Just as another friendly reminder,
"guided notes" like these are always yours to keep.
Never turn them into me.

Guided notes — Game theory

Today: How traditional game theory actually works and our first two "equilibrium" concepts

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Recall the simple, *simultaneous* (not sequential), *one-shot* (not repeated) penalty kick game introduced last time and the "**payoff matrix**" we used to represent the two *players* in that game, their two *strategies*, and their *payoffs* for each possible outcome.

PAYOFF MATRIX

KICKER \ GOALIE	JUMP TO KICKER'S LEFT	JUMP TO KICKER'S RIGHT
KICK LEFT	0, 1	1, 0
KICK RIGHT	1, 0	0, 1

We can use payoff matrices for games with *more than two players*, but we won't worry about that yet (because that makes the matrices more complicated).

Adding *more than two strategies for each player* doesn't involve any additional complication ->

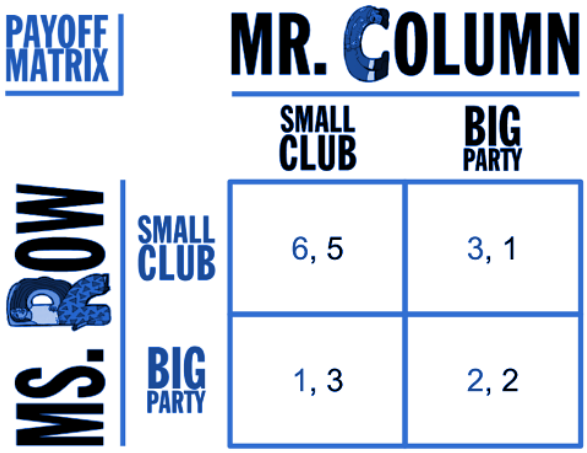
PAYOFF MATRIX

KICKER \ GOALIE	JUMP TO KICKER'S LEFT	CENTER	RIGHT
KICK TO LEFT	,	,	,
CENTER	,	,	,
RIGHT	,	,	,

What should happen in this game, according to game theory? Or, to put it another way: What's the "**equilibrium**" of this game? As we'll learn, there are *many* equilibrium concepts in game theory. (By the end of the semester, we'll have learned about "dominant-strategy" equilibriums … "iterated dominant-strategy" or "iterated-dominance" equilibriums … "pure-strategy Nash" equilibriums … "subgame perfect Nash" equilibriums … "mixed-strategy Nash" equilibriums … "evolutionarily stable" equilibriums … and "Bayesian Nash" equilibriums.) The first equilibrium concept we'll learn is a **"dominant-strategy" equilibrium**.

The penalty kick game doesn't have a "dominant-strategy" equilibrium because neither player has a "dominant" strategy, so let's use a different game to introduce and explain those terms.

Consider a simultaneous, one-shot game in which two people are each going out for the night. Each person can go to either a small club or a big party. The "payoff matrix" to the right shows those players, their possible strategies, and the payoffs we'll assume (in utils of utility, let's say).



For the in-class activity we'll do, we'll see that each player has a **"dominant" strategy** (i.e., a strategy that is the best response, regardless of the strategy chosen by the other player) and therefore this game has a **"dominant-strategy" equilibrium** (i.e., an equilibrium in which each player has a dominant strategy and chooses it).

To find any "iterated-dominance" (or, equivalently, "iterated dominant-strategy") equilibriums, we use our **"iterated deletion of dominated strategies" method**. As that name suggests, we go through player's strategies and successively/iteratively delete any "dominated" strategies.[1] If we're left with a unique combination of players' strategies, then the game is "dominance solvable," and what we've found is the game's **"iterated-dominance" equilibrium**.

Note that the "iterated-dominance" equilibrium concept, in particular, as well as most of the other equilibrium concepts we'll learn, relies on assuming that:

(1.) each player is "rational" (i.e., they know the options available to them, they have consistent preferences over those options, and they choose their most-preferred option)

(2.) each player knows the other players are rational (i.e., "common knowledge of rationality"); and

(3.) each player knows the other players' payoffs (or at least their best responses to any strategy).

We'll implicitly assume we're making those assumptions whenever we talk about these equilibrium concepts so we don't have to explicitly make those assumptions every time, but those seem like big assumptions(!), and we'll eventually try to relax some of the rationality and/or informational assumptions.

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[1] What's a "dominated" (as opposed to "dominant") strategy, you may ask? Well, thank goodness you have those glossary handouts you're bringing to class until you're fluent in game theory!

Freakonomics star defines "game theory"

The following is a transcript of a Freakonomics podcast by Stephen Dubner and Steven Levitt.

DUBNER: Alright, Levitt, define "game theory" for me.

LEVITT: I would define "game theory" as the study of the strategic interactions between a small number of adversaries, usually two or three competitors.

DUBNER: So that sounds pretty simple, doesn't it? Levitt has written several papers that involve game theory—mostly papers about sports and gambling and cheating, things like that. So how does it actually work? Well, here it gets a bit more complicated…

LEVITT: Yeah, so game theory…The promise of game theory…Or…Or…So one of the predictions of game theory is that when you are in, oh well let's see, OK, wait a minute, wait let me say again. I would describe game theory as a mathematical formalization…So, OK let me start over…

DUBNER: Yeah, actually, you know what, wait…

LEVITT: Let me talk about my frustration with game theory, and then I'll go back and say that I actually have written papers [where] game theory does apply.

DUBNER: Yep, yep.

LEVITT: So my applications of game theory, and there are a handful of them, have essentially all been to sports. Really my…Sorry. So, so my…So my uh…Uh, let me say it again…Now, there are very particular predictions that theory make about how [baseball] pitchers should [mix]…There are very particular predictions about how pitchers should [mix] their pitches. Let me start over. Let me, let me talk, let me just like think differently about it.

DUBNER: OK.

LEVITT: So when a pitcher sometimes throws fastballs, and sometimes throws curveballs, it must be the case that, in the end, the pitcher must be indifferent between whether the guy…God, you know, game theory sucks so bad because it's so hard.

DUBNER: [Laughs]

LEVITT: I mean it's really…I mean because everything is backwards in game theory. I don't even think it's worth talking about, because like **the predictions they're just, they're just impossible to describe without going into what equilibria is.**

Another glossary Game theory

As previously mentioned in another similar handout, game theory has its own language that you'll become fluent in by the end of this course. Here are some more key terms we'll cover. Continue to bring these handouts to class until you feel you've mastered all these terms.

Term	Definition
Player's dominant strategy	A player's "dominant" strategy is the strategy that is the best response, regardless of the strategies of the other players
Strongly (or strictly) vs. weakly dominant strategies	A player's strategy is a "strongly" (or "strictly") dominant strategy if its payoff is strictly greater than the payoff from any other strategy, regardless of the strategies chosen by the other players. The player's strategy is only a "weakly" dominant strategy if: (1.) its payoff is greater than or equal to the payoff from any other strategy, regardless of the strategies chosen by the other players, and (2.) its payoff is strictly greater than the payoff from any other strategy for at least one strategy that the other players could choose
Game's dominant-strategy equilibrium	An equilibrium in which each player has a dominant strategy and chooses it. As with all other concepts of equilibrium we'll discuss, such an equilibrium may or may not exist, depending on the game
Strongly vs. weakly dominant-strategy equilibriums	For a game with a dominant-strategy equilibrium, that equilibrium is said to be the "strongly dominant-strategy" equilibrium if the players all have strongly dominant strategies, whereas the equilibrium is said to be a "weakly dominant-strategy" equilibrium if at least one player only has a weakly dominant strategy
Dominant vs. dominated strategies	In contrast to a dominant strategy defined above, a player's strategy is "dominated" if the player has other strategies whose payoffs are greater than (if strongly dominated) or at least greater than or equal to (if weakly dominated) that strategy's payoff, regardless of what the other player does.
Game's iterated dominant-strategy (or iterated-dominance) equilibrium	An equilibrium found by deleting strongly or weakly dominated strategies until only one pair of strategies remains. Depending on whether the deleted strategies are all strongly dominated or some of them are only weakly dominated, the equilibrium is said to be either an iterated "strongly" or "weakly" dominant-strategy equilibrium. Some games may have more than one iterated weakly dominant-strategy equilibrium. And again, as with all other concepts of equilibrium we'll discuss, such an equilibrium may or may not exist, depending on the game
Pareto superiority or dominance or efficiency	An outcome is "Pareto superior" or "Pareto dominant" or "Pareto efficient" relative to another outcome if at least one player can be made better off without making any other players worse off. This welfare criterion, which is popular with economists, is named after Italian economist Vilfredo Pareto