please keep going from there.

Bo: Now, for each torque, we can substitute in the equation for torque Billy just mentioned. … The angle in each of those torque equations is 90 degrees because each “r”, the distance from the axis of rotation to where the force is applied, is horizontal and each force is vertical. The sine of 90 degrees equals one. … So now we have r for the object times the mass of the object times acceleration due to gravity minus r for the stick times the mass of the stick times the acceleration due to gravity plus r for normal force 2 times normal force 2 all equal to zero. Move everything but r 2 times force normal 2 over to the other side and divide by r 2 to solve for normal force 2. … Now we need the distances or "r" values for the stick, the object, and normal force 2.

Mr.p: [

$$\Rightarrow r_o F_{g_o} \sin\theta_o - r_s F_{gs} \sin\theta_s + r_2 F_{N_2} \sin\theta_2 = 0 \;\&\; \theta_o = \theta_s = \theta_2 = 90^\circ \;\&\; \sin\left(90^\circ\right) = 1$$

$$\Rightarrow r_o m_o g - r_s m_s g + r_2 F_{N_2} = 0 \Rightarrow r_2 F_{N_2} = r_s m_s g - r_o m_o g \Rightarrow F_{N_2} = \frac{r_s m_s g - r_o m_o g}{r_2}$$

]

Billy, please determine those three torque "r" values.

- Billy: Well, 15 cm equals 6 cm plus r for the object, therefore r for the object equals 15 minus 6 or 9 cm. … 15 cm plus r for the meterstick equals 50 cm, therefore r for the meterstick equals 50 minus 15 or 35 cm. … 15 cm plus r for force normal number 2 equals 92 cm, therefore r for force normal number 2 equals 92 minus 15 or 77 cm. … We can now substitute all our values in to the equation for which Bo just solved. Force normal number 2 equals the quantity 25 times 0.093 times 9.81 minus 9 times 0.25 times 9.81 all divided by 77 which equals {calc} 0.12804 or 0.13 with two significant digits. And the units are … Newtons. Dang it. We need to convert all the r values from cm to meters because newtons are kilogram *meters* per second squared. So multiply all r values by 1 meter over 100 cm and substitute all those new r values back in to the equation and we get {calc} hey, we get the same answer. That's weird.

- Bobby: … Oh, that is because, if you substitute all the units into the equation, you can see the centimeters cancel out and you end up with kilogram meters per second squared. So we did not actually have to convert to meters. That is weird.

Mr.p: [$15 = 6 + r_o \Rightarrow r_o = 15 - 6 = 9cm \times \frac{1m}{100cm} = 0.09m$ &

$15 + r_s = 50 \Rightarrow r_s = 50 - 15 = 35cm \times \frac{1m}{100cm} = 0.35m$ &

$15 + r_2 = 92 \Rightarrow r_2 = 92 - 15 = 77cm \times \frac{1m}{100cm} = 0.77m$

$$F_{N_2} = \frac{r_s m_s g - r_o m_o g}{r_2} = \frac{(0.35)(0.093)(9.81) - (0.09)(.25)(9.81)}{0.77} = 0.12804 \approx 0.13N$$
] It is a good habit to convert your values to base SI units, however, you are correct that we actually did not need to convert the r values to meters in this particular problem. … We have solved for force normal number 2, anybody know how we can now solve for force normal number 1? {hear Billy} That is correct Billy. We can use the equation we previously placed in our equation holster. {hear Billy} Sure, go ahead.

Billy: (Oh, we use the equation holster! …May I do it?) We can solve the equation for force normal number 1 and substitute in known values to get, force normal number 1 equals 0.25 times 9.81 minus 0.12804 plus 0.093 times 9.81 which equals {calc} 3.23679 or 3.2 newtons with 2 significant digits!

Mr.p: [$-m_o g + F_{N_1} - m_s g + F_{N_2} = 0 \Rightarrow F_{N_1} = +m_o g - F_{N_2} + m_s g$

$\Rightarrow F_{N_1} = (0.25)(9.81) - 0.12804 + (0.093)(9.81) = 3.23679 \approx 3.2N$] Correct.

Now realize we could also have solved for force normal number 1 by summing the torques and picking a different axis of rotation. This is because the meterstick is at rest and therefore in static equilibrium, which means the net torque acting on the meterstick about any axis of rotation equals zero. … But, you know we have to check our predictions, right? [Switch to force sensors] I have replaced the meterstick supports with force sensors and you can see the force sensor readings currently read zero. Adding the meterstick … and then then 250 grams of mass gives values for the two normal forces which, I think you would agree, means… The physics works, the physics works, uh huh, uh huh, the physics works! ... Thank you very much for learning with me today, I enjoyed learning with you.