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B.Sc. Hons. Math (Semester – 6th)

MATHEMATICAL MODELLING

Subject Code: BMAT1-620

Paper ID: [18131228]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a. Give an example where mathematical modelling is used.

b. Use the linear congruence relation: $x_{n+1} = (ax_n + b) \bmod (c)$ to generate 5 random numbers using a = 5, b = 1, and c = 8.

c. Find the inverse Laplace transform of

$$F(s) = \frac{1}{s-3} - \frac{16}{s^2-9}$$

d. Define "Degeneracy" in linear programming problems.

e. What are the limitations of the mathematical modelling?

f. Define the convex set with an example.

g. Define generating function.

h. Write down the expressions of Legendre's and Chebyshev polynomials.

i. When an optimization problem is said to be constrained or unconstrained?

j. Let $V = 4x^2yz^3$ at a given point P (1, 2, 1), then find the potential V at P and also verify whether the potential V satisfies the Laplace equation.

Section – B **(5 marks each)**

Q2. Find the area trapped between the two curves $y = x^2$ and $y = 6 - x$ and the x and y axes.

Q3. Use the middle square method to generate

a. 10 random number using $x_0 = 1009$.

b. 20 random number using $x_0 = 653217$.

Q4. Find the solution using laplace transform method $y'' - 3y' + 2y = 4t + e^{3t}$ when $y(0) = 1$ and $y'(0) = -1$

Q5. Derive the Malthusian model of population growth.

Q6. Find the D'Alembert solution to the homogeneous wave equation.

Section – C **(10 marks each)**

Q7. Use simplex method to minimize $z = x_2 - 3x_3 + 2x_5$

Subject to

$3x_2 - x_3 + 2x_5 \leq 7,$

$-2x_2 + 4x_3 \leq 12,$

$-4x_2 + 3x_3 + 8x_5 \leq 10,$

$x_2 \geq 0,\ x_3 \geq 0,\ x_5 \geq 0,$

Q8. Consider a small harbor with unloading facilities for ships. Only one ship can be unloaded at any one time. Ships arrive for unloading of cargo at the harbor, and the time between the arrival of successive ships varies from 15 to 145 min. The unloading time required for a ship depends on the type and amount of cargo and varies from 45 to 90 min. Answers to the following questions:

(i) What are the average and maximum times per ship in the harbor?

(ii) What is the length of the longest queue?

Q9. Define Bessel's function $J_n(x)$ and prove the following recurrence relation

$$\frac{d}{dx}\left[x^n J_n(x)\right] = x^n J_{n-1}(x)$$

$$or$$

$$\int x^n J_{n-1}(x)dx = x^n J_n(x)$$